Seasonality in the Frequency of Price Change and Optimal Monetary Policy

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Abstract

The implications for optimal monetary policy of introducing seasonality in the frequency of price change in the baseline New Keynesian model are studied. In the resulting model, both the parameters of the Phillips curve and the weight on inflation stabilization in the welfare criterion vary seasonally. I show that for a plausible calibration, even a modest degree of seasonality in the frequency of price change gives rise to large seasonal differences in the equilibrium responses of the output gap and inflation to cost-push shocks. The effects on welfare, however, are small under both discretionary and commitment policy.

Keywords: Price Setting; Staggering; Seasonality; Optimal Monetary Policy

JEL codes: E31; E32

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1 Introduction

Staggered price and wage change is a key feature of the New Keynesian DSGE model, forming the basis for the model to generate sufficient degrees of monetary non-neutrality to nominal shocks. In the standard version of the model, the frequency of price and wage change is constant over time. But recent empirical evidence cast doubt on the plausibility of this assumption. Nakamura and Steinsson (2008) find a strong seasonal component in both consumer and producer prices in the U.S. economy, the frequency of price change being the highest in the first quarter and then declining monotonically over the year. In the euro area, consumer prices are more likely to change at the beginning of the year and after the summer, and less likely to change during the summer months, while most of the seasonality in producer prices is due to a spike in the frequency of price change in the first quarter of the year (Dhyne et al., 2005, Vermeulen et al., 2012).

The literature also points to a high degree of seasonality in the adjustment of wages. In the euro area, the frequency of wage change is the highest in the first quarter of the year and the lowest in the last quarter (Druant et al., 2009). A similar pattern is reported for the U.S. economy by Olivei and Tenreyro (2007), who find that most wages are renegotiated in the fourth quarter and become effective in the first quarter of the subsequent year. Olivei and Tenreyro (2010) extend the analysis to encompass Japan and several European countries. They find a very high degree seasonal synchronization in Japan, almost all wages being changed during the Shunto between March and April each year, while seasonality is less pronounced in Germany, France, and the U.K.

Motivated by these empirical findings, several recent studies have relaxed the assumption in the standard New Keynesian model that the frequency of price and wage change is constant over time. Olivei and Tenreyro (2007) find that seasonality in the frequency of wage change in the Calvo (1983) model also leads to seasonal differences in the output response to monetary policy shocks. Söderberg (2013) shows—in the context of a simple two-period model with Taylor (1980) prices—that the way the output response is affected by seasonality crucially depends on whether prices are strategic complements or strategic substitutes. When prices are strategic complements, seasonality in the frequency of price change only has a limited effect on the response of output to monetary policy shocks.¹ Using an estimated DSGE model of the euro

¹ Söderberg (2013) uses the term “nonuniform staggering” when referring to the concept of seasonality in the frequency of price change.
area where the frequency of wage change varies seasonally, Juillard et al. (2013) obtain results that are in line with the analytical results in Söderberg (2013). Despite a substantial degree of seasonality in the frequency of wage change in their economy, there are only small seasonal differences in the response of output to monetary policy shocks.

In the light of these studies, the effects of seasonality in the frequency of price and wage change on the economy’s response to monetary policy shocks are quite well-understood. Seasonality has only a limited effect on the response of output when prices are strategic complements, which, as is well-known in the literature, is necessary for the model to produce real effects from nominal shocks that are of realistic magnitude and persistence. In this paper, I instead turn to the question of how seasonality in the frequency of price change affects the optimal conduct of monetary policy in the presence of cost-push shocks that create a trade-off between stabilizing the output gap and inflation.

Besides the assumption of seasonality in the frequency of price change, the model is identical to the baseline New Keynesian model presented in, e.g., Woodford (2003, ch. 3). The resulting model differs from the baseline model in two important aspects. First, while the Phillips curve is of the same form as in the baseline model, the parameters in the Phillips curve vary seasonally, leading to seasonal differences in how inflation is affected by changes in the output gap, expected inflation, and cost-push shocks. Second, the relative weight on inflation stabilization in the utility-based welfare criterion varies seasonally. Yet, as it turns out, the optimal targeting rules under both discretionary and commitment policy are seasonally invariant. In fact, the central bank conducts policy to attain the same seasonally-invariant relation between the output gap and inflation as in the baseline model where the frequency of price change is constant. In other words, any seasonality observed in the outcome of policy is solely a result of the seasonality embedded in the Phillips curve. In a calibrated version of the model, it is shown that even a modest degree of seasonality in the frequency of price change leads to large seasonal differences in the equilibrium responses of the output gap and inflation under both discretionary and commitment policy; the effects on welfare, however, are small.

The remainder of this paper is organized as follows. In the section below, the model is presented. In Section 3 the Phillips curve in the economy is derived and Section 4 presents the welfare criterion. In Section 5 the optimal targeting rules are derived and the economy’s response to cost-push shocks is characterized. In Section 6 the effect on welfare is evaluated. Section 7 concludes.
2 The model

In this section, the behavior of households and firms in the economy is described. The economy is populated by a large number of infinitely-lived households and a large number of firms, each producing a specific good. Following Woodford (2003), firms belong to different industries, each industry forming its own labor market where firms only utilize labor of a particular type. Prices are set in a staggered fashion a la Calvo (1983), but all firms in the same industry are assumed to change their prices at the same time.

Since I allow for seasonality in the frequency of price change, the time \( t \) value of any endogenous variable will, for given initial values of the predetermined variables and a given sequence of exogenous disturbances, not only be a function of time itself, but also of the season at that time. It is therefore convenient to index variables by season. Suppose that one year has \( S \) seasons and let \( z_t(q) \) denote the time \( t \) value of the variable \( z \) for period \( t \) occurring in season \( q = \{1, 2, ..., S\} \). It is convenient to define the sequences

\[
s = (1, 2, ..., S, 1, 2, ..., S, 1, 2, ...)
\]

and

\[
s^q = (s_k)_{k=q-1}^{\infty}.
\]

The value of \( z \) at time \( t \) can be then be written as \( z_t(s^q) \) for period 0 occurring in season \( q \).

2.1 Households

The representative household derives utility from consumption of the different goods according to the aggregator

\[
C_t(s^q_t) = \left( \int_0^1 C_{it}(s^q_I) \frac{d}{d\bar{s}_I} \right)^{-1},
\]

where \( C_{it}(s^q_I) \) denotes the household’s consumption of the good produced by firm \( i \in [0, 1] \). The household seeks to maximize the utility function

\[
\sum_{t=0}^{\infty} \beta^t \left\{ \frac{1}{1 - \sigma} C_t(s^q_t)^{1-\sigma} - \frac{1}{1 + \varphi} \int_0^1 N_{it}(s^q_I)^{1+\varphi} dI \right\},
\]
where $\beta \in (0, 1)$ is the subjective discount factor and $N_{it}(s_i^q)$ is the number of working hours the household works at firm $i$, subject to the budget constraint

$$
\int_0^1 C_{it}(s_i^q) P_{it}(s_i^q) \, dq + Q_{it}(s_i^q) B_{t-1}(s_{t-1}^0) = B_{t-1}(s_{t-1}^0) + \int_0^1 W_{it}(s_i^q) N_{it}(s_i^q) \, dq + \Phi_{it}(s_i^q),
$$

(5)

where $P_{it}(s_i^q)$ is the price of good $i$, $Q_{it}(s_i^q)$ is the price of a one-period bond that pays off one in the next period, $B_{t}(s_i^q)$ is the household’s end of period bond holdings, $W_{it}(s_i^q)$ denotes the nominal wage paid in the industry which firm $i$ belongs to, and $\Phi_{it}(s_i^q)$ represents dividends from firm ownership and lump-sum transfers.\(^2\)

The first-order conditions yield a set of demand equations

$$
C_{it}(s_i^q) = \left( \frac{P_{it}(s_i^q)}{P_{t}(s_i^q)} \right)^{-\epsilon} C_t(s_i^q), \quad \forall i \in [0, 1]
$$

(6)

where $P_{t}(s_i^q) = \left[ \int_0^1 P_{it}(s_i^q)^{1-\epsilon} \, dq \right]^{1-\epsilon}$ is the aggregate price level, the consumption Euler equation

$$
Q_{t}(s_i^q) = \beta E_t \left\{ \left( \frac{C_{t+1}(s_{t+1}^q)}{C_t(s_i^q)} \right)^{-\sigma} \frac{P_{t}(s_i^q)}{P_{t+1}(s_{t+1}^q)} \right\},
$$

(7)

and the labor supply condition

$$
\frac{N_{it}(s_i^q)}{C_t(s_i^q)^{-\alpha}} = \frac{W_{it}(s_i^q)}{P_{t}(s_i^q)}.
$$

(8)

### 2.2 Firms

Good $i$ is produced by a monopolist with technology

$$
Y_{it}(s_i^q) = A_t(s_i^q) N_{it}(s_i^q)^{1-\alpha},
$$

(9)

where $Y_{it}(s_i^q)$ is output, $A_t(s_i^q)$ denotes technology, and $N_{it}(s_i^q)$ is the input of labor in production.

In season $q$, the firm gets to change its price with probability $1 - \theta(q)$, where $\theta(q) \in [0, 1]$ for all $q$. With probability $\theta(q)$ the price remains fixed. A firm that gets to change its price in period $t$ sets its optimal price $P_{it}^*(s_i^q)$ to maximize its expected discounted profit stream while

\(^2\) It is assumed that the representative household supplies all types of labor being used in the different industries.
the price remains fixed, i.e., the firm chooses $P_t^* (s^q_t)$ to maximize

$$E_t \sum_{k=0}^{\infty} \prod_{j=1}^{k} \left( \theta \left( s^q_{t+j} \right) \right) Q_{t,t+k} \left( s^q_{t+k} \right) \left\{ P_t^* (s^q_t) Y_{t+k|t} \left( s^q_{t+k} \right) - \Psi_{t+k|t} \left( s^q_{t+k} \right) \right\}.$$ \hspace{1cm} (10)

Here $Q_{t,t+k} \left( s^q_{t+k} \right) = \beta^k \left( C_{t+k} \left( s^q_{t+k} \right) / C_t \left( s^q_t \right) \right)^{-\sigma} \left( P_t \left( s^q_t \right) / P_{t+k} \left( s^q_{t+k} \right) \right)$ is the nominal stochastic discount factor between periods $t$ and $t+k$,

$$Y_{t+k|t} \left( s^q_{t+k} \right) = \left( \frac{P_t^* (s^q_t)}{P_{t+k} \left( s^q_{t+k} \right)} \right)^{-\varepsilon} Y_{t+k} \left( s^q_{t+k} \right),$$ \hspace{1cm} (11)

in which $Y_t \left( s^q_{t+k} \right) \equiv C_t \left( s^q_{t+k} \right)$, is the firm’s demand in period $t+k$ when the price was last reset in period $t$, and

$$\Psi_{t+k|t} \left( s^q_{t+k} \right) = T_{t+k} \left( s^q_{t+k} \right) W_{t+k|t} \left( s^q_{t+k} \right) \left( \frac{Y_{t+k|t} \left( s^q_{t+k} \right)}{A_{t+k} \left( s^q_{t+k} \right)} \right)^{1/\alpha},$$ \hspace{1cm} (12)

is the firm’s nominal total cost in period $t+k$ when the price was last reset in period $t$. In the last expression, $T_t \left( s^q_t \right)$ represents a time-varying wage tax factor (a subsidy if $T_t \left( s^q_t \right) < 1$), and $W_{t+k|t} \left( s^q_{t+k} \right)$ is the nominal wage (in the industry which the firm belongs to) in period $t+k$ when (all) the firms in the industry last reset their prices in period $t$.\(^3\)

The first-order condition is given by

$$E_t \sum_{k=0}^{\infty} \prod_{j=1}^{k} \left( \theta \left( s^q_{t+j} \right) \right) Q_{t,t+k} \left( s^q_{t+k} \right) Y_{t+k|t} \left( s^q_{t+k} \right) \left\{ P_t^* (s^q_t) - \lambda P_{t+k} \left( s^q_{t+k} \right) MC_{t+k|t} \left( s^q_{t+k} \right) \right\} = 0,$$ \hspace{1cm} (13)

where $\lambda = \varepsilon / (\varepsilon - 1)$ is the firm’s desired gross markup, and $MC_{t+k|t} \left( s^q_{t+k} \right)$ denotes the firm’s real marginal cost in period $t+k$ when the price was last reset in period $t$.

3 Inflation dynamics

In this section, I derive the Phillips curve and discuss how inflation dynamics is affected by seasonality in the frequency of price change.

\(^3\) The proceeds from the tax (or the financing of the subsidy) are transferred lump sum to (or from) the households.
3.1 The Phillips curve

As shown in Appendix A, the “supply side” of the economy, can be represented by the following log-linearized equations:

\[
p_t^s (s_q^t) = (1 - \chi (s_q^t)) [p_t (s_q^t) + \nu \widehat{mc}_t (s_q^t)] + \chi (s_q^t) p_{t+1}^s (s_q^{t+1}),
\]

\[
p_t (s_q^t) = \theta (s_q^t) p_{t-1} (s_q^{t-1}) + (1 - \theta (s_q^t)) p_t^s (s_q^t),
\]

\[
\widehat{mc}_t (s_q^t) = \widehat{\pi}_t (s_q^t) + \omega x_t (s_q^t),
\]

where \( \widehat{mc}_t (s_q^t) \) is the log-deviation of the average real marginal cost in the economy from its steady state value, \( x_t \) denotes the output gap, \( \nu = \frac{1}{1+\varepsilon(\varphi+\alpha)/(1-\alpha)} \), \( \omega = \sigma + (\varphi + \alpha) / (1 - \alpha) \), and

\[
\chi (s_q^t) = \frac{\sum_{n=0}^{S-1} \prod_{j=1}^{n+1} \theta (s_{q+j}^t) \beta_j}{\sum_{n=0}^{S-1} \prod_{j=1}^{n} \theta (s_{q+j}^t) \beta_j},
\]

The optimal reset price is a weighted average of contemporaneous variables and next period’s optimal reset price; the price level is a weighted average of the price level in the previous period and this period’s optimal reset price. These expressions are of the same form as in the standard Calvo model, but the weights are time-varying reflecting seasonality in the frequency of price change. The relation between marginal cost and the output gap is seasonally-invariant and identical to that obtained without seasonality.

Combing equations (14) to (16) yields the Phillips curve

\[
\pi_t (s_q^t) = \kappa (s_q^t) x_t (s_q^t) + \beta^* (s_q^t) E_t \pi_{t+1} (s_q^{t+1}) + \xi (s_q^t) u_t (s_q^t),
\]

where \( \pi_t (s_q^t) = p_t (s_q^t) - p_{t-1} (s_q^{t-1}), u_t (s_q^t) = \nu \hat{\pi}_t (s_q^t) \),

\[
\kappa (s_q^t) = \frac{(1 - \theta (s_q^t)) (1 - \chi (s_q^t))}{\theta (s_q^t)},
\]

\[
\beta^* (s_q^t) = \frac{1 - \theta (s_q^t)}{1 - \theta (s_{q+1}^t)} \chi (s_q^t),
\]

in which \( \phi = \nu \omega \) and \( \xi (s_q^t) = \kappa (s_q^t) / \phi \). Seasonality in the frequency of price change does not change the form of the Phillips curve, but it introduces seasonal variations in both the slope

4. Throughout, lower case letters denote the log of a variable and hats denote the log deviation of a variable from its steady state value.

5. The output gap is defined as the log deviation of the actual level of output from the hypothetical efficient level of output chosen by a social planner.
of the curve, \( \kappa(s_t^q) \), and the weight on next period’s expected inflation, \( \tilde{\beta}(s_t^q) \). In addition, the pass-through of the cost-push shock, \( \tau_t(s_t^q) \), into inflation is different in different seasons as \( \xi(s_t^q) \) varies seasonally.\(^6\)

The parameter \( \phi \) measures the degree of strategic complementarity in price setting. If \( \phi < 1 \), prices are strategic complements; if \( \phi > 1 \), prices are strategic substitutes. As is well-known, the higher the degree of strategic complementarity, i.e., the lower the value of \( \phi \), the flatter the Phillips curve, and the larger the real effects from nominal shocks, such as monetary policy shocks.\(^7\) Using a model with Taylor pricing, Söderberg (2013) shows that the degree of strategic complementarity also plays a crucial role for how seasonality in the frequency of price change affects the response of output to nominal shocks: When the degree of strategic complementarity is high, even high degrees of seasonality in the frequency of price change only have small effects on the response of output. It is straightforward to verify that this result carries over to the present model with Calvo pricing: The flattening of the Phillips curve due to prices being strategic complements not only amplifies the effects of nominal shocks, but also reduces the difference in the slope of the curve between seasons, thus reducing the seasonal differences in the response of output.

### 3.2 A two-season example

To gain intuition on how seasonality in the frequency of price change affects inflation dynamics, it is instructive to consider a special case when there are only two seasons \((S = 2)\). I also assume that \( \theta(2) = 1 - \theta(1) \), implying that the mean duration of prices always are two seasons (one year).\(^8\)

Suppose that \( \theta(1) < \theta(2) \), so that a majority of the prices are changed in the first half of the year; it follows from (17) that \( \chi(1) > \chi(2) \).\(^9\) In other words, firms are more forward-looking—i.e., they care more about \( p_{t+1}^* \) and less about \( p_t \) and \( mc_t \)—when changing the price in the first half of the year. The reason is that when a firm gets to change its price in the first half of the year, it realizes that the probability of getting to change the price again in the second half of

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6. In this model, the cost-push shock arises due to time-variations in the wage tax. But cost-push shocks can also be introduced through other mechanisms, e.g., time-variations in firms’ desired markup, see Gali (2008, ch. 5).
7. See, e.g., Woodford (2003, ch. 3) for a discussion of the role of strategic complementarity in generating real effects from nominal shocks.
8. The mean duration is defined as \( \mu = (1 - \theta(1))\mu(1) + (1 - \theta(2))\mu(2) \) where \( \mu(1) = [(1 - \theta(2)) \times 1 + \theta(2) (1 - \theta(1)) 2 \times 2 + \theta(2) (1 - \theta(1)) (1 - \theta(2)) \times 3 + \ldots] \) is the mean duration of a price set in the first half of the year, and \( \mu(2) \) the corresponding expression for a price set in the second half of the year.
9. See Appendix B for proofs of the inequalities in this section.
the year is relatively low and the firm is therefore more forward-looking. When the firm instead
gets to change the price in second half of the year, it knows that the probability of getting to
change the price again in the first half (of next year) is relatively high, and the firm is more
myopic.

How does these seasonal differences in price setting affect inflation dynamics? It is straight-
forward to show that \( \kappa(1) > \kappa(2) \), i.e., the Phillips curve is steeper and movements in the
output gap have larger effects on inflation in the first half of the year. By (16), a positive
output gap leads to an increase in marginal cost, and resetting firms set higher prices. But
because firms are more forward-looking in the first half of the year, the price that they set is,
for a given price level, lower in the first half of the year than in the second half of the year. This
is however more than compensated for by the fact that more firms are adjusting their prices in
the first half of the year, leading to a higher price level and a larger effect on inflation in that
season. By the same token, the pass-through of the cost-push shock into inflation is larger in
the first half of the year, i.e, \( \xi(1) > \xi(2) \).

It is also straightforward to show that \( \tilde{\beta}(1) > \tilde{\beta}(2) \). In other words, inflation is more affected
by expected future inflation in the first half of the year. Firms are more forward-looking when
they reset their prices in the first half of the year, which increases the influence of expected
inflation on inflation today. The effect is further amplified by the fact that more firms are
adjusting their prices in the first half of the year. Hence, inflation is not only more sensitive
to current, but also to future, economic developments in the first half of the year when the
frequency of price change is high.

4 The welfare criterion

In the previous section, the Phillips curve when the frequency of price change varies between
seasons was derived. In this section, the welfare criterion is derived by taking a second-order
approximation to the representative household’s utility function. It is assumed that the steady
state value of \( T_t (s^0_t) \) is set such that the distortion from monopolistic competition is neutralized
in the steady state.

Without imposing any specific assumptions about the nature of price adjustment in the
economy, welfare—conditional on period 0 occurring in season q—can be written as

\[ \mathcal{W}^q = -\frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \omega (x_t (s^q_t))^2 + \frac{\xi}{\nu} \Delta_t (s^q_t) \right\}, \]  

(21)

where \[ \Delta_t (s^q_t) = \text{var}_t \{ p_{it} (s^q_t) \} \] measures the cross-sectional dispersion of prices in period t.\(^{10}\)

As shown in Appendix C, the price dispersion term evolves according to

\[ \Delta_t (s^q_t) = \theta (s^q_t) \Delta_{t-1} (s^q_{t-1}) + \frac{\theta (s^q_t)}{1 - \theta (s^q_t)} (\pi_t (s^q_t))^2. \]  

(22)

Taking the discounted sum of these values for periods \( t \geq 0 \), ignoring terms independent of policy, one obtains

\[ \sum_{t=0}^{\infty} \beta^t \Delta_t (s^q_t) = \sum_{t=0}^{\infty} \beta^t \frac{1}{\xi (s^q_t)} (\pi_t (s^q_t))^2. \]  

(23)

Using (23), (21) can be written as

\[ \mathcal{W}^q = -\frac{\omega}{2} E_0 \sum_{t=0}^{\infty} \beta^t L_t (s^q_t), \]  

(24)

where

\[ L_t (s^q_t) = (x_t (s^q_t))^2 + \frac{1}{\lambda (s^q_t)} (\pi_t (s^q_t))^2 \]  

(25)

is the period loss function, in which \( \lambda (s^q_t) = \kappa (s^q_t) / \epsilon \). In this formulation, \( 1/\lambda (s^q_t) \) is the relative weight on inflation stabilization.\(^{11}\) Because \( \lambda (s^q_t) \) is proportional to \( \kappa (s^q_t) \), the relative weight on inflation stabilization is high in periods when \( \kappa (s^q_t) \) is low, i.e., when the Phillips curve is flat. In the context of the two-season model presented above, inflation stabilization is given a higher weight in the loss function in the second half of the year when the frequency of price change is low. This is due to the fact that a given level of inflation leads to a higher degree of price dispersion—which is what ultimately affects welfare—in the season when few firms are adjusting their prices. To see this, note that inflation has both an extensive margin (how many firms that are adjusting) and an intensive margin (the size of the price change by the firms that are adjusting). When the frequency of price change is low, the extensive margin is weak. For a given level of inflation, firms adjusting in the second half of the year must therefore have changed their prices by more relative to the (average) unchanged prices. And the more the

10. See, e.g., Woodford (2003, ch. 6) for a derivation of (21).

11. In the standard Calvo model without seasonality, it is commonplace to instead refer to \( 1/\lambda (s^q_t) \) as the relative weight on output gap stabilization. But this interpretation is improper here because the welfare function cannot be scaled with a time-varying \( \lambda (s^q_t) \).
newly set prices deviate from the unchanged prices, the higher the degree of price dispersion. Therefore, inflation is more detrimental for welfare in the second half of the year.

5 Monetary policy

Having derived the welfare criterion in the previous section, I now turn to the issue of optimal monetary policy. The optimal targeting rules under both discretion and commitment are derived, and the economy’s optimal response to cost-push shocks is characterized.

5.1 Optimal discretionary policy

First, consider the case when the central bank is unable to commit to future policy actions. In this case, the policy problem reduces to a series of static optimization problems where the central bank chooses $x_t(s^q_t)$ and $\pi_t(s^q_t)$ to minimize the period loss function in (25) subject to the Phillips curve in (18). The optimality condition is given by

$$x_t(s^q_t) = \frac{\kappa(s^q_t)}{\lambda(s^q_t)} \pi_t(s^q_t)^2.$$ (26)

Since both $\kappa(s^q_t)$ and $\lambda(s^q_t)$ vary seasonally, the optimality condition seemingly implies a seasonally-varying targeting rule. However, using the fact that $\lambda(s^q_t) = \kappa(s^q_t)/\epsilon$, (26) can be written as

$$x_t(s^q_t) = -\epsilon \pi_t(s^q_t),$$ (27)

which is seasonally-invariant. In fact, the targeting rule is identical to that obtained without seasonality in the frequency of price change. The central bank conducts policy to attain the same constant relation between inflation and the output gap as when the frequency of price change is constant.

Suppose that $\tilde{\pi}_t(s^q_t)$ follows the AR(1)-process

$$\tilde{\pi}_t(s^q_t) = \rho \tilde{\pi}_{t-1}(s^q_{t-1}) + \epsilon_t,$$ (28)

where $\epsilon_t$ is a seasonally-invariant white-noise process with constant variance $\sigma^2_{\epsilon}$. Figure 1 plots responses of the output gap and inflation to a one standard deviation increase in $\epsilon_t$. For this
Figure 1: Impulse responses of the output gap and inflation to the cost-push shock under discretion for the baseline calibration. No seasonality in the legend indicates no seasonality in the frequency of price change. Season 1 indicates that the shock occurred in the first half of the year when $\theta = 0.45$; Season 2 indicates that the shock occurred in the second half of the year.

exercise, the parameter values $\alpha = 1/3$, $\beta = 0.985$, $\epsilon = 6$, $\sigma = 1$, $\varphi = 3$, $\rho = 0.64$, and $\sigma_r = 0.1$ have been used. The value of $\beta$ implies an annual steady state interest rate of approximately 3%. The calibration of $\epsilon$ yields a 20% desired markup. The values of $\sigma$ and $\varphi$ imply that the intertemporal elasticity of substitution is 1 and that the labor supply elasticity is $1/3$. The autoregressive coefficient of the cost-push shock, $\rho$, indicates a substantial degree of persistence in the cost-push shock. The value of $\sigma_r$ is set so that the equilibrium fall in the output gap, in response to the cost-push shock, is roughly one percentage point when the frequency of price change is constant. The values of the deep parameters are standard in the literature and imply $\phi = 0.1935$, indicating a substantial degree of strategic complementarity.

When there is no seasonality in the frequency of price change, the central bank in response to the exogenous inflationary pressure stemming from the cost-push shock creates a negative output gap to accommodate some of the inflationary pressure. To illustrate the effects of seasonality, I also plot the responses when $\theta$ is set to 0.45, implying a modest degree of seasonality in the frequency of price change. The fact that the targeting rule is seasonally invariant means that all the seasonality observed in the equilibrium responses is a result of the seasonality embedded in the Phillips curve. Compared to the case when the frequency of price change is constant, both the output gap and inflation are less stabilized if the shock occurs in first half of the year, but more stabilized if the shock occurs in the second half of the year. As discussed in Section 3.2, the pass-through of the cost-push shock into inflation is higher in the first half of the year when the frequency of price change is high; this worsens the central bank’s trade-off between output gap and inflation stabilization, so that both are stabilized less. For given inflation expectations,
the trade-off worsens further as the coefficient on expected inflation is larger, so the effect of the next period’s expected inflation on inflation today is magnified. This is, however, to some extent counteracted by a fall in inflation expectations, as the private sector realizes that the pass-through of the cost-push shock into inflation will be lower in the next period when the frequency of price change is low. The worsening of the trade-off is further dampened by the Phillips curve being steeper in the first half of the year, making it is less costly for the central bank, in terms of the output gap, to attain a given reduction in inflation. The difference in the equilibrium responses observed in the figure are the net of these effects.

Despite the modest degree of seasonality in the frequency of price change, there is a substantial degree of seasonality in the equilibrium responses to the cost-push shock. The responses of the output gap and inflation are 28 percent larger in magnitude if the shock occurs in the first half of the year than if it occurs in the second half of the year. To understand this result, recall from Section 3.1 that the flattening of the Phillips curve due to prices being strategic complements also reduces the difference in the slope of the curve between seasons. Therefore, the trade-off between output gap and inflation stabilization in the first half of the year, when the pass-through of the cost-push shock into inflation is high, becomes less favorable compared to the trade-off in the second half of the year when the pass-through is low. As a consequence, the higher the degree of strategic complementarity, the less inflation and the output gap is stabilized in the first half of the year compared to the second half of the year, leading to a greater degree of seasonality in the equilibrium responses. Therefore, in this economy, with a substantial degree of strategic complementarity, even a modest degree of seasonality in the frequency of price change gives rise to large seasonal differences in the equilibrium responses.

To further illustrate this point, Figure 2 shows impulse responses to the same cost-push shock when assuming a linear production function and an economy-wide labor market, implying $\phi = 4$. The output gap and inflation are stabilized less on average. In the absence of strategic complementarities, the pass-through of the cost-push shock into inflation is higher on average, worsening the central bank’s trade-off between output gap and inflation stabilization in both seasons. But at the same time, the Phillips curve is steeper on average, improving the trade-off.

13. Since the model is purely forward-looking, optimal discretionary policy is in practice the solution to a static optimization problem, and the relative difference in the responses between seasons is therefore the same in every period.
14. With a linear production function ($\alpha = 0$) and an economy-wide labor market, $\nu = 1$ and $\omega = \sigma + \varphi$. See, e.g., Gali (2008, ch. 3) for a derivation of the baseline New Keynesian model under the assumption of an economy-wide labor market.
Figure 2: Impulse responses of the output gap and inflation to the cost-push shock under discretion without strategic complementarity. No seasonality in the legend indicates no seasonality in the frequency of price change. Season 1 indicates that the shock occurred in the first half of the year when $\theta = 0.45$; Season 2 indicates that the shock occurred in the second half of the year.

Here, the first effect dominates and there is less stabilization in both seasons. However, more important for the analysis here, the degree of seasonality in the equilibrium responses is, in both absolute and relative terms, considerably lower. The responses of the output gap and inflation are only 4 percent larger in magnitude if the shock occurs in the first half of the year than if it occurs in the second half of the year.

5.2 Optimal commitment policy

Having analyzed optimal policy under discretion in the previous section, I now turn to optimal policy under commitment, i.e., when the central bank commits to a state-contingent policy plan in period 0. The Lagrangian for this problem is given by

$$
\mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ (x_t (s_t^q))^2 + \frac{1}{\lambda (s_t^q)} (\pi_t (s_t^q))^2 \\
\quad + \mu_t (s_t^q) \left[ \pi_t (s_t^q) - \kappa (s_t^q) x_t (s_t^q) - \bar{\beta} (s_t^q) \pi_{t+1} (s_{t+1}^q) - \xi (s_t^q) u_t (s_t^q) \right] \right\}.
$$

Eliminating the multipliers, the optimality condition can be written as

$$
x_t (s_t^q) = \Lambda (s_{t-1}^q) \frac{1}{\beta} x_{t-1} (s_{t-1}^q) - \frac{\kappa (s_t^q)}{\lambda (s_t^q)} \pi_t (s_t^q),
$$

where

$$
\Lambda (s_{t-1}^q) = \frac{1/\kappa (s_{t-1}^q)}{1/ \left( \kappa (s_t^q) \beta (s_{t-1}^q) \right)}.
$$
Figure 3: Impulse responses of the output gap and inflation to the cost-push shock under commitment for the baseline calibration. No seasonality in the legend indicates no seasonality in the frequency of price change. Season 1 indicates that the shock occurred in the first half of the year when $\theta = 0.45$; Season 2 indicates that the shock occurred in the second half of the year.

The parameter $\Lambda (s_{t-1})$ is a measure of the gain from commitment policy in period $t-1$. The nominator is the output gap cost of a unit reduction in period $t-1$ inflation attained through a reduction in the output gap in period $t-1$, while the denominator is the corresponding cost if the reduction in inflation is attained through a commitment to reduce the output gap in period $t$. Thus, the higher the value of $\Lambda (s_{t-1})$, the higher the gain from the central bank’s ability to commit. Equation (31) seemingly implies that the gain from commitment policy varies between seasons. But using (19) and (20), one finds—just as in the standard Calvo model where the frequency of price change is constant—that $\Lambda (s_{t-1})$ is constant and equal to $\beta$.\textsuperscript{15} Imposing this result and $\varphi (s_{t}) = \phi (s_{t})$, (30) can be written as

$$x_{t} (s_{t}) = x_{t-1} (s_{t-1}) - \epsilon \pi_{t} (s_{t}),$$

(32)

which is identical to the targeting rule obtained when there is no seasonality in the frequency of price change. Hence, also the targeting rule under commitment is seasonally-invariant and identical to that obtained when frequency of price change is constant.

Figure 3 plots responses of the output gap and inflation to the cost-push shock under commitment for the same calibration as before. Under commitment, the central bank achieves a better trade-off between output gap and inflation stabilization in the period when the shock occurs, by committing to a prolonged period of negative output gaps. Therefore, both the out-

\textsuperscript{15} To gain some intuition for this result, consider the two-season model presented in Section 3.2. On the one hand, $\kappa (1) > \kappa (2)$, so it is cheaper, in terms of the output gap, to reduce inflation in the first half of the year. On the other hand, $\beta (1) > \beta (2)$, so a reduction in expected inflation has a larger affect on current inflation in the first half of the year. The first effect reduces the gain from commitment policy, while the second one increases it. As it turns out, these two effects exactly cancel each other out and $\Lambda (1) = \Lambda (2) = \beta$.\textsuperscript{15}
put gap and inflation are in the initial period stabilized more than under discretion, regardless of the season in which the shock occurs. But, as in the case of discretion, the economy is less stabilized if the shock occurs in the season when the frequency of price change is high, the responses of the output gap and inflation being 19 percent larger in magnitude if the shock occurs in the first half of the year than if it occurs in the second half of the year. While this difference is smaller than that observed under discretion, there is still a substantial degree of seasonality in the equilibrium responses also under commitment.

As a result of the history-dependence embedded in optimal commitment policy, the cyclical fluctuations observed in the output gap in subsequent periods are less pronounced than under discretion. Suppose for instance that the shock occurs in the first half of the year. In the second period, exogenous inflationary pressure is lower and inflation falls, but the output gap is, as result of the central bank’s first-period commitment, further reduced, and remains more suppressed than that observed when the frequency of price change is constant. Under discretion, in contrast, the output gap is adjusted to be proportional to contemporaneous inflation, resulting in an output gap in the second period that is less suppressed than that observed when the frequency of price change is constant.

6 Welfare

In the previous section, it was shown that even a modest degree of seasonality in the frequency of price change gives rise to large seasonal differences in the equilibrium responses of the output gap and inflation to cost-push shocks. I now turn to the question of how this affects welfare in the economy. In Section 4, an expression for welfare conditional on period 0 occurring in season \( q \) was derived. Here, I calculate an unconditional measure of welfare by averaging conditional welfare over all possible realizations of the season in period 0:

\[
W = -\frac{1}{S}E_0 \sum_{q=1}^{S} W^q
\]

\[
= -\frac{\omega}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{1}{S} \sum_{q=1}^{S} \left[ (x_t(q))^2 + \frac{1}{\lambda(q)} (\pi_t(q))^2 \right] \right\}.
\]

(33)

Let \( var_q(x) \equiv var(x_t(q)) \) denote the variance of the output gap in season \( q \) and \( var_q(\pi) \) the corresponding expression for inflation. If \( \beta \) is close to unity, the average per-period welfare loss
Table 1: Welfare calculations under discretion.

<table>
<thead>
<tr>
<th>$\theta$</th>
<th>$\text{var}_1(x)$</th>
<th>$\text{var}_2(x)$</th>
<th>$\text{var}(x)$</th>
<th>$\text{var}_1(\pi)$</th>
<th>$\text{var}_2(\pi)$</th>
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<td>0.7764</td>
<td>1.0230</td>
<td>1.2696</td>
<td>0.7764</td>
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<td>0.5916</td>
<td>1.0926</td>
<td>1.5935</td>
<td>0.5916</td>
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</tr>
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<td>0.4397</td>
<td>1.2100</td>
<td>1.9803</td>
<td>0.4397</td>
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<td>1.3774</td>
<td>2.4392</td>
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</table>

Note: The table shows the components of the loss function under discretion for the baseline calibration. The variances and the welfare loss are computed relative to the case when $\theta = 0.5$. The volatility of inflation shows the same pattern as the volatility of the output gap. But the

The volatility of inflation can be approximated as

$$\frac{\omega}{2} \left\{ \text{var}(x) + \bar{\text{var}}(\pi) \right\}, \quad (34)$$

where

$$\text{var}(x) = \frac{1}{S} \sum_{q=1}^{S} \text{var}_q(x) \quad (35)$$

is the average of the variances of the output gap across seasons, and

$$\bar{\text{var}}(\pi) = \frac{1}{S} \sum_{q=1}^{S} \left( \frac{1}{\lambda(q)} \right) \text{var}_q(\pi) \quad (36)$$

is a weighted average of the variances of inflation across seasons, the weight in season $q$ corresponding to the relative importance of inflation stabilization in that season.

Table 1 reports values of the components of the loss function in the two-season model under discretion. Lower values of $\theta$ are associated with a greater volatility of the output gap in the first half of the year when the pass-through of the cost-push shock into inflation is high, and lower volatility in the second half of the year when the pass-through is low. But the higher volatility in the first season dominates, leading to a higher average volatility of the output gap, affecting welfare negatively. Intuitively, when the frequency of price change varies between seasons this leads to a dispersion of the output gap response between seasons (cf. Figure 1). Even though the average response is not much affected, this reduces welfare because the loss function is quadratic. In other words, the smaller deviations in the periods when the frequency of price change is low do not compensate for the larger deviations in the periods when the frequency of price change is high.

The volatility of inflation shows the same pattern as the volatility of the output gap. But the
Table 2: Welfare calculations under commitment.

<table>
<thead>
<tr>
<th>$\theta$</th>
<th>$\text{var}_1(x)$</th>
<th>$\text{var}_2(x)$</th>
<th>$\text{var}(x)$</th>
<th>$\text{var}_1(\pi)$</th>
<th>$\text{var}_2(\pi)$</th>
<th>$1/\lambda_1$</th>
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<th>$\text{var}(\pi)$</th>
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Note: The table shows the components of the loss function under commitment for the baseline calibration. The variances and the welfare loss are computed relative to the case when $\theta = 0.5$. 

As is evident in Table 2, the volatility of inflation is not much affected by commitment policy. But, the difference in output gap volatility between the seasons is considerably lower than under discretion—the volatility now being almost as high in the second as in the first half of the year—reflecting the looser relation between the magnitude of output gap deviations and the season in which the deviations occur under history-dependent policy. But because the volatility in the first half of the year is considerably lower than under discretion, average volatility is reduced. Therefore, while the welfare loss increases as $\theta$ falls also under commitment, the increase is more subdued than under discretion. When $\theta = 0.45$, the loss is 0.21% higher than in the case when the frequency of price change is constant.

16. As discussed in Section 4, this is due to the fact that inflation has a weaker effect on price dispersion the higher the frequency of price change.
17. An illuminating special case is when all firms adjust their prices in the first half of the year. Consequently, the volatility of inflation is very high in the first half of the year, yet this has no negative effects on welfare since all firms set the same price and no price dispersion therefore arises.
7 Conclusions

Empirical evidence suggest that the frequency of price change is not constant over the year, but varies seasonally. In this paper, I provide a generalization of the baseline New Keynesian model that incorporates this feature of data, and study the implications for optimal monetary policy. It is shown that when prices are strategic complements, even a modest degree of seasonality in the frequency of price change gives rise to large seasonal differences in the equilibrium responses of the output gap and inflation to cost-push shocks.

Despite the seasonal differences in the equilibrium responses, optimal policy is seasonally-invariant insofar that the central bank under both discretion and commitment targets the same seasonally-invariant relation between the output gap and inflation as when the frequency of price change is constant. In this sense, policy is not more difficult to implement in this model than in the baseline New Keynesian model, as the conduct of optimal policy does not require the central bank to have precise, or in fact any, knowledge of how the model’s parameters vary seasonally.

When the central bank follows a targeting rule, it in practice implements policy by adjusting its policy instrument—typically the nominal interest rate—until the optimal relation between the target variables is attained. Often in the literature, it is instead assumed that the central bank commits to a Taylor-like interest rate rule. While such rules rarely are fully optimal, they are thought to offer a good description of the behavior of real-world central banks. An interesting question for future research is whether there are welfare gains from letting the response coefficients of such rules vary seasonally when there is seasonality in the frequency of price change. The results in Juillard et al. (2013) suggest that there are such gains. But their result is obtained in a larger model, with more shocks and friction, using an ad hoc welfare function that also penalizes interest rate volatility and does not account for the fact that the welfare cost of inflation volatility varies between seasons. It is unclear how the results from their analysis would have changed if it had been conducted using a utility-based welfare function.
References


Appendix A

In this Appendix, I derive equations (14), (15), and (16) in Section 3.

Log-linearization of (13), yields

\[
E_t \sum_{k=0}^{\infty} \prod_{j=1}^{k} \left( \theta \left( s_{t+j}^q \right) \beta \right) \left[ p_t^s (s_t^q) - p_{t+k} \left( s_{t+k}^q \right) \right] = 0, \quad (A.1)
\]

where

\[
\widehat{m}_{t+k|t} \left( s_{t+k}^q \right) = \widehat{\gamma}_{t+k} \left( s_{t+k}^q \right) + \frac{\varphi}{1 - \alpha} \left[ \widehat{y}_{t+k|t} \left( s_{t+k}^q \right) - a_{t+k} \left( s_{t+k}^q \right) \right] + \sigma \widehat{y}_{t+k} \left( s_{t+k}^q \right) - \frac{1}{1 - \alpha} q_{t+k} \left( s_{t+k}^q \right), \quad (A.2)
\]

Log-linearization of (8), and imposing \( \widehat{\epsilon}_t = \widehat{y}_t \), yields

\[
\hat{w}_{it} \left( s_t^q \right) - \hat{p}_t \left( s_t^q \right) = \varphi \hat{\eta}_{it} \left( s_t^q \right) + \sigma \hat{y}_t \left( s_t^q \right) = \frac{\varphi}{1 - \alpha} \left( \hat{y}_{it} \left( s_t^q \right) - a_t \left( s_t^q \right) \right) + \sigma \hat{y}_t \left( s_t^q \right), \quad (A.3)
\]

where the second line makes use of (9). Since all firms in the same industry adjust their prices at the same time, it follows that

\[
\hat{w}_{t+k|t} \left( s_{t+k}^q \right) = \hat{\eta}_{t+k} \left( s_{t+k}^q \right) + \frac{\varphi}{1 - \alpha} \left[ \hat{y}_{t+k|t} \left( s_{t+k}^q \right) - a_{t+k} \left( s_{t+k}^q \right) \right] + \sigma \hat{y}_{t+k} \left( s_{t+k}^q \right)
\]

\[
= \hat{p}_{t+k} \left( s_{t+k}^q \right) + \frac{\varphi}{1 - \alpha} \left[ \hat{y}_{t+k} \left( s_{t+k}^q \right) - a_{t+k} \left( s_{t+k}^q \right) \right] + \sigma \hat{y}_{t+k} \left( s_{t+k}^q \right) - \frac{\varphi \epsilon}{1 - \alpha} \left[ p_t^s \left( s_t^q \right) - p_{t+k} \left( s_{t+k}^q \right) \right], \quad (A.4)
\]

where the second line makes use of (11). Substitution of (A.4) into (A.2), yields

\[
\widehat{m}_{t+k|t} \left( s_{t+k}^q \right) = \gamma_{t+k} \left( s_{t+k}^q \right) + \left( \sigma + \frac{\varphi + \alpha}{1 - \alpha} \right) \hat{y}_{t+k} \left( s_{t+k}^q \right)
\]

\[
- \frac{1}{1 - \alpha} q_{t+k} \left( s_{t+k}^q \right) - \frac{\epsilon (\varphi + \alpha)}{1 - \alpha} \left( p_t^s \left( s_t^q \right) - p_{t+k} \left( s_{t+k}^q \right) \right), \quad (A.5)
\]

Substitution of (A.5) into (A.1), yields

\[
E_t \sum_{k=0}^{\infty} \prod_{j=1}^{k} \left( \theta \left( s_{t+j}^q \right) \beta \right) \left[ \left( 1 + \frac{\epsilon (\varphi + \alpha)}{1 - \alpha} \right) \left[ p_t^s \left( s_t^q \right) - p_{t+k} \left( s_{t+k}^q \right) \right] - \widehat{m}_{t+k} \left( s_{t+k}^q \right) \right] = 0, \quad (A.6)
\]
where
\[\tilde{m}_t(s_t^q) = \tilde{r}_t(s_t^q) + \left(\sigma + \frac{\varphi + \alpha}{1 - \alpha}\right) \tilde{y}_t(s_t^q) - \frac{1 + \varphi}{1 - \alpha} \tilde{a}_t(s_t^q).\] (A.7)

Solving for \(p_t^*(s_t^q)\) in (A.6), yields
\[
p_t^*(s_t^q) = (1 - \chi(s_t^q)) E_t \sum_{k=0}^{\infty} \prod_{j=1}^{k} \left(\theta(s_{t+j}^q) \beta\right) \left[p_{t+k} \left(s_{t+k}^q\right) + \nu \tilde{m}c_{t+k} \left(s_{t+k}^q\right)\right],
\] (A.8)

where \(\nu = 1/[1 + \epsilon (\varphi + \alpha)/(1 - \alpha)],\)
\[
1 - \chi(s_t^q) = \frac{1}{1 + \nu \tilde{m}c_t(s_t^q)} = \frac{1}{\sum_{k=0}^{\infty} \prod_{j=1}^{k} \left(\theta(s_{t+j}^q) \beta\right) \sum_{n=0}^{\infty} \prod_{j=1}^{n} \left(\theta(s_{t+j}^q) \beta\right)},
\] (A.9)

and accordingly
\[
\chi(s_t^q) = \frac{\sum_{n=0}^{\infty} \prod_{j=1}^{n+1} \left(\theta(s_{t+j}^q) \beta\right)}{\sum_{n=0}^{\infty} \prod_{j=1}^{n} \left(\theta(s_{t+j}^q) \beta\right)}.
\] (A.10)

Alternatively, (A.8) can be written as
\[
p_t^*(s_t^q) = (1 - \chi(s_t^q)) \left[p_t(s_t^q) + \nu \tilde{m}c_t(s_t^q)\right]
+ (1 - \chi(s_t^q)) E_t \sum_{k=1}^{\infty} \prod_{j=1}^{k} \left(\theta(s_{t+j}^q) \beta\right) \left[p_{t+k} \left(s_{t+k}^q\right) + \nu \tilde{m}c_{t+k} \left(s_{t+k}^q\right)\right].
\] (A.11)

Leading (A.8) one period and taking time \(t\) expectations on both sides, yields
\[
E_t p_{t+1}^*(s_{t+1}) = (1 - \chi(s_{t+1}^q)) E_t \sum_{k=0}^{\infty} \prod_{j=1}^{k} \left(\theta(s_{t+j}^q) \beta\right) \left[p_{t+k+1} \left(s_{t+k+1}^q\right) + \nu \tilde{m}c_{t+k+1} \left(s_{t+k+1}^q\right)\right]
= (1 - \chi(s_{t+1}^q)) \frac{1}{\theta(s_{t+1}^q) \beta} E_t \sum_{k=1}^{\infty} \prod_{j=1}^{k} \left(\theta(s_{t+j}^q) \beta\right) \left[p_{t+k} \left(s_{t+k}^q\right) + \nu \tilde{m}c_{t+k} \left(s_{t+k}^q\right)\right].
\] (A.12)

Combining (A.11) and (A.12), one obtains
\[
p_t^*(s_t^q) = (1 - \chi(s_t^q)) \left[p_t(s_t^q) + \nu \tilde{m}c_t(s_t^q)\right] + \frac{1 - \chi(s_t^q)}{1 - \chi(s_{t+1}^q)} \left(\theta(s_{t+1}^q) \beta\right) E_t p_{t+1}^*(s_{t+1}^q),
\] (A.13)

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which making use of the fact that

\[
\frac{(1 - \chi(s^q_{t+j+1}))(1 - \chi(s^q_{t+1}))}{1 - \chi(s^q_{t+1})} = \frac{1 - \prod_{j=0}^{S-1} \left( \theta \left( s^q_{t+j} \right) \beta \right) \sum_{n=0}^{S-1} \prod_{j=1}^{n} \left( \theta \left( s^q_{t+j} \right) \beta \right)}{1 - \prod_{j=0}^{S-1} \left( \theta \left( s^q_{t+j+1} \right) \beta \right)} = \frac{1 - \prod_{j=0}^{S-1} \left( \theta \left( s^q_{t+j+1} \right) \beta \right) \sum_{n=0}^{S-1} \prod_{j=1}^{n} \left( \theta \left( s^q_{t+j} \right) \beta \right)}{1 - \prod_{j=0}^{S-1} \left( \theta \left( s^q_{t+j+1} \right) \beta \right) \sum_{n=0}^{S-1} \prod_{j=1}^{n} \left( \theta \left( s^q_{t+j} \right) \beta \right)} \frac{\chi(s^q_t)}{\theta(s^q_{t+1})} \beta, \tag{A.14}
\]

where the second line follows from the property that

\[
\prod_{j=0}^{S-1} \left( \theta \left( s^q_{t+j} \right) \beta \right) = \prod_{j=0}^{S-1} \left( \theta \left( s^q_{t+j+z} \right) \beta \right) \tag{A.15}
\]

for all \( z \) such that \( t + j + z \geq 0 \), can be written as

\[
p^*_t(s^q_t) = (1 - \chi(s^q_t)) \left[ p_t(s^q_t) + \nu \tilde{m} \tilde{c}_t(s^q_t) \right] + \chi(s^q_t) E_t p^*_t(s^q_{t+1}). \tag{A.16}
\]

The aggregate price level in the economy is given by

\[
P_t(s^q_t) = \int_0^1 \left( P_t(s^q_t) \left( 1 - \varepsilon \right) \right)^{\frac{1}{1-\varepsilon}} dt, \tag{A.17}
\]

which using the fact that only a fraction \( 1 - \theta(s^q_t) \) of the firms adjust their prices in period \( t \), can be written as

\[
P_t(s^q_t) = \left[ \theta(s^q_t) \int_0^1 P_{t-1}(s^q_{t-1})^{1-\varepsilon} dt + (1 - \theta(s^q_t)) \left( P^*_t(s^q_t) \right)^{1-\varepsilon} \right]^{\frac{1}{\varepsilon-1}} \tag{A.18}
\]

Log-linearization of the above equation, yields

\[
p_t(s^q_t) = \theta(s^q_t) p_{t-1}(s^q_{t-1}) + (1 - \theta(s^q_t)) p^*_t(s^q_t). \tag{A.19}
\]

The efficient allocation is obtained by letting the social planner maximize the household’s
utility function in (4), subject to the resource constraints

\[ C_{it}(s^q_i) = A_t(s^q_i) N_{it}(s^q_i)^{1-\alpha}. \] (A.20)

for all \( i \in [0, 1] \). Solving the optimization problem, one finds that the efficient level of output, \( Y^e_t(s^q_t) \), must satisfy

\[ Y^e_t(s^q_t) (\sigma + \frac{\varphi + \alpha}{1-\alpha}) = (1 - \alpha) A_t(s^q_t)^{\frac{1+\varphi}{1-\alpha}}. \] (A.21)

Log-linearization of the above equation, yields

\[ \left( \sigma + \frac{\varphi + \alpha}{1-\alpha} \right) \hat{y}^e_t(s^q_t) = \frac{1 + \varphi}{1 - \alpha} a_t(s^q_t). \] (A.22)

Combining (A.7) and (A.22), one obtains

\[ \hat{m}_{ct}(s^q_t) = \hat{\tau}_t(s^q_t) + \left( \sigma + \frac{\varphi + \alpha}{1-\alpha} \right) x_t(s^q_t), \] (A.23)

where \( x_t(s^q_t) = y_t(s^q_t) - y^e_t(s^q_t) \).

Appendix B

In this Appendix, the inequalities in Section 3.2 when \( \theta(1) < \theta(2) \) and \( \beta \in (0, 1) \) are proved.

B.1 Proof of \( \chi(1) > \chi(2) \)

The inequality can using (17) be written as

\[ \theta(2) \beta \left[ \frac{1 + \theta(1) \beta}{1 + \theta(2) \beta} \right] > \theta(1) \beta \left[ \frac{1 + \theta(2) \beta}{1 + \theta(1) \beta} \right] \] (B.1)

or rearranging as

\[ \theta(2) > \left[ \frac{1 + \theta(2) \beta}{1 + \theta(1) \beta} \right]^2 \theta(1). \] (B.2)

When \( \theta(1) < \theta(2) \), the squared term is larger than unity and the inequality is satisfied.
B.2 Proof of $\kappa(1) > \kappa(2)$

The inequality can using (19) be written as

$$\frac{(1 - \theta(1))(1 - \chi(1))}{\theta(1)} > \frac{(1 - \theta(2))(1 - \chi(2))}{\theta(2)}$$  \hspace{1cm} (B.3)

or rearranging as

$$\frac{\theta(2)}{\theta(1)} \frac{1 - \theta(1)}{1 - \theta(2)} > \frac{1 - \chi(2)}{1 - \chi(1)}.$$  \hspace{1cm} (B.4)

which using (17) can be written as

$$\frac{\theta(2)}{\theta(1)} \frac{1 - \theta(1)}{1 - \theta(2)} > \frac{1 + \theta(2)\beta}{1 + \theta(1)\beta}.$$  \hspace{1cm} (B.5)

Multiplying both sides of the above expression with $[(1 - \theta(2))(1 + \theta(1)\beta)]/\theta(2)$ and expanding the polynomials in the nominators of the resulting expression, one obtains

$$\frac{1}{\theta(1)} + \beta - 1 - \theta(1)\beta > \frac{1}{\theta(2)} + \beta - 1 - \theta(2)\beta$$  \hspace{1cm} (B.6)

or

$$\frac{1}{\theta(1)} - \frac{1}{\theta(2)} > \beta [\theta(1) - \theta(2)].$$  \hspace{1cm} (B.7)

When $\theta(1) < \theta(2)$, the left hand side is positive and the right hand side is, since $\beta \in (0,1)$, negative and the inequality is satisfied.

B.3 Proof of $\tilde{\beta}(1) > \tilde{\beta}(2)$

The inequality can using (20) be written as

$$\frac{1 - \theta(1)}{1 - \theta(2)} \frac{\chi(1)}{\theta(1)} > \frac{1 - \theta(2)}{1 - \theta(1)} \frac{\chi(2)}{\theta(2)},$$  \hspace{1cm} (B.8)

which using (17) and rearranging can be written as

$$\left[\frac{\theta(2)}{\theta(1)} \frac{1 - \theta(1)}{1 - \theta(2)}\right]^2 > \left[\frac{1 + \theta(2)\beta}{1 + \theta(1)\beta}\right]^2,$$  \hspace{1cm} (B.9)

which is always satisfied when $\theta(1) < \theta(2)$ since (B.5) is satisfied under same proviso.
Appendix C

Let $E_t \{p_{lt}(s_t^q)\} = \int_0^1 p_{lt}(s_t^q) \, dt$ denote the cross-sectional mean of (log) prices in period $t$. The cross-sectional dispersion of prices can then be written as

$$
\Delta_t (s_t^q) = E_t \{p_{lt}(s_t^q) - E_t \{p_{lt}(s_t^q)\}\}^2
= E_t \{p_{lt}(s_t^q) - p_t (s_t^q)\}^2
= E_t \{p_{lt}(s_t^q) - p_{t-1} (s_{t-1}^q) - \pi_t (s_t^q)\}^2
= \theta (s_t^q) E_t \{p_{lt-1}(s_t^q) - p_{t-1} (s_{t-1}^q) - \pi_t (s_t^q)\}^2
\quad + (1 - \theta (s_t^q)) E_t \{p_t^q (s_t^q) - p_{t-1} (s_{t-1}^q) - \pi_t (s_t^q)\}^2
= \theta (s_t^q) \Delta_{t-1} (s_{t-1}^q) + \frac{\theta (s_t^q)}{1 - \theta (s_t^q)} (\pi_t (s_t^q))^2,
$$

(C.1)

where the second line follows from the fact that up to a first order approximation $p_t (s_t^q) = E_t \{p_{lt}(s_t^q)\}$, and the second last line makes use of (A.19).

Iterating (C.1) backwards, the degree of price dispersion at time $t$ can—ignoring terms independent of policy—be written as

$$
\Delta_t (s_t^q) = \sum_{n=0}^t \prod_{j=n+1}^t \left( \theta \left(s_j^q\right) \right) \frac{\theta (s_n^q)}{1 - \theta (s_n^q)} (\pi_n (s_n^q))^2.
$$

(C.2)

Taking the discounted sum of these values, one obtains

$$
\sum_{t=0}^{\infty} \beta^t \Delta_t (s_t^q) = \sum_{t=0}^{\infty} \beta^t \sum_{n=0}^t \prod_{j=n+1}^t \left( \theta \left(s_j^q\right) \right) \frac{\theta (s_n^q)}{1 - \theta (s_n^q)} (\pi_n (s_n^q))^2
= \sum_{t=0}^{\infty} \beta^t \sum_{k=0}^t \prod_{j=1}^k \left( \theta (s_{t+j})^q \right) \frac{\theta (s_{t+k})}{1 - \theta (s_{t+k})} (\pi_t (s_{t+k})^q)^2
= \sum_{t=0}^{\infty} \beta^t \frac{\theta (s_t^q)}{1 - \theta (s_t^q)} (1 - \chi (s_t^q)) (\pi_t (s_t^q))^2
= \sum_{t=0}^{\infty} \beta^t \frac{1}{\xi (s_t^q)} (\pi_t (s_t^q))^2,
$$

(C.3)

where the third line makes use of (A.9).