Abstract
As the struggle against low inflation intensifies, renewed attention is focusing on the potential instability of the relationship between labor market demand pressure and inflation. A weaker Phillips curve has mainly been documented for the United States. Since it is unlikely that this phenomenon is limited to a single country, more international evidence is required. We analyse changes in the slope of the Phillips curve in eleven OECD countries (including the United States for comparison). Bayesian VAR models with time varying parameters indicate that relationship between inflation and unemployment has strengthened rather than weakened. Shocks to unemployment typically have significant effects on inflation in 2018, indicating that the Phillips curve is still alive and well. The statistical method may matter for the results as rolling window estimation shows a weakened relationship in six out of ten non-US countries.

Keywords: Inflation, Phillips Curve, Bayesian time-varying parameter VARs
JEL classifications: F41, E31
1 Introduction

Inflation remained stagnant through the recent period of falling unemployment, which lead policy makers and researchers alike to question whether the effects of labor market pressure on wages and prices have fallen in recent years. Given the apparent lack of responsiveness of inflation to changes in unemployment from the Great Recession and onward, several observers have even announced the death of the Phillips curve (Hall 2013, Krugman 2015, Cochrane 2019). If the relationship between unemployment and inflation has weakened, it is important to document this since it affects the ability of central banks to achieve their inflation targets.

A vast majority of the empirical studies of the stability of the Phillips curve focus on U.S. data. If the inflationary effects of labour demand pressure have fallen, this phenomenon is unlikely to be restricted to a single country. This paper contributes to the compilation of international evidence by investigating changes in the slope of the Phillips curve in eleven OECD countries. We apply models with explicitly time varying parameters as well as rolling window estimation.

Previous studies of recent changes in the slope of the Phillips curve in other countries than the United States provide a mixed picture. We argue that the choice of empirical method matters for the results as studies using models with time varying parameters have rejected the hypothesis that the Phillips curve has flattened more often than studies relying on rolling window estimation (or structural breaks). We apply both methods to identical data in order to investigate whether the results differ between the two approaches. Rolling regressions indicate a weakening of the Phillips curve in six of the ten non-US countries, compared to only two countries when time varying
parameters are used. Furthermore, the Phillips curve is alive and well in 2018 eight of ten countries in 2018 according to the model with time varying parameters. Relying instead on rolling window estimation, the hypothesis that shocks to labor demand pressure as measured by unemployment have significant effects on inflation in 2018 is rejected twice as often.

Among recent studies of the stability of the Phillips curve in other countries than the United States, Oinonen and Paloviita (2014) and Riggi and Vendetti (2015) conclude that the slope of the Euro zone Phillips curve has increased rather than decreased since the financial crisis. Both papers employ models with time varying parameters. France, Italy, and Spain are included in three recent studies, two of which find a steeper Phillips curve (Bonam et al., 2018, Bulligan and Viviano, 2017). Again, the empirical approaches rely on explicitly time varying parameters. On the other hand, Bell and Blanchflower (2018) as well as Rusticelli (2014) document a flattening of the UK Phillips curve. These studies use rolling window estimation to capture changes over time. In Section 5 we categorize recent studies of the stability of the Phillips curve according to empirical methods and results. There appears to be differences between the findings depending on which statistical method that is used in previous studies as well as for our data.

The scope of this paper is limited to documenting changes in the reduced form Phillips curve in other developed countries than the United States. A weaker reduced form relationship between unemployment and inflation may for instance be observed because supply shocks have become more important or because that the central bank has successfully stabilized inflation at target. We do not attempt to distinguish between different explanations for changes in the slope of the Phillips curve.

As argued by Fuhrer (2013) and others, it is important to control for
changes in monetary policy regimes by including data on expected inflation when studying changes in the slope of the Phillips curve. Consistent data on expected inflation is only available since 1990 and for six of the countries. Estimation of expectation augmented Phillips curves for this smaller sample produces similar results as the baseline specification with only inflation and unemployment. Because models with time varying parameters may display long memory if the parameters are assumed to follow random walk processes, it is interesting that the two methods still produce different results when earlier decades with potentially strong reduced form relationships between inflation and unemployment are excluded. Hence, the finding that the slope of the Phillips curve remains stable according to the models with time varying parameters is not due to lingering effects from a strong Phillips curve relationship in e.g. the 1960s.

The remainder of the paper is organized as follows. Section 2 presents the statistical methods and Section 3 describes the data. Section 4 contains the empirical results, while Section 5 is devoted to analyzing whether empirical methods and findings are related in previous studies of changes in the slope of the Phillips curve. Section 6 concludes.

2 Statistical models

The most common procedure when estimating the Phillips curve is to regress current period inflation on current period unemployment or output gap. However, the maximum effect of changes in demand pressure typically occurs with a lag, which is only captured if lagged variables are included in the empirical specification. The dynamics may differ over time and between countries. Furthermore, reversed causality may be a problem since
unexpected changes in inflation affects unemployment in many theoretical models. We have therefore chosen to focus on the impulse response functions from VAR models that allows for flexible dynamics and causality in both directions.

We estimate Bayesian VAR models with time-varying parameters and stochastic volatility as well as standard frequentist VARs with twelve year rolling window. The length of the window is chosen so that the last observation only covers the period since the onset of the Great Recession in 2007. The main output from this procedure consists of a large number of impulse response functions (IRFs). For each country and time period, the maximum effect of unemployment shocks on inflation is collected and analyzed. In addition, we show the IRFs in 2005 and 2018 in order to investigate whether there are significant effects of unemployment on inflation. If the effect is smaller in 2018 than in 2005 we conclude that the Phillips curve has flattened.

2.1 A VAR with time-varying parameters

The empirical specification follows Primiceri (2005) and the subsequent application of his model by Arratibel and Michaelis (2014). Following the notation of Primiceri, a multivariate model with $k$ lags is set up as:

$$ y_t = c_t + B_{1t}y_{t-1} + B_{2t}y_{t-2} + \ldots + B_{kt}y_{t-k} + u_t, \quad t = 1, \ldots, T \quad (1) $$

where $y_t$ is a $n \times 1$ vector of endogenous variables, $c_t$ is a vector of time-varying constants, and $B_{it}, i = 1, \ldots, k$ are $n \times n$ matrices containing time-varying coefficients for all lags. The error terms $u_t$ are heteroscedastic with the variance covariance matrix $\Omega$, which can be factored as:
\[ \Omega_t = A_t^{-1} H_t \left( A_t^{-1} \right)' , \quad H_t = \Sigma_t \Sigma_t' \] (2)

where \( \Sigma_t \) is a time-varying matrix with error variances on the diagonal and \( A_t \) is a time-varying lower triangular matrix with ones on the diagonal that contains the simultaneous relationships between the variables:

\[ A_t = \begin{bmatrix}
1 & 0 & \ldots & 0 \\
0 & 1 & \ldots & \ldots \\
\ldots & \ldots & \ldots & 0 \\
0 & \ldots & \ldots & 1
\end{bmatrix} \quad (3) \]

\[ \Sigma_t = \begin{bmatrix}
\sigma_{1,t} & 0 & \ldots & 0 \\
0 & \sigma_{2,t} & \ldots & \ldots \\
\ldots & \ldots & \ldots & 0 \\
0 & \ldots & 0 & \sigma_{n,t}
\end{bmatrix} \quad (4) \]

Equations (1) and (2) imply that the VAR can be written as

\[ y_t = c_t + B_1 t y_{t-1} + B_2 t y_{t-2} + \ldots + B_{kt} t y_{t-k} + A_t^{-1} \Sigma_t \varepsilon_t , \] (5)

where the variance-covariance matrix of \( \varepsilon_t \) is diagonal: \( V(\varepsilon_t) = I_n \). The time-varying VAR can then be expressed as

\[ y_t = X_t B_t + A_t^{-1} \Sigma_t \varepsilon_t , \] (6)

\[ X_t' = I_n \otimes [1, y_{t-1}', \ldots, y_{t-k}'] , \] (7)

where \( \otimes \) denotes the Kronecker product and \( B_t \) is a vector containing all right hand side coefficients of equation (5): \( B_t = vec (c_t, B_{1t}, \ldots, B_{kt}) \). Equation (6) summarizes the processes for the time-varying coefficients. The
time-varying coefficients \((B_t)\) and simultaneous relationships \((A_t)\) are assumed to evolve according to driftless random walk procedures. The covariance matrix containing the standard deviations is assumed to evolve according to a geometric driftless random walk.

Denoting \(a_t\) as a vector of non-zero, non-one elements in \(A_t\) (stacked by rows), and \(\sigma_t\) as the vector containing diagonal elements of \(\Sigma_t\), the state equations of the model can be expressed as:

\[
B_t = B_{t-1} + \nu_t \tag{8}
\]
\[
a_t = a_{t-1} + \xi_t \tag{9}
\]
\[
\log \sigma_t = \log \sigma_{t-1} + \eta_t \tag{10}
\]

The innovations of the model follow a jointly normal distribution and have a joint variance covariance matrix \(V\).

\[
\begin{bmatrix}
\varepsilon_t \\
\nu_t \\
\xi_t \\
\eta_t
\end{bmatrix} \sim N(0, V), \quad V = \begin{bmatrix}
I_n & 0 & 0 & 0 \\
0 & Q & 0 & 0 \\
0 & 0 & S & 0 \\
0 & 0 & 0 & W
\end{bmatrix} \tag{11}
\]

The diagonal of \(V\) contains the hyperparameters of the model, which describe the tightness of the parameter distributions. \(I_n\) is the variance matrix of \(\varepsilon_t\) from equation (5). \(Q, S\) and \(W\) contain the variances of the innovations from equations (8) to (10) and are defined as positive definite. \(S\) is assumed to be block diagonal, which implies that the simultaneous relationships between the variables can be estimated independently equation per equation.
2.2 Priors

Choices of priors follow earlier literature, mainly Primiceri (2005), Cogley and Sargent (2005) and Arratibel and Michaelis (2014). The prior distributions of the initial states are estimated by OLS estimation of a fixed coefficient VAR on a training sample covering the first five years. The point estimates of $B_0$ and $A_0$ from the OLS regressions are set as means of the distributions for initial states of these parameters respectively ($\hat{B}_{OLS}$ and $\hat{A}_{OLS}$). Conventionally, the variances are set to four times the variance of the OLS point estimates. The mean of the prior distribution for log $\sigma$ is set to the logarithm of the OLS point estimate ($\hat{\sigma}_{OLS}$), while the covariance matrix is set as an identity matrix $I_n$.

The hyperparameters $Q$, $S$, and $W$ adjust the variances of the state equation innovations, i.e. the tightness of the parameter distributions. We follow Cogley and Sargent (2005) and Arratibel and Michaelis (2014) and define $Q$ and $S$ as inverse Wishart distributions, and $W$ as an inverse-Gamma distribution. Also in line with these authors, we restrict $W$ to be diagonal. This partly departs from Primiceri, who similarly defines $W$ as a scaled identity matrix, but sets all hyperparameters as belonging to the inverse Wishart distribution family. Given the limited sample size in this study, we choose the same approach as Arratibel and Michaelis (2014), who motivate their choice mainly as a means to reduce dimensionality of the model. The priors can be summarized as follows:
\[
B_0 \sim N \left( \hat{B}_{OLS}, \ 4\text{Var} \left( \hat{B}_{OLS} \right) \right) \\
A_0 \sim N \left( \hat{A}_{OLS}, \ 4\text{Var} \left( \hat{A}_{OLS} \right) \right) \\
\log \sigma_0 \sim N \left( \hat{\sigma}_{OLS}, \ 4I_n \right) \\
Q \sim IW \left( k_Q^2 \tau \text{Var} \left( \hat{B}_{OLS} \right), \tau \right) \\
W \sim IG \left( (1 + \text{dim} (W)) I_n, (1 + \text{dim} (W)) \right) \\
S_b \sim IW \left( k_S^2 (1 + \text{dim} (S_b)) \text{Var} \left( \hat{S}_{bOLS} \right), (1 + \text{dim} (S_b)) \right)
\]

(12)

\( S_b \) denotes the different blocks of \( S \), \( \tau \) equals the number of observations in the training sample, and \( \hat{A}_{bOLS} \) refers to the respective blocks of the OLS estimate of simultaneous relationships (\( \hat{A}_{OLS} \)). The priors for initial states of the coefficients, simultaneous relations and standard deviations are assumed to be normally distributed. Together with the evolution of coefficients in (8) to (10), this implies that the sequence of parameters will be normally distributed.

The scaling matrices in \( Q \), \( W \) and \( S_b \) are constant fractions of their OLS estimates, where the OLS estimates are multiplied by corresponding degrees of freedom and scaling factors \( k_Q \), \( k_W \), and \( k_S \). Degrees of freedom in each scaling matrix is set to size of training sample for \( Q \) and to one plus the dimension of the corresponding matrix for \( W \) and \( S_b \). These numbers correspond to the minimum levels required to have proper priors, implying that the resulting priors only impose weak restrictions on the posterior. Given the short period of the training sample, and hence the possibility of imprecise OLS estimates, it seems reasonable to avoid an approach that puts large weight on the priors. The small sample size of the training sample also motivates setting the parameters \( k_S = 0.1 \) and \( k_W = 0.01 \) quite conservatively, i.e. in a manner that keeps priors rather diffuse. Primiceri (2005) uses the same calibration and calls the priors “not flat but diffuse and unin-
formative”. However, as shown by Reusens and Croux (2015), the standard priors for $k_Q$ (the degree of time variation) implies that the parameters vary little over time. Overall, our choice of priors is in line with earlier literature (Cogley and Sargent, 2001, Stock and Watson 1996).

### 2.3 Estimation and identification

Denoting the history of variable $x$ from time 1 to $T$ as $x^T = [x'_1, ..., x'_T]'$ and similarly the simultaneous relations and stochastic volatility are contained in $B^T$, $A^T$, and $\sigma^T$. Bayesian methods are used to estimate these unobservable states and the hyperparameters of the joint covariance matrix $V$, i.e. the joint posterior distribution of $(B^T, A^T, \sigma^T, V)$. Letting $\theta$ denote a vector of the unknown parameters $(\theta = (B^T, A^T, \sigma^T, V))$ and representing the data as $Y^T = (y'_1, ..., y'_T)'$, prior information about the distribution of these parameters is $\pi(\theta)$, while the likelihood function is $f(Y^T|\theta)$. The posterior distribution $\pi(\theta|Y^T)$ is deduced by updating prior information about the parameters with information given by data.

With stochastic volatility the state space model is non linear, which implies that the posterior moments do not have closed forms. Therefore, numerical methods are required to evaluate the likelihood function and deduce the posterior distribution of the parameters. We use Gibbs sampling for this purpose. The sampling procedure is carried out in four steps, beginning with draws of the coefficient states, continuing with draws of covariance and volatility states and finally the hyperparameters. Further details on the sampling procedure can be found in Carter and Kohn (1994).

To approximate the posterior distributions, 10 000 iterations of the Gibbs sampler is run, with an initial burn-in period of 30 000 observations to assure
convergence to the ergodic distribution. To break autocorrelation of the
draws, only every 10th draw is considered.

Shocks are identified through the lower-triangular matrix $A_t$. By con-
struction, such a set-up imposes a recursive structure on the contempor-
aneous effects with the ordering going from the top to the bottom variables.
The ordering of variables is $[u_t, \pi_t]$. Hence, shocks to unemployment has zero
contemporaneous effects on inflation, while shocks to inflation are allowed to
affect unemployment within a period. When expected inflation is included,
the other shocks have zero contemporaneous effects on this predetermined
variable.

3 Data

Few empirical economic relationships appear in as many different forms as
the Phillips curve. We focus on two versions, using standard data on inflation
and unemployment in the baseline specification and then adding data on
expected inflation. Quarterly data on CPI and unemployment is collected
from the OECD database Main Economic Indicators.\footnote{In the cases where the available timer series are longer for harmonized unemployment
than for total unemployment, the former is used. This concerns Italy, Spain, Sweden, and
the United Kingdom.} While there are other
measures of inflation that are theoretically more appealing in a Phillips
curve setting, consistent data for a reasonably wide selection of countries
and sample periods are available only for CPI. Similarly, we use total rather
than short-term unemployment since consistent data across countries are
only available for the most recent period in case of the latter measure. The
choice of countries is dictated by the availability of data at least as far back
as 1985.

Proxies for inflation expectations are typically country specific. The proxy for expected inflation available for the widest selection of countries and longest time period is the Consensus Economics survey of professional forecasts for CPI inflation. This data set starts in 1989 for the following countries: Canada, France, Germany, Japan, Netherlands, United Kingdom, and United States. Specifically, our proxy for expected inflation is the mean of the survey forecasts of CPI inflation for period $t$ collected in period $t-4$. Forecasts two years ahead are available, but only for an even more limited sample.

4 Empirical results

The main specification is a bivariate Bayesian VAR model with time varying parameters estimated using long time series on inflation and unemployment. Previous studies of the stability of the Phillips curve typically use rolling window regressions. In order to facilitate comparison of the results across studies and also between statistical methods, we also estimate frequentist VAR models using a 12-year window. Previous studies have typically used a 15-year window. We have chosen a slightly shorter time span to ensure that the last observation only captures the period since the Great Recession, 2007-2018, when the question of the potential weakening of the Phillips curve is most relevant.

The mechanisms behind the determination of inflation may differ between monetary policy regimes, in particular between periods before and after inflation targeting was introduced. Time varying models are well suited to take this into consideration since not only the dynamics but also the in-
intercept is allowed to change over time. In addition, models with expected inflation are estimated for the countries where data are available. Time varying intercepts also virtually eliminate the potential problem of non-stationarity in the data as impulse response functions converge.\footnote{Convergence is absent or questionable for seven of the 68 impulse response functions in question.} We have nevertheless included data on expected inflation as a robustness check.

Given two data sets and two statistical methods, we estimate four specifications: Models with time varying parameters and rolling window regressions using the longest time series on unemployment and inflation available in the OECD database Main Economic Indicators for all eleven countries, and expectations augmented Phillips curves since 1990 for the six countries where Consensus data on expected inflation is available.

### 4.1 Baseline specification

Time varying two-variable VAR models with only unemployment and inflation are estimated for eleven OECD countries including the United States. The TVP-VAR yields one set of impulse response functions (IRFs) for each time period and country, as does the rolling window estimation. These IRFs are shown in Appendix A. Figures 1a to 1d illustrate the output from the models in the case of Germany. There are four impulse response functions: for the observations 2018Q2 and 2005Q1, estimated using time varying parameters and a twelve year rolling window. The point estimate of the maximum effect is smaller in 2018 than in 2005 according to the rolling window, while the opposite is found using time varying parameters. Hence, the Phillips curve has flattened according to the rolling window, but not according to the TVP model. Furthermore, the effect in 2018 is significant
using the TVP-VAR, but not according to the rolling window (figure 1b). For comparison, we also run standard simple OLS regressions of inflation on contemporaneous unemployment (see the final column of Table 2). According to this approach, the German Phillips curve was dead in 2005 as well as in 2018 as the slope coefficients are insignificant in both periods.

Given that we estimate a total of 44 impulse response functions, the results have to be summarized in a comprehensible way. Tables 1 to 3 present three different aspects of the IRFs: The maximum effects of shocks to unemployment on inflation in 2005 and 2018 (Table 1), qualitative indications of whether the effect is significant in 2005(2018) according to the two methods and whether the maximum effect has increased or decreased between these years (Table 2), and finally averages of the time series of maximum effects.
over the entire sample period for the two methods, as well as the correlation between these time series (Table 3).

Table 1 contains the maximum effect of shocks to unemployment on inflation according to the two methods in 2005Q1 and 2018Q2 (the last observation in the sample). This is typically a negative number since it is the minimum of the corresponding impulse response function. As shown in the final row, the average across countries excluding the United States is 0.87 in 2005 and 0.94 in 2018 when the model with time varying parameters is used, compared to 0.19 in 2005 and 0.14 in 2018 according to the rolling window estimation. The TVP VAR hence indicates much larger effects than the rolling window. Furthermore, the average effect has increased between 2005 and 2018 according to the TVP VAR, but fallen according to the rolling window. Changes in the maximum effect of shocks to unemployment on inflation are analyzed further below. In Section 3 we show that systematic differences in the results depending on the choice of method may be detected also in previous research. In case of the United States, the effects of shocks to unemployment on inflation are smaller and have fallen in recent years according to both approaches, hence confirming the standard finding.

Table 2 summarizes several qualitative aspects of the impulse response functions in 2005 and 2018. The first two column show whether the effects of shocks to unemployment on inflation are significant (+) or insignificant (−) in 2005 and 2018 according to the two methods. All countries except the United Kingdom display significant effects in 2005 according to the TVP model, while the rolling window indicates insignificant effects in five of the ten non-US countries. In 2018, the TVP model shows insignificant effects also for New Zealand, while insignificant results are now found for four of the ten countries using the rolling window. Models with explicitly time varying
Table 1: Maximum effect of unemployment on inflation

<table>
<thead>
<tr>
<th>Country</th>
<th>TVP\textsuperscript{a}</th>
<th>Rolling\textsuperscript{b}</th>
</tr>
</thead>
<tbody>
<tr>
<td>Year</td>
<td>2005 2018</td>
<td>2005 2018</td>
</tr>
<tr>
<td>Australia</td>
<td>-1.262 -1.441</td>
<td>-0.504 -0.198</td>
</tr>
<tr>
<td>Canada</td>
<td>-0.501 -0.536</td>
<td>-0.152 -0.202</td>
</tr>
<tr>
<td>France</td>
<td>-1.251 -1.034</td>
<td>-0.074 -0.233</td>
</tr>
<tr>
<td>Germany</td>
<td>-0.480 -0.647</td>
<td>-0.333 -0.070</td>
</tr>
<tr>
<td>Italy</td>
<td>-0.524 -0.667</td>
<td>-0.033 -0.042</td>
</tr>
<tr>
<td>Japan</td>
<td>-0.859 -1.065</td>
<td>-0.144 -0.089</td>
</tr>
<tr>
<td>New Zealand</td>
<td>-0.434 -0.278</td>
<td>-0.299 -0.075</td>
</tr>
<tr>
<td>Spain</td>
<td>-0.590 -0.593</td>
<td>-0.122 -0.173</td>
</tr>
<tr>
<td>Sweden</td>
<td>-2.575 -2.783</td>
<td>-0.220 -0.311</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>-0.388 -0.331</td>
<td>-0.029 -0.021</td>
</tr>
<tr>
<td>United States</td>
<td>-0.508 -0.259</td>
<td>-0.204 -0.032</td>
</tr>
<tr>
<td>Average non-US</td>
<td>-0.886 -0.938</td>
<td>-0.191 -0.141</td>
</tr>
</tbody>
</table>

\textsuperscript{a}TVP denotes the Bayesian VAR model with time varying parameters. The columns contain the minimum of the impulse response functions in 2005 and 2018.

\textsuperscript{b}Rolling denotes rolling estimation of frequentist VAR models with a 12 year window. The columns contain the minimum of the impulse response functions in 2005 and 2018.

parameters apparently reject the null hypothesis that the effect of shocks to unemployment on inflation is zero more often that the rolling window.

The main conclusion from this column is however that Phillips curve as defined here is however alive and well except in the United Kingdom and New Zealand.

The third column in Table 2 shows whether the slope of the Phillips curve (defined as the maximum effect of shocks to unemployment on inflation) has increased (+) or decreased (−) between 2005 and 2018. The models with explicitly time varying parameters indicate that the Phillips curve has flattened only in three of the ten non-US countries (France, New Zealand, and the United Kingdom). In contrast, estimation using rolling windows results in a flatter Phillips curve in six of the ten non-US cases. Hence the model with time varying parameters is less prone to show a flatter Phillips curve.
curve than the more commonly used rolling window. Although we obviously do not know which empirical approach that is more accurate, it is clear that the stylized fact of a flatter Phillips curve in recent years is generally rejected for our data set and the Bayesian VAR model with the time varying parameters.

Turning to the results for specific countries, both methods indicate a steepening of the Phillips curve in Canada, Italy, and Sweden, while the effect of shocks to unemployment on inflation has decreased between 2005 and 2018 in New Zealand and the United Kingdom. In the cases of Australia, Germany, Japan, and Spain, the TVP model produces a steeper Phillips curve, while the rolling window indicates a flattening. In addition, the results for U.S. data confirm the standard finding of a weaker relationship.

Time varying models produce time series of the slope of the Phillips curve as defined here. Figures 2a and 2b show the two time series from the
Table 2: Qualitative summary of the findings

<table>
<thead>
<tr>
<th>Country</th>
<th>2005a</th>
<th>2018b</th>
<th>Changec</th>
<th>Regressiond</th>
</tr>
</thead>
<tbody>
<tr>
<td>Australia</td>
<td>++</td>
<td>++</td>
<td>+/−</td>
<td>−/+</td>
</tr>
<tr>
<td>Canada</td>
<td>++</td>
<td>++</td>
<td>+/+</td>
<td>++</td>
</tr>
<tr>
<td>France</td>
<td>+/−</td>
<td>+/+</td>
<td>−/+</td>
<td>+/+</td>
</tr>
<tr>
<td>Germany</td>
<td>+/+</td>
<td>+/−</td>
<td>+/−</td>
<td>−/−</td>
</tr>
<tr>
<td>Italy</td>
<td>+/−</td>
<td>+/+</td>
<td>+/+</td>
<td>−/+</td>
</tr>
<tr>
<td>Japan</td>
<td>+/−</td>
<td>+/−</td>
<td>+/−</td>
<td>−/−</td>
</tr>
<tr>
<td>New Zealand</td>
<td>+/+</td>
<td>−/−</td>
<td>−/−</td>
<td>+/−</td>
</tr>
<tr>
<td>Spain</td>
<td>+/−</td>
<td>+/+</td>
<td>+/−</td>
<td>−/+</td>
</tr>
<tr>
<td>Sweden</td>
<td>+/+</td>
<td>+/+</td>
<td>+/+</td>
<td>−/+</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>−/−</td>
<td>−/−</td>
<td>−/−</td>
<td>−/−</td>
</tr>
<tr>
<td>United States</td>
<td>+/+</td>
<td>−/−</td>
<td>−/−</td>
<td>−/−</td>
</tr>
</tbody>
</table>

a2005: +/− denotes that the effect of shocks to unemployment on inflation in 2005 is significant/insignificant according to the TVP model/rolling window estimation.
b2018: +/− denotes that the effect of shocks to unemployment on inflation in 2018 is significant/insignificant according to the TVP model/rolling window estimation.
cChange +/− denotes an increase/decrease in the maximum effect between 2005 and 2018 according to the TVP model/rolling window estimation.
dRegression: +/− denotes that the effect of shocks to unemployment on inflation in 2005/2018 is significant/insignificant according to a standard OLS regression of contemporaneous unemployment on inflation.

TVP model and the rolling window for Germany and France (see Appendix A1 for the remaining graphs). First, the TVP model appears to yield larger and more variable effects of shocks to unemployment on inflation than the rolling window. Second, it is not obvious that these two times series capture similar movements over time in the slope of the Phillips curve. Table 3 shows descriptive statistics of these maximum effects as estimated by the two approaches and confirms that the effects from the TVP model are indeed larger in every case, but only slightly more variable on average.

Finally, column 3 in Table 3 shows the correlations between the slope of the Phillips curve estimated using the TVP model and the rolling window. The average correlation is only 0.094, indicating that the two statistical
approaches captures movements that unrelated.

Table 3: Descriptive statistics: Maximum effects

<table>
<thead>
<tr>
<th>Country</th>
<th>mean TVP(rolling)(^a)</th>
<th>std TVP(rolling)(^b)</th>
<th>corr(^c)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Australia</td>
<td>-1.166 (−0.616)</td>
<td>0.204 (0.307)</td>
<td>-0.708</td>
</tr>
<tr>
<td>Canada</td>
<td>-0.609 (−0.385)</td>
<td>0.103 (0.184)</td>
<td>0.342</td>
</tr>
<tr>
<td>France</td>
<td>-0.900 (−0.232)</td>
<td>0.395 (0.160)</td>
<td>0.326</td>
</tr>
<tr>
<td>Germany</td>
<td>-0.476 (−0.247)</td>
<td>0.317 (0.109)</td>
<td>-0.218</td>
</tr>
<tr>
<td>Italy</td>
<td>-0.583 (−0.217)</td>
<td>0.132 (0.105)</td>
<td>-0.172</td>
</tr>
<tr>
<td>Japan</td>
<td>-1.557 (−0.399)</td>
<td>0.490 (0.287)</td>
<td>0.744</td>
</tr>
<tr>
<td>New Zealand</td>
<td>-0.366 (−0.296)</td>
<td>0.068 (0.129)</td>
<td>0.440</td>
</tr>
<tr>
<td>Spain</td>
<td>-0.638 (−0.213)</td>
<td>0.086 (0.071)</td>
<td>0.344</td>
</tr>
<tr>
<td>Sweden</td>
<td>-2.326 (−0.395)</td>
<td>0.375 (0.281)</td>
<td>-0.158</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>-0.260 (−0.099)</td>
<td>0.046 (0.107)</td>
<td>-0.630</td>
</tr>
<tr>
<td>United States</td>
<td>-0.443 (−0.349)</td>
<td>0.192 (0.335)</td>
<td>0.724</td>
</tr>
<tr>
<td>Average</td>
<td>-0.848 (−0.313)</td>
<td>0.219 (0.189)</td>
<td>0.094</td>
</tr>
</tbody>
</table>

\(^a\) mean TVP(rolling) is the average maximum effect as estimated using the TVP model (rolling window estimation).

\(^b\) std TVP(rolling) is the standard deviation of the maximum effect as estimated using the TVP model (rolling window estimation).

\(^c\) corr the correlation between the maximum effects as estimated using the TVP model and the rolling window. Bold numbers denotes significance at the 5 percent level.

Using the longest available sample of (quarterly) data on only inflation and unemployment, we find more evidence of a stronger Phillips curve than of a weakened relationship.

### 4.2 Expectation augmented Phillips curves

As noted by Fuhrer (2013) and others, including a measure of inflation expectations in the Phillips curve may improve estimation of changes in the slope of the Phillips curve. The mean inflation conditional on a given unemployment rate is affected by the expected inflation rate. In this section, survey data on expected inflation is added to the baseline specification. Because of data limitations, the sample period is shortened to 1990Q1 to
2018Q2 and the set of countries is reduced to five plus the United States.

Tables 4 shows the maximum effect of shocks to unemployment on inflation in 2005 and 2018, estimated using time varying parameters and a rolling window. Compared to the baseline specifications without expected inflation, the effects are generally smaller for this data set. This is especially true for the TVP model, where the average slope of the Phillips curve is reduced by more than 50 percent.

Turning to the two time periods, the average effect is now smaller in 2018 than in 2005 for the TVP models, while the opposite is found for the rolling window. These averages are however not completely comparable across specifications since the set of included countries differ. The models with time varying parameters still indicate larger effects of unemployment on inflation both in 2005 and in 2018 than the rolling window estimations.

Because the parameters in the TVP model are assumed to be random walks and previous shocks therefore have permanent effects, the suspicion arises that large effects of unemployment shocks on inflation in e.g. the 1980s may still affect the estimates for 2018 when a long sample period is used. This type of long memory is not possible when for the rolling windows since the estimates for 2018 are based solely on the last twelve years of the sample. The shorter sample period used in this section prevents strong Phillips curve effects up to 1990 from affecting the results for 2005 and 2018 using the TVP model. The estimated slope of the Phillips curve is indeed smaller in every single case except the United States in 2018 when earlier decades are excluded from the sample.

Table 5 contains qualitative findings, among all whether there are significant effects in 2005 and 2018 and whether the Phillips curve has flattened between 2005 and 2018. First, it is clear that adding expected inflation to
Table 4: Effects in 2005 and 2018 with expected inflation

<table>
<thead>
<tr>
<th>Country</th>
<th>TVP^a</th>
<th>Rolling^b</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2005</td>
<td>2018</td>
</tr>
<tr>
<td>Canada</td>
<td>-0.211</td>
<td>-0.249</td>
</tr>
<tr>
<td>France</td>
<td>-0.559</td>
<td>-0.556</td>
</tr>
<tr>
<td>Germany</td>
<td>-0.497</td>
<td>0.017</td>
</tr>
<tr>
<td>Japan</td>
<td>-0.596</td>
<td>-0.651</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>-0.144</td>
<td>-0.148</td>
</tr>
<tr>
<td>United States</td>
<td>-0.483</td>
<td>-0.407</td>
</tr>
<tr>
<td>Average non-US</td>
<td>-0.401</td>
<td>-0.317</td>
</tr>
</tbody>
</table>


the models has minor effects on the qualitative finding. Significant effects of shocks to unemployment on inflation are found for virtually the same countries and methods in 2005 with and without expected inflation. For 2018, significance is now lost for Germany and the United States in 2018 using the TVP model and Germany and the United Kingdom using the rolling window. Hence, the TVP model still appears to be more prone to indicate a significant relationship in 2018 than the rolling window, and overall there is slightly less evidence of a significant Phillips curve in 2018 for this shorter sample.

Concerning the change between 2005 and 2018, the maximum effect has still increased in three non-US countries and fallen in two according to the TVP model. The rolling window however indicates a steeper Phillips curve in four out of five cases, compared to only two out of five without expected inflation.

The final column of Table 5 shows whether there are significant effects
of unemployment on inflation in an OLS regression with these contemporaneous variables, a constant, and expected inflation, in 2005 and 2018. First, the Phillips curve as defined here is alive in two of the five non-US countries. Second, the qualitative findings are reasonably similar between this simple, standard test and two methods used in this paper. Third, Japan is the country where adding data on expected inflation has the largest effects on the results.

Table 5: Qualitative summary of the findings with expected inflation

<table>
<thead>
<tr>
<th>Country</th>
<th>2005(^a)</th>
<th>2018(^b)</th>
<th>Change(^c)</th>
<th>Regression(^d)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Canada</td>
<td>+/+</td>
<td>+/+</td>
<td>+/+</td>
<td>+/+</td>
</tr>
<tr>
<td>France</td>
<td>+/+</td>
<td>+/+</td>
<td>-/+</td>
<td>+/+</td>
</tr>
<tr>
<td>Germany</td>
<td>+/+</td>
<td>-/-</td>
<td>-/-</td>
<td>-/-</td>
</tr>
<tr>
<td>Japan</td>
<td>+/-</td>
<td>+/-</td>
<td>+/+</td>
<td>+/-</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>-/-</td>
<td>-/-</td>
<td>+/+</td>
<td>-/-</td>
</tr>
<tr>
<td>United States</td>
<td>+/+</td>
<td>+/-</td>
<td>-/-</td>
<td>-/-</td>
</tr>
</tbody>
</table>

\(^a\)2005 +/- denotes that the effect of shocks to unemployment on inflation in 2005 is significant/insignificant according to the TVP model/rolling window estimation.

\(^b\)2018:+/- denotes that the effect of shocks to unemployment on inflation in 2018 is significant/insignificant according to the TVP model/rolling window estimation.

\(^c\)Change +/- denotes an increase/decrease in the maximum effect between 2005 and 2018 according to the TVP model/rolling window estimation.

\(^d\)Regression +/- denotes that the effect of shocks to unemployment on inflation in 2005/2018 is significant/insignificant according to a standard OLS regression of contemporaneous unemployment on inflation.

Finally we show descriptive statistics for the time series on maximum effects as estimated using the TVP model and the rolling window. As in the baseline specification, the two approaches capture movements in the slope of the Phillips curve as the measures are uncorrelated (the only significant correlation coefficient is negative). The model with time varying parameters still indicates larger and more volatile effects of unemployment on inflation than the rolling window estimation on average across the sample.

When comparing Tables 4 to 6 with the equivalent Tables in Section 4.1,
Table 6: Descriptive statistics: Maximum effects with expected inflation

<table>
<thead>
<tr>
<th>Country</th>
<th>mean TVP(rolling)(^a)</th>
<th>std TVP(rolling)(^b)</th>
<th>corr(^c)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Canada</td>
<td>-0.240 (-0.173)</td>
<td>0.057 (0.075)</td>
<td>0.098</td>
</tr>
<tr>
<td>France</td>
<td>-0.567 (-0.168)</td>
<td>0.011 (0.051)</td>
<td>-0.127</td>
</tr>
<tr>
<td>Germany</td>
<td>-0.150 (-0.177)</td>
<td>0.184 (0.071)</td>
<td>0.058</td>
</tr>
<tr>
<td>Japan</td>
<td>-0.731 (-0.212)</td>
<td>0.129 (0.090)</td>
<td>-0.049</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>-0.128 (-0.057)</td>
<td>0.028 (0.100)</td>
<td>0.234</td>
</tr>
<tr>
<td>United States</td>
<td>-0.500 (-0.204)</td>
<td>0.165 (0.085)</td>
<td><strong>0.711</strong></td>
</tr>
<tr>
<td>Average non-US</td>
<td>-0.386 (-0.165)</td>
<td>0.096 (0.079)</td>
<td>-0.083</td>
</tr>
</tbody>
</table>

\(^a\) mean TVP(rolling) is the average maximum effect as estimated using the TVP model (rolling window estimation).

\(^b\) std TVP(rolling) is the standard deviation of the maximum effect as estimated using the TVP model (rolling window estimation).

\(^c\) corr is the correlation between the maximum effects as estimated using the TVP model and the rolling window. Bold numbers denotes significance at the 5 percent level.

it is clear that incorporating data on expected inflation and shortening the sample to the period 1990–2018 only has minor effects on the main results given this empirical framework. The Phillips curve has flattened or even disappeared in recent year the United States, but there is less evidence of this phenomenon in other countries.

5 Results vs. empirical method in previous studies

Models with explicitly time varying parameters and rolling window estimation are the two most common statistical methods in studies of changes in the slope of the Phillips curve over time. For our data sets, the findings differ systematically between these two approaches. Relying on rolling window regressions appears to increase the likelihood of concluding that the Phillips curve has flattened. This observation prompts the question whether the
results in previous studies also differ depending on the choices of methods. In lieu of resources to actually replicate the research in question, we settle for sorting results versus statistical methods in recent studies of changes in the slope of the Phillips curve and check whether the finding of a flatter Phillips curve is more common in studies that employ rolling windows. The set of studies consists of all investigations of time variation in or stability of the Phillips curve identified by Google Scholar since 2011. Furthermore, only studies incorporating data up to 2007 or later, other countries than the United States, and using either models with explicitly time varying parameters or rolling window estimation are shown in Table 7. These two methods cover a vast majority of the existing studies. There are however also a few papers that apply other statistical approaches (such as Markov switching models).

Seven studies use rolling window estimation to investigate whether the slope of the Phillips curve has changed over time. Only one of these papers finds that the Phillips curve has steepened. A total of 18 cases are investigated, only one of which displays an increase in the slope of the Phillips curve (Canada in the study by Chletsos et al., 2016).

Eight studies focus on models with explicitly time varying parameters. Two of these find a flattening of the Phillips curve and another two document different results for different countries. 15 cases are investigated, ten of which show increases in the slope of the Phillips curve. Hence, a tendency towards differentiated results depending on statistical method is discernible also in previous studies.

Given the diversity of the results in Table 7 it is difficult to draw conclusions regarding changes in the slope of the Phillips curve in individual countries. Germany is the only country for which the findings are most
consistent as both studies that include German data reveal a weakening of the Phillips curve. Furthermore, these two papers employ models with time varying parameters, which is the method that appears to be more prone to indicate a flattening of the relationship. Our results mainly confirm this conclusion as the slope of German Phillips curve has fallen in three of our four specifications. The overall picture in Table 7 remains diverse across

<table>
<thead>
<tr>
<th>Studies using a rolling window</th>
<th>Included countries and findings</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bell and Blanchflower, 2018</td>
<td>UK (-)</td>
</tr>
<tr>
<td>Chletos et al., 2016</td>
<td>Canada (+)</td>
</tr>
<tr>
<td>Chowdhury and Sarkar, 2017</td>
<td>Brazil, India(-)</td>
</tr>
<tr>
<td></td>
<td>South Africa and Russia, no P.C.</td>
</tr>
<tr>
<td>Jasova et al., 2018</td>
<td>Panel of 22 advanced economies (-)</td>
</tr>
<tr>
<td>Muto and Shintani, 2014</td>
<td>Japan (-)</td>
</tr>
<tr>
<td>Riggi and Santoro, 2015</td>
<td>Italy (-)</td>
</tr>
<tr>
<td>Rusticelli, 2014</td>
<td>Canada, Germany, Greece,Ireland (-)</td>
</tr>
<tr>
<td></td>
<td>Japan, Portugal, Spain, UK (-)</td>
</tr>
<tr>
<td></td>
<td>France, Italy (0)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Studies using TVP</th>
<th>Included countries and findings</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bonam et al., 2018</td>
<td>France, Italy, and Spain (+)</td>
</tr>
<tr>
<td></td>
<td>Holland (0)</td>
</tr>
<tr>
<td></td>
<td>Germany (-)</td>
</tr>
<tr>
<td>Bulligan and Viviano, 2017</td>
<td>France, Italy, and Spain (+)</td>
</tr>
<tr>
<td></td>
<td>Germany (-)</td>
</tr>
<tr>
<td>Kabund et al., 2019</td>
<td>South Africa (+)(+)</td>
</tr>
<tr>
<td>Karagedikli and McDermott, 2017</td>
<td>New Zealand (+)</td>
</tr>
<tr>
<td>Machado and Portugal, 2014</td>
<td>Brazil (-)</td>
</tr>
<tr>
<td>Oinonen and Paloviita, 2014</td>
<td>Euro Area (+)</td>
</tr>
<tr>
<td>Riggi and Vendetti, 2015</td>
<td>Euro Area (+)</td>
</tr>
<tr>
<td>Szafranek, 2017</td>
<td>Poland (-)</td>
</tr>
</tbody>
</table>

(+) denotes that the slope of the Phillips curve has increased.
(-) denotes that the slope of the Phillips curve has decreased.
countries with some systematic differences between the two most common empirical approaches for studying changes in the slope of the Phillips curve.

6 Conclusions

Before announcing the death of the Phillips curve, evidence from other countries than the United States should be collected. We contribute to the formation of stylized facts concerning changes in the slope of the Phillips curve by estimating time varying Phillips curves using data for 10 OECD countries (plus the United States for comparison).

In more than half of the countries studied, the Phillips curve appears to be alive and well: Australia, Canada, France, Italy, Spain, and Sweden. In these cases, the estimated effect of changes in labor demand pressure on inflation remains significantly negative in 2018 for all the empirical approaches that we have employed. There are however also two countries where all our empirical approaches indicate that the Phillips curve as defined here has indeed disappeared: New Zealand and the United Kingdom. For Germany and Japan, the models with explicitly time varying parameters document a significant relationship between shocks to unemployment and inflation in 2018, while rolling window estimation results in an insignificant effect.

A second issue is whether the Phillips curve has flattened since the financial crisis. Again, the results differ between countries and statistical methods. In four cases, a steeper rather than flatter Phillips curve is found using all approaches: Canada, Italy, Spain, and Sweden. New Zealand is the only country where the estimated effect is consistently reduced between 2005 and 2018. Furthermore, the rolling window estimation but not the model with time varying parameters yields a flattening of the Phillips curve.
in three cases: Australia, Germany, and Japan. The UK Phillips curve flattened in the 1980s according to both approaches, but has changed little since the financial crisis.

Overall, there appears to be systematic differences between the two most common statistical methods for studying time variation in the Phillips curve. When a long sample period is used, estimation using rolling windows results in a flattening of the Phillips curve more often than the models with explicitly time varying parameters. This relationship between statistical method and findings is detectable also in previous studies of changes in the slope of the Phillips curve. Since this difference between the methods disappear when the sample is reduced to 1990-2018, we suspect that the long memory of the random walk process for the parameters in the time varying model produces strong effects of shocks to unemployment on inflation also in 2018 when earlier decades with a more pronounced Phillips curve are included in the sample.

Because central banks rely on the Phillips curve to control inflation, a weaker connection between labor demand pressure and inflation renders monetary policy less potent. However, we find more evidence of a strengthened Phillips curve than of a weakened relationship between unemployment and inflation. Hence, our results indicate that the reason for the low global inflation lies elsewhere.

References


Appendix A1: Maximum effects of shocks to unemployment on inflation, TVP vs rolling window estimation, main specification
Appendix A2: Maximum effects on inflation, TVP and rolling windows, data on expected inflation and shorter samples

Figure A2a: Canada  Figure A2b: France  Figure A2c: Germany

Figure A2d: Japan  Figure A2e: United Kingdom  Figure A2f: United States

Rolling window  TVP
Appendix A3: Impulse response functions of inflation to unemployment shocks, 2005Q1 and 2018Q2, main specification

Figure A3a: IRF 2018, TVP, Australia

Figure A3b: IRF 2018, Rolling window, Australia

Figure A3c: IRF 2005, TVP, Australia

Figure A3d: IRF 2005, Rolling window, Australia

Figure A3e: IRF 2018, TVP, Canada

Figure A3f: IRF 2018, Rolling window, Canada
Figure A3ae: IRF 2005, TVP, Spain
Figure A3af: IRF 2005, Rolling window, Spain

Figure A3age: IRF 2018, TVP, Sweden
Figure A3ah: IRF 2018, Rolling window, Sweden

Figure A3ai: IRF 2005, TVP, Sweden
Figure A3aj: IRF 2005, Rolling window, Sweden

Figure A3ak: IRF 2018, TVP, UK
Figure A3al: IRF 2018, Rolling window, UK
Appendix A4: IRFs in 2005 and 2018, with data on expected inflation, sample 1990–2018
Figure A4u: IRF 2018, TVP, US

Figure A4v: IRF 2018, Rolling window, US

Figure A4w: IRF 2005, TVP, US

Figure A4x: IRF 2005, Rolling window, US