Dynamic Banking with Endogenous Risk Based Funding Cost:

Value Maximization, Risk-taking, Responses to Regulation and Credit Contraction

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Abstract
We develop a stochastic dynamic model of bank value maximization under limited liability and in which bankruptcy can occur. Main issues are banks’ optimal responses to regulation and credit-losses. We show that risk-neutral banks behave as if they were risk-averse when they are under-capitalized. Risk-taking is always below that of single period value maximization under limited liability. We also show that banking regulations often have significant and adverse second-order effects through banks’ dynamic adjustment to regulations. The model gives rise to endogenous capital buffers and shows that it takes time to re-build bank capital after a credit-loss. That makes the model suitable to analysis of situations as the current post financial crisis period with large macroeconomic disturbances and credit contraction.

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1 Introduction

The financial crisis 2007-2008, which started with significant credit-losses in the sub-prime lending and considerable disturbances on the interbank credit market, made it clear that many banks did not have liquidity and capital enough to cushion such large disturbances and that the financial system is fragile.\(^2\) The crisis significantly reduced economic growth, world-wide. In many countries financial collapses were looming but were avoided only through government support of the banking industry, leading to soaring government budget deficits and even unsustainable debt levels in some countries. Such a crisis, of course, created a strong demand for policies that can prevent, or at least reduce the risk for, recurrence of banking crises.

The changes in regulations and policies were pushed through when regulators and policy makers were in a crisis mood and when they regarded it important that new regulatory measure were put in place rapidly. However, at that time the literature on banking regulation was– and still is- in an initial stage and mainly oriented towards micro prudential policies. The reason for that probably is that it was developed in the wake of the Savings & Loans crash in the late 1980’s in which excessive risk-taking was perceived as the main problem. There were no significant macroeconomic effects of that crisis. Risk-taking was certainly an issue also for the 2007-2008 crisis but as the crisis evolved it led to severe disturbances on the whole banking sector and significant macroeconomic effects that lingered on (and still lingers on) for several years. Analyses of such courses of events require other types of models than those used to explore micro-prudential regulation, namely truly dynamic models.

\(^2\) Acharya, Gujral, Kulkarni and Shin (2011) report that bank leverage, the ratio between assets and common equity, increased by more than 30 percent between the first quarter 2000 until the fourth quarter 2007.\(^2\) The increase was lower among government-sponsored enterprises, GSEs, such as Fannie Mae and Freddy Mac, only by about 6 percent but notably from an already high leverage level of approximately 40.
This paper develops a dynamic model of a bank. The model’s focus is on the relationships among bank capital, size of lending, risk-taking, dividend payouts and prolonged credit contraction. Essentially the bank borrows funds which it lends to borrowers. The cost of bank borrowing is endogenous. Bank capital serves as collateral for bank debt. The analysis is in the framework of an equilibrium model of a single bank that engages in non-scalable relationship banking.

Relationship banks represent a significant share of the banking industry. A bank that deals with relationship banking has got a range of potential borrowers who differ in terms of willingness and ability to repay loans. How far the bank goes on the risk/return ladder of potential borrowers determines the bank’s level of risk (compare Stiglitz and Weiss (1981)). In this paper borrowers differ only with regard to credit risk. In the short-run, credit risk is increasing in the number of borrowers. We refer to that as the bank is non-scalable. That is in contrast to banks who invest in traded assets. In that case scalability is probably a good approximation. An advantage of our approach is that a non-scalable bank has an optimum size -in contrast to scalable banks where the sizes of individual banks are indeterminate. Our approach allows analysis of bank’s responses to various external factors on the individual bank level.

Two types of asymmetric information are central for the analysis. The first is that the bank has an advantage over individual investors in evaluating proposals for loans implying that there is a margin between the return the bank earns on investments and the return individual investors would earn, should they invest themselves. The advantage comes, e.g., from greater experience in project evaluation, lower costs of project evaluation and monitoring. The second type is that it is very costly for individual investors to evaluate and monitor the bank’s current investment portfolio. Therefore they are not prepared to provide
new equity capital to the bank; instead they want to be first in line to get their reimbursement from profit and bank capital, if that is necessary.

Our analysis gives important results that cannot be expected to go away in general equilibrium analysis. Examples of such results are: banks keep endogenous capital buffers, banks behave as if they were risk-averse even if bank value is evaluated under risk-neutrality, the response of bank lending to credit losses is non-linear with only small adjustments of the size of lending to small shocks at the stochastic equilibrium while reductions in lending are larger than proportional to the size of shocks when shocks make banks significantly under-capitalized and it takes time to build capital buffers after significant credit losses. The latter means that there will be prolonged credit contraction after a significant negative shock to bank capital. The implication is that recapitalization of banks is an important element of macroeconomic policy when systematic shocks have made many banks significantly under-capitalized.

The paper is organized as follows. Section 2 gives a short description of models of banking in relation to the analysis in this paper. Section 3 presents empirical facts about banks’ liability and asset sides. Section 4 contains a description of the model as well as analyses of bank behavior under different conditions. First, the core dynamics of a bank’s value maximization problem is described and analyzed without any government regulation. We refer to that maximization problem as “A First-Best Case” since it involves no regulation. Second, we explore the maximization problem both with a restriction on dividend payouts and with capital adequacy regulation. We refer to those cases as “Second-Best Cases” since they involve government regulation, which impose additional constraints on the value-maximization problem. In order to obtain numerical results on the relations among bank buffer capital and dividend payouts and scale of lending, the interest rate the bank has to pay

3 The model in this paper is an adaptation and extension of the analysis is in Radner (1998) to banking.
on its borrowing is analyzed. In Section 5 we describe how we implement the model numerically in discrete time. We present results on banks’ value and their expected life-time under capital adequacy regulation. Finally, concluding comments are in Section 6.

2 Literature

The early literature on banks and financial intermediation had its focus on how banks are an endogenous response to frictions on financial markets. The current literature on banking and financial intermediation can broadly be said to have two branches; one explores theoretical foundations of banking and micro-prudential policies, while the other branch takes the existence of banks and intermediaries as given and deals with macro issues such as the role of credit in the business cycle dynamics and with macro-prudential regulation.


The other branch is a retake on the relation between financial markets and the real economy. Early significant contributions are Bernanke and Gertler (1989), Bernanke, Gertler and Gilchrist (1999) and Kiyotaki and Moore (1997). Those contributions deal mainly with establishing links between financial conditions and the real economy, thus exploring propagation and amplification of business cycle swings through financial markets. Such mechanisms are lacking in real business cycle (RBC) analyses, although it seems as if disturbances are not propagated and amplified, the real disturbances required to generate significant swings and persistent deviations from long-run equilibrium need to be both large and persistent. But large and persistent real disturbances are rarely observed. Bringing in frictions on financial markets into the analyses is a way to reconcile the conflict between relatively small and short-lived disturbances and significant business cycle swings.
The emphasis in the early analyses was on firms’ net worth and how net worth affects possibilities to raise capital for investment. The financial crisis of 2007-2008 has moved the focus from firms to banks and how banks’ balance sheets affect supply of bank loans. Recent contributions related to the topic of this paper are: Repullo and Suarez (2013), (RS), Hellman, Murdock and Stiglitz (2000), (HMS), Brunnermeier and Sannikov (2014), (BS), Phelan (2014), (P), Peura and Keppo (2006) and Peura (2003).

RS investigate the performance of the Basle I rules relative to Basle II rules. Basle I rules are more rigid than Basle II rules. The framework essentially is a two period model with three types of agents: Entrepreneurs that need financing, a bank/intermediary, and investors. Entrepreneurs finance their investment projects by borrowing from the bank and investors finance the bank through equity and debt capital. The outcomes of investment projects are stochastic. A crucial financial friction is that debt is only for one period while equity is for two periods. Debt is covered by deposit insurance which implies that the bank can borrow at the risk-free interest rate. Equity bears some risk and is therefore more costly than debt but there is a capital adequacy constraint which limits the amount of debt-capital the bank can raise.

At the outset there is no asymmetric information between investors and the bank about final investment projects, implying that the Myers and Majluf (1984) type of problem with equity financing does not appear and the bank can choose its capital structure so as to maximize present value of expected dividend payouts less financing costs. After one period the bank needs to refinance entrepreneurs, but at that moment, the bank has got better information than investors regarding the profitability of ongoing investment projects and the bank can raise only debt capital. The implication is that the repayment to the bank after the first period determines how much equity capital remains. That puts a limit on how much debt-capital the bank can raise and how many investment projects it can finance in the second
period. If credit losses are very large, the bank goes bust, if credit losses are significant, a small number of projects can be financed staying within the bound of the capital adequacy regulation and if losses are minor, a large number of investments can be financed and the bank may even be able to pay dividends. An interesting result is that banks raise more equity capital in the first period than necessary, i.e., they build an endogenous capital buffer in order to be less affected by potential credit losses and to not have to pass on highly profitable investments in the second period. In this paper the bank has buffer capital because it reduces the bank’s borrowing cost. On the different Basle rules the RS result is that the swings are more pronounced under the risk-based Basle II capital adequacy rules than under the flat Basle I rules. However, Basle II rules are better in bringing down bank failures than Basle I rules.

The objective of the contribution by HMS is to explore policies to prevent banks from opportunistic behavior, i.e., invest in risky assets that are only profitable because of limited liability. In their model banks’ external financing is government insured deposits and banks maximize expected present value of dividends. Competition for funds leads to lower franchise value, i.e., the value of owning a bank, which will induce banks to invest in the risky asset rather than safe lending. HMS show that a capital adequacy constraint may solve the problem since such a regulation forces banks to have more “skin in the game”, but it may be a costly solution. A less costly solution is to use a combination of capital adequacy regulation and a cap on the interest rate banks can offer to depositors. The latter measure counters the loss of franchise value and therefore the capital adequacy can be somewhat less strict which reduces banks’ cost of capital. A feature of the model is that it lacks dynamic effects from actions today on tomorrow’s decision: investing in the risky asset and the realization of the bad state lead to immediate bankruptcy. Therefore the model cannot be used
to study situations such as the current where banks are undercapitalized over a prolonged period of time.

BS and P work with somewhat similar dynamic stochastic general equilibrium models, with a financial friction, to explore endogenous systemic risk. The friction is that there are agents, experts in BS, and banks in P, which cannot issue equity. Both models are scalable in the sense that levels of size and wealth are not essential. The evolutions of central variables such as productivity, price of capital and the constrained agents’ marginal value of wealth and bank capital are expressed as stochastic growth rates which are affected by a common disturbance factor.\(^4\) In both models there are risk-neutral households with endowments of capital that they can use for the production of consumption goods. The constrained agents are more productive than the households, from which they borrow funds. Experts’ and banks’ leverage is affected by the marginal value of wealth and bank capital; high marginal value leads to low expert and bank borrowing and investment. Hence, the constraint on equity issuance has an efficiency cost.

Inclusion of intermediaries represents a novel component of dynamic stochastic general equilibrium models. The optimization problem solved by experts in BS and banks in P, coincide in the form with bank optimization in this paper.\(^5\) Technically, it amounts to finding optimizing strategies for experts’ consumption and leverage depending on expert wealth (equity capital). The trade-offs are as follows. Current consumption erodes expert wealth and lowers levels of future consumption. Increased leverage increases expected flow of net revenue but it also increases net revenue volatility which is negative. The trade-offs in P regarding banks are similar.

\(^4\) The growth rates are in terms of geometric Brownian motions with a common Wiener process.

\(^5\) A difference is that the evolution of bank capital in Peura and Keppo (2006) and in this paper is not in growth rates. Technically, it is represented by arithmetic Brownian motions.
From a modeling point of view one may note that the dynamic stochastic utility maximization problem for experts in BS and banks in P in which leverage or scale of investment and dividend payouts are chosen simultaneously, is not solved explicitly. In contrast, in this paper we solve such an optimization problem with both leverage and dividend payouts as controls while BS and P get around the problem by introducing an artificial variable, marginal value of expert wealth, in the BS case and marginal value of bank capital in P. That variable is like a price of experts’ wealth and bank capital, although expert wealth and bank capital are not marketed, which -in fact- is the basic friction in those models. The marginal values of expert wealth and bank capital are equilibrium outcomes to which experts and banks adjust. The interpretation of that variable is not obvious since it should also be the outcome of expert and bank optimization.\(^6\)

The forms of the evolvement of price of capital and marginal value of experts’ wealth or bank capital, is geometric Brownian motions in which parameters are functions of aggregate experts’ wealth and bank capital. That gives a very strong result, namely that the volatility of the price of capital is sensitive to the distribution of wealth and capital. When experts’ net worth is low, volatility of the price of capital is low and increasing in experts’ net worth. At some level of net worth, and above that level, experts hold all capital. The volatility of the price of capital decreases from the minimum level of experts’ net worth at which experts hold all capital, up to the level of experts’ net worth where experts consume all additional wealth. When experts’ share of wealth is below the latter level, negative productivity shocks give rise to feedback effects through decreased price of capital. That is, systemic risk is high at intermediary levels of expert wealth or bank capital. It also implies that the non-linearity of responses to shocks comes entirely from general equilibrium effects.

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\(^6\) One issue is how experts and banks learn about their marginal value of wealth and bank capital, to which they are assumed to adjust. Another issue is whether the marginal value of wealth and bank capital coincides with marginal value of wealth and bank capital as obtained from the “first principle” expert and bank maximization problems.
on the price of capital and marginal values of expert wealth or bank capital. Increased
wariness by experts and banks to take on risk plays no role in the BS and P analyses. That is,
they explore only the general equilibrium channel for how credit losses affect lending and
investment. In this paper, that channel is not present but results regarding non-linear responses
to credit losses are still similar which implies that, in this paper, those results come solely
from bank optimization.

stochastic optimization. They explore dividend and recapitalization strategies of a bank with a
stream of profit which evolves as an arithmetic (in contrast to a geometric) Brownian motion
with fixed drift and volatility parameters. Bank capital is eroded by dividends and credit
losses. The bank goes bankrupt when bank capital reaches zero. The first part of the problem,
the optimal dividend strategy, follows the analysis by Jeanblanc-Picqué and Shiryaev (1995).
When dividends can be paid out in discrete amounts, the strategy is to not pay dividends when
bank capital is below a certain threshold, and to pay dividends so as to never exceed that
threshold. The recapitalization part is the novel contribution. For that part of the problem
there are two frictions. The first is that it takes time to raise new equity capital and during that
period of time, the bank is unable to pay dividends. The second friction is that there is an “ice-
berg” cost to equity issuance. The larger the issue the more costly it is. The solution in that
part then is never to let bank capital fall short of a threshold level and whenever bank capital
break through that level, issue new equity so as to restore that level of capital.

The analysis in this paper deals with a bank that simultaneously choses size of
lending and dividend payouts so as to maximize expected present value of dividends. A
difference from the reviewed recent contributions is that the bank’s borrowing cost is
endogenous and sensitive to the level of bank buffer capital. There is no deposit insurance
available which reflects the situation of banks that finance themselves on the wholesale
market. In relation to BS and P, this paper solves the bank’s first principle maximization problem, i.e., maximization of expected present value of dividends which allows exploration of bank response to policy such as regulation at the individual bank level. In relation to Peura and Keppo (2006) and Peura (2003), the size of the bank’s balance sheet is explored in this paper. The mechanism for an endogenous capital buffer is in this paper different from that in RS. In relation to HMS the analysis here is truly dynamic, allowing banks to be far from optimally capitalized for long periods of time. Several results, in this paper, are obtained through simulations in discrete time. By varying the length of time periods we obtain results on effects of maturity mismatch, i.e., borrowing short and lending long.

3. Empirical background

Banks’ level of risk is intimately linked to the composition of their balance sheets. The risk on the asset side stems from the quality of assets in terms of credit risk. The risk on the liability side lies mainly with the substantial liquidity risk due to refinancing needs. The composition of liabilities also determines the effects of regulations, deposit insurance and expectations on bail-outs. Were a bank to borrow only in the form of deposits, eligible for deposit insurance, asset risk would not have a significant effect on the bank’s borrowing costs. However, banks do not only borrow in that form. Figure 1 displays the composition of European banks’ liabilities.

The aggregate data on European bank liabilities 2008 – 2013 reveals that deposits from the public accounts for 35 – 45 percent of total liabilities. Large shares of European banks’ borrowing are therefore not covered by deposit insurance and lenders would have to rely on banks’ risk management and collateral, i.e., secured debt, or on government bailout policies. If and how a bailout will take place cannot be known with certainty. Banks’ risk managements and the value of collateral are also uncertain factors. Lenders therefore require risk premiums on lending to banks.
Figure 1: European banks’ deposits to other credit institutes, their outstanding debt certificates, deposits from the public and their equity in the years 2008 through 2013. The “other” field is just the residual between total liabilities and the sum of the specified posts.

The main part of bank liabilities are funds borrowed at the market which means that there are risk premiums on banks’ funding on the market that depend on the banks’ risk-level, and the risk in the banking system. According to the aggregate ECB data depicted in Figure 1 roughly 20 percent of liabilities are publicly traded bank certificates. Figure 2 shows the difference between different Swedish bank\(^7\) certificates and the 3-month T-bill. The graph shows two important things. First, banks pay more than ten basis points in risk-premium relative the Swedish government. Second, there is a considerable variation of risk-premiums within the banking sector. The difference between the lowest and the highest is on average

\(^7\) Swedish banking market is much consolidated with only four major banks and two minor that borrow on the Certificate market. SHB who is the bank that borrows cheapest by wide margin is the Swedish bank that by far cleared the financial crises the best not even cutting back on dividends.
more than seven basis points. Given this empirical evidence, the bank’s funding cost, in this paper, is sensitive to the bank’s risk-taking.

Figure 2: Swedish banks’ borrowing rate on their 3-month Certificate and Swedish 3-month T-bill rate.

Another interesting feature of banks’ balance sheets is the relationship between cash and equity. If equity is going to be used to cushion credit losses it cannot be tied up in illiquid assets. Figure 1 reveals that banks keep substantial amounts of cash in their asset portfolio. The data from ECB shows that in the aggregate, European banks’ ratio of cash to equity after Lehman was a little over 40 percent. Since then the ratio has increased to levels in the range of 70 – 80 percent, which is shown in Figure 3. One possible interpretation of the aggregate holdings of cash to equity of European banks is that banks have used cash to cover for the more expensive liquidity after the Lehman crash.
In the model in this paper we assume that 100 percent of equity is kept in cash which is a slight exaggeration but not very far from reality. However, the assumption simplifies the model. It would perhaps be reasonable to assume that equity may be stored in T-bills at the risk-free interest rate, but whether banks get a zero return or the risk-free return does not affect the model significantly.

4. Model

4.1 Value maximization –First-Best Case

Consider a bank that borrows funds and uses the borrowed funds for lending and trading which we refer to as bank investment.\(^8\) The bank has bank capital, \(k\), that is, common equity and retained earnings, which it uses as collateral for bank debt. We assume that bank capital is

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\(^8\) The existence of banks could e.g. be justified through CSV. Our banks can go bankrupt which means that the model from for example Williamson (1986) needs to be expanded to allow correlation between agents as in Larsson (2010). Another possibility is that banks are audited by a regulator, and that investors monitor the regulation and its actions rather than the bank itself. This latter explanation seems to be present in banking markets in India, Iyer, Puri and Ryan (2012).
not part of bank investment and it can only be held as cash.\(^9\) The return on bank investment is proportional to the size of investment, \(z\), but uncertain. The degree of uncertainty is increasing with the size of investment. The stochastic disturbances are normally distributed. Increasing uncertainty limits the size of the bank.\(^ {10} \)

We assume that the bank’s gross cost of borrowed funds increases in its debt and that it is non-increasing in the amount of bank capital. Moreover, for each level of bank capital the gross borrowing cost is a convex function of bank debt/investment. Hence, gross borrowing cost (principal and risk-adjusted interest) is denoted:

\[ \rho(z, k) \text{ where } \rho_z > 0, \rho_{zz} > 0 \text{ and } \rho_k \leq 0. \]

In the simulations below, we determine \( \rho \), endogenously.

The bank’s owners maximize the expected present value of all future dividends \( w_t, t \in [0, T] \), from the bank, from the starting point at time \( 0 \), with equity capital \( k_0 \), up to the point where there remains no equity capital, i.e., the point at which the bank goes bankrupt, \( T \), if that ever occurs. We write the value of the bank, \( V \), as follows.

\[ V(k_0) = E_0 \int_0^T w_t e^{-\mu t} dt, \quad (1) \]

where \( E_0 \) is the expectation operator and \( \mu \) is the risk-free rate of discount and

\[ T = \inf \{ t | k_t \leq 0 \}. \quad (2) \]

That is, \( T \) is the first time \( k_t \leq 0 \) which is the time of bankruptcy. We also impose the conditions that

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\(^9\) Allowing for bank capital to be invested in risk-free assets would alter the analysis slightly but the essence of the results would be unchanged. Another potential alternative strategy for a bank would be to use bank capital for its lending. However, that reduces the benefits from leverage. We show in Appendix that the expected profit with that strategy is in fact dominated by the levered strategy using equity as collateral.

\(^ {10} \) An alternative modeling strategy to limit bank size would be to assume that lending/investment risk grows at a constant pace but that the demand curve for loans is downward sloping. Yet another possibility is to assume that the internal administrative cost for providing credit is increasing and convex in the size of lending/investment. The strategy chosen here emphasize uncertainty of bank returns. We believe that is an important component in the short- and medium range.
\[ V(0) = 0 , \] (3)

which is the bank’s limited liability constraint. In addition we impose the condition that the value of the bank should not be smaller than bank capital. Hence, \( V(k_t) \geq k_t \). If bank capital at any moment in time would exceed the value of the bank, the bank’s owners would immediately, at no cost, liquidate the bank.

The evolution of equity capital \( k_t \) is described by a controlled Brownian motion.

\[
dk_t = \{m(z_t,k_t) - w_t\}dt + \sqrt{v(z_t)} \times d\epsilon_t
\] (4)

Where (i) \( \{m(z_t,k_t) - w_t\} \) is the drift and \( m(z_t,k_t) \equiv Rz_t - \rho(z_t) \) in which \( R \) is defined as \( R = 1 + r \) where \( r \) is the expected (but uncertain) rate of return on bank investment. For each \( k \), \( m(z_t,k_t) \) is a given concave function of \( z_t \). (ii) \( v(z_t) \) is the variance of return to bank investment and \( d\epsilon_t \) is the normally distributed stochastic disturbance to the return to bank investment. We assume that \( v_z(z_t) > 0 \) and \( v_{zz}(z_t) > 0 \), i.e., the variance of the return on investment is increasing at an increasing rate with the scale of bank investment.¹¹ Note that since bank capital, \( k_t \), evolves stochastically, the time at which the bank goes bankrupt, \( T \), has a probability distribution.

Note that \( z_t \) and \( w_t \) are controlled by the bank and they represent its tools for maximizing the value on currently invested capital \( k_t \). We assume that the bank cannot receive capital injections. That is \( w_t \geq 0 \), (compare Myers and Majluf (1984)).

The Bellman principle gives the following necessary condition for the value of the bank, the scale of the bank’s assets and dividends.

\[
\mu V(k_t) = \max_{z_t, w_t} \{(1 - V'(k_t))w_t + mV'(k_t) + \frac{1}{2}v(z_t)V''(k_t)\}
\] (5)

¹¹ Technically there is nothing that binds the uncertainty to bank revenue. It is therefore possible to interpret the uncertainty as interest rate uncertainty which is a relevant issue for inter-bank markets.
It follows that the optimal choices of $z_t$ and $w_t$ only depend on $k_t$. The actual time elapsed lacks significance. In the following we therefore omit the time index.

In order to characterize the solution to the maximization problem we first note that $V(k) \geq 0$ and that $V'(k) \geq 0$. We continue by exploring the optimal dividend policy. Note that if $V'(k) > 1$, $w = 0$ is the optimal choice and if $V'(k) < 1$, $w$ would be chosen as large as possible. If the stochastic disturbance instantaneously increases bank capital up to a point where $V'(k)$ hypothetically would be below 1, the optimal dividend policy is to instantaneously pay a large dividend so that $V'(k) = 1$. That implies that bank capital never remains larger than the level at which $V'(k) = 1$. We refer to that level of bank capital as $b$.

Jeanblanc-Piqué and Shiryaev (1995) and Radner (1998) refer to a dividend policy whereby no dividends are paid out when capital is below a certain level and as much as possible is paid out when capital is above that level, as an overflow policy. The intuition for such a dividend policy is simple: if the value of the bank increases by less than the increase in equity capital it is better to give the capital back to the owners.\footnote{Tobin’s q is less than one.} Below we return to the characterization of $z$ and $b$.

Having established that for $k \leq b$, $w = 0$ is the maximizing choice we continue by characterizing the optimal sizes of bank investment. Hence, for $k \leq b$, the optimal choice of size of bank investment needs to satisfy the following condition.

\[
m_z V'(k) + \frac{1}{2} v_z(z) V''(k) = 0 \tag{6}
\]

Using (6) in (5) we obtain,

\[
\mu V = m V' - \frac{m_v}{v_z} V', \tag{7}
\]

where $z$ satisfies (6) or more conveniently written,
\[ \mu V = h(z(k), k) V', \quad (7') \]

where \( h(z(k), k) = m(z(k), k) - \frac{m_z(z(k), k)}{v_z(z(k))} v(z(k)) \). Assuming that the value function \( V \) is concave and noting that \( \frac{m_z}{v_z} = \frac{1}{2} \frac{V''}{V'} \), \( h \) may be given an interpretation as the certainty equivalent drift. That is, the non-stochastic drift that has the same value to bank owners as the stochastic drift of bank profit. We assume that \( \mu - h_k(z(k), k) > 0 \). That is, the rate of discount is larger than the gain in certainty equivalent drift as bank capital increases. Note that \( (7') \) holds for every \( k < b \). We therefore have,

\[ \mu V' = h V'' + h_z V' \dot{z}(k). \quad (8) \]

By using (6) one more time to eliminate \( V'' \), we obtain the evolution of bank investment as a function of bank capital.

\[ \dot{z}(k) = \frac{2h \frac{m_z}{v_z} - h_k + \mu}{h_z}. \quad (9) \]

Note that (9) does not involve the unknown function \( V(k) \) and the unknown value \( b \). The solution to (9) is obviously key to solving for the optimal \( V(k) \). Note, that \( V'(0) > 1 \). It therefore follows from \( (7') \) that the initial condition \( V(0) = 0 \) is satisfied only if \( h(z(0), 0) = 0 \). That in turn implies that the optimal \( z(0) \), i.e., the initial condition that together with (9) determines \( z(k), k < b \), maximizes \( \frac{m}{v} \). That is, \( z(0) \) maximizes the risk standardized current period profit at zero bank capital. That is clearly different from the optimum choice of size in

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13 An alternative way to calculate bank value is to use \( V(k) = \int_0^k V'(s) ds \).
a static model with a limited liability constraint.\textsuperscript{14} \(z(0)\) together with (9) traces out \(z(k)\) for \(k < b\). That also gives the equilibrium level of bank capital in relation to bank lending, \(k/z(k)\) for \(k < b\).

Next step is to characterize the optimal bank value. It follows from the optimal dividend policy, the optimal sizes of bank investment and the stochastic equilibrium value of bank capital, \(b\). We proceed as follows. From (6) we obtain,

\[
\frac{V''(k)}{V'(k)} = -\frac{2m_z}{v_z} \equiv H(z(k),k),
\]

which has the solution

\[
V''(k) = e^{-\int_k^b H(z(s),s)ds} \quad \text{for} \quad k \leq b. \tag{11}
\]

\(V'(k)\) is the marginal value of bank capital in this model; compare marginal value of expert wealth in BS and marginal value of bank capital in P.

Note, (i) that (11) implies \(V'(k) > 1\) as long as \(-\int_k^b H(z(s),s)ds > 0\). We have already established that \(V'(b) = 1\) at the capital level where it is optimal to commence to pay dividends. (ii) Note that for every \(k < b\), \(V'(k)\) increases in \(b\) up to the value of \(b\) where \(H(z(k),k)\) changes sign from being negative to positive. Hence, (ii) implies that the value-maximizing \(b\) should satisfy the following equation. \(H(z(b),b) = 0\).

An observation: \(H(z(k),k) = 0\) only if \(m_z = 0\), which is that the current period drift term in (5) is, maximized. That in turn (from (6)) implies that \(V''(b) = 0\) and \(h(z(b),b) = m(z(b),b)\). The latter yields, \(V = \frac{m(z(b),b)}{\mu}\), the value of the bank when it is optimally capitalized. That is, the bank’s value is the present value of the drift when the bank

\textsuperscript{14} Maximization of expected current period profit under limited liability amounts to finding the \(z\) that maximizes \(E \max \left\{ 0, k + Rz - \rho(z,k) + v(z)de \right\}\) which is the end of period expected wealth.
is optimally capitalized. The value of the bank for \( k > b \) is:

\[
V(k) = (k - b) + \frac{m(z(b), b)}{\mu}.
\]

For \( k \leq b \) the bank’s value is given by (7).

At levels of equity capital below \( b \) the bank behaves as if it were risk averse, although its valuation of future earnings actually is risk neutral. That means that maximizing the one period problem will not be optimal unless the discount factor goes to zero (the rate of discount goes to infinity). It is optimal to take less risk today in order to raise the probability to “be in the money” tomorrow in order to get future dividends.

Sensitivity of credit volume to shocks to bank capital is given by (9) for bank capital \( k < b \). As will be shown below, in our numerical analysis, \( z(k) \) is an increasing concave function in \( k \), implying that the sensitivity to shocks is lower the closer is \( k \) to \( b \). The intuition for that result is fairly straightforward: for levels of bank capital close to \( b \) the bankruptcy risk is small and a negative shock on bank capital does therefore not need to be countered by a large decrease in the credit volume. On the other hand when bank capital is low the bankruptcy risk is more pronounced and the bank needs to counter a negative shock to bank capital with a relatively large decrease in the credit volume.

A conclusion is that at the level of capital where the bank pays dividends, it adjusts its balance sheet so as to maximize expected earnings, without regard to its own limited liability constraint. Bank limited liability enters in two ways: (i) through bank value where zero is a lower bound. Should bank owners have to fully shoulder bank losses, the bankruptcy value of the bank would be lower and that would lower bank value for all levels of bank capital but it would not directly affect size of lending and choice of capital buffer. (ii) Through the cost of bank borrowing: If external lenders realize that the bank will not service its debt, i.e., pay principal and interest, in all states of the world, rational investors demand a risk-premium, above the risk-free return, on the return from lending to the bank. Through the
latter avenue the bank’s risk-taking is held back. In contrast to one-period models there will be no direct effect on risk-taking of bank limited liability.

Sources of bank excessive risk-taking –if one believes that banks take inappropriately large risks- should probably be sought in mispriced bank borrowing resulting from deposit insurance schemes in which banks’ pay insurance premiums that do not reflect risks appropriately, in government explicit or implicit bail-out rules that lower bank investors’ risk and in corporate governance of banks where managers’ incentives may be tilted towards excessive risk-taking and misaligned with owners’ interests.

It is easy to see that in case the government pursues a full bailout policy to avoid bank-runs, this leads, in the model in this paper, to extreme risk-taking since the marginal profit on lending is constant. Hence, banks will want to lever up without bound. That mimics the result in HMS where they assume that banks’ lenders are insured.

4.2 Restriction on Bonuses and Dividends –A Second-Best Case

A type of regulation that has been considered and also will be implemented within the Basle III regulation is constraints on dividends and bonuses. While it is obvious that such constraints prevent erosion of bank capital in situations where bank capital has fallen significantly, an issue is how such a regulation might affect the level of capitalization at which banks choose to pay dividends. To explore that issue without making the analysis too complex, we impose a constraint on the rate of dividend payouts which is not contingent on the bank’s leverage or capital adequacy. The regulation considered does not exactly mirror the Basle III regulation but results highlight an important result namely that constraints on payment of dividends and bonuses will have second-order effects, which can be significant. In our analysis the bank will hoard less capital, i.e., the desired level of buffer capital will be reduced, and earn less since the return to capital will necessarily be diminished.
We write the bank’s value maximization problem as that in the first-best case but with the following additional constraint.\(^{15}\)

\[
w \leq \bar{w}
\]

That is, in the continuous time model, over-capitalization can only be reduced over a period of time. If we would have considered a discrete time model the constraint corresponds to a cap, each period, on dividends and bonuses.

The mathematical problem changes slightly when there is such a restriction on the maximum size of dividends. Equation (7') changes to:

\[
\mu V_i = \bar{w} + \{h(z(k), \bar{w}) V_i', i = 1, 2
\]

The subscript \(i\) denotes bank value in the two regions: subscript 1 where the bank does not pay dividends and 2 where the bank pays dividends. That subscript convention is also followed for the size of bank investment, \(z\).

Suppose the threshold for bank capital between not paying dividends and paying dividends is \(b^e\). Bank value will evolve according to (5), with \(w = 0\) when \(k < b^e\) and with \(w = \bar{w}\) when \(k \geq b^e\).\(^{16}\) In accordance with that, bank investment, \(z_2(k)\) for \(k > b^e\) develops according to the following differential equation.

\[
\dot{z}_2(k) = \frac{2(h - \bar{w}) \frac{m}{V_z} - h_z + \mu}{h_z}
\]

The optimal switching point \((b^e)\) between not paying dividends and paying dividends and bonuses remains to be characterized.

---

\(^{15}\) Within the context of the model in this paper where there is no other explicit production factor than bank capital there is no distinction between dividends and bonuses since. However, in a richer model with banking expertise as an additional production factor, an issue is whether bonuses should be a part of the capital share of total return or if it is best viewed as part of return to labor.

\(^{16}\) In the continuous time case the constraint is always binding for \(k \geq b^e\).
First, recall that the switching point satisfies $V_1'(b^c) = V_2'(b^c) = 1$. It is also the case that $V_1(b^c) = V_2(b^c)$. From (7') it then follows that:

$$h(z_1(b^c), b^c) = h(z_2(b^c), b^c) \Rightarrow z_1(b^c) = z_2(b^c).$$

For a particular initial condition, $z_2(b^c)$ for the differential equation (14), located along the $z_1(k)$ path, three things can happen:

1. When $z_2(b^c)$ is sufficiently large $z_2(k)$ will cross $z^*(k)$, where $z^*(k) \equiv \arg \max_z \{m(z,k)\}$, at a level of bank capital which is economically meaningful. Note that $z_2(k)$ above $z^*(k)$ is not optimal since it implies drift and volatility combinations which are dominated by $\{m(z^*(k), k), v(z^*(k))\}$.

2. For a sufficiently low $z_2(b^c)$, $z_2(k)$ will follow a declining path and reach zero for a finite $k$. Such a path is not optimal.

3. For some intermediary initial condition, $z_2(b^c)$, $z_2(k)$ never hits zero and reaches $z^*(k)$ at a level of bank capital where $V_2(k) \leq k$. Such an initial condition and a path for $z_2(k)$ satisfies the Bellman equation and is in fact optimal if $z_2(k)$ reaches $z^*(k)$ at a level of bank capital for which $V_2(k) = k$.

Note that the stricter the dividend constraint, the lower is the capital level at which the bank pays dividends. Intuitively, when dividend payments are small the bank should also be small. When the dividend constraint is very loose, close to allowing the reduction of excessive capital instantaneously, the capital level at the switch-point between not paying and paying dividends, should be close to the first-best level.

To illustrate the optimal controls and the optimal value function we solve the differential system numerically and plot the optimal paths of size of lending/investment as
functions of bank capital. In order to do that we need to specify the dependence of the bank’s borrowing cost on the size of its lending and bank capital.

4.3 Capital Adequacy Constraint

A pure capital adequacy regulation stipulates a lower bound on the ratio of reserve capital, measured as common equity tier 1 capital, and risk weighted assets. As our model is a bare-bone model with no other distinction between bank liability than that between equity capital and retained earnings, and debt, and with only one type of asset, the constraint takes the following form.

\[
\frac{k}{z} \geq \gamma, \tag{17}
\]

where \( \gamma \) is the capital adequacy requirement. Perfect maturity matching between assets and debt implies that banks will never violate the capital adequacy constraint since cutting down on lending and investment is feasible when bank capital has suffered a negative shock.

Hence, maximum bank value satisfies (5) given no violation of (17). It is easy to see that for the case where (17) is non-binding the solution for dividends and bank investment coincides with the unconstrained case. When (17) binds bank investment becomes proportional to \( k \).

4.4 Numerical Solutions

4.4.1 Equilibrium Cost of Bank Funds

A first step to obtain numerical solutions is to calculate the equilibrium interest rate or funding cost the bank has to pay on borrowed funds. The central argument in the Meyers and Majluf (1984) complication of obtaining outside and also inside equity funding is that investors who are not directly involved in the bank’s management has limited information about the bank’s prospective profitability and they may therefore be worried that they are
offered conditional repayments that are favorable to the bank but not to themselves’. Being first in line to be repaid and to obtain a fixed repayment, if that is feasible, are means to reduce the problem with asymmetric information. However, the risk-premium required by outside investors is still an issue.

As a benchmark, we assume that there is no information asymmetry between the bank and its lenders regarding the current period situation. Hence, lenders and the bank have the same beliefs about the bank’s borrower’s repayments and the bank’s ability to repay its lenders. We also assume that the bank’s lenders require a risk premium that equalizes the expected repayment by the bank to the return on risk-free lending. The latter assumption could easily be exchanged for some other level of return.

We explore the statutory gross repayment, $\rho$ for a loan, $z$ taken by the bank at time $t$ and repaid at time $t$. At time $t$ the bank receives the expected return $z e^r t$ on its investment. The return on investments is uncertain and the deviation from the expected value is normally distributed, $N(0, \nu)$ where $\nu = \nu(z) t$ is the variance. The amount of equity capital, which serves as collateral for the debt, is $k$. Hence, $\rho$ satisfies the following equation:

$$
\frac{1}{\sqrt{2\pi\nu}} \int_{-k}^{\rho-k} (k + x) e^{-\frac{(x-k)^2}{2\nu}} dx + \frac{1}{\sqrt{2\pi\nu}} \int_{\rho-k}^{\rho} e^{-\frac{(x-k)^2}{2\nu}} dx = ze^r t
$$

(15)

The first term on the left hand side is the repayment under bankruptcy weighted with the probability of bankruptcy. The second term is statutory repayment multiplied with the probability that the bank can service its obligations and the right hand side is the return on a risk-free investment.

It is clear that, for fixed values of $\mu$ and $r$, and a given function $\nu(z)$, $\rho$ is a function of $z$, $k$ and $t$. That is, $\rho = \rho(z,k,t| r,\mu,\nu(z))$. One observation can be made, namely if $\nu = \sigma^2 z t$, that is if the variance is proportional to the size of bank investment, then $\rho$ is
homogeneous of the first degree in \( z \) and \( k \). That would imply that the bank’s value maximization problem has no solution. Here, we assume that the volatility \( v(z) \) is a strictly convex function of \( z \).

There is no explicit solution for (15) in terms of \( \rho(z,k,t) \). Therefore, we solve for the borrowing-cost numerically. We calculate the \( \rho(z,k) \) function for a loan of duration \( t = 1 \) so as to capture the risk in lending to the bank and to get risk-premiums at the same magnitudes as the risk-free rate of interest \( \mu \) and the gross return to lending/investment, \( r \). The equation used in the calculations is presented in Appendix.

In the numerical solutions to (15) and in the numerical analysis of the bank’s value maximization problem we assume that the variance of the repayment is the following explicit function:

\[
v(z) = \beta^2 e^{2\alpha z}.
\] (16)

Parameter values \( \beta \) and \( \alpha \), are chosen such that the variance does not increase too fast with lending and that there is some variance for very low values of lending. It would have been natural that the variance is zero at zero bank investment. However, without some variance for arbitrary low levels of lending, problems with existence might occur since the starting value for bank investment is the value that maximizes the ratio of drift to variance. Hence, zero variance is not a tractable feature. We view the assumption of a positive variance at \( z = 0 \) as similar to some fixed cost. That is, even if the bank lends to a very few, it would be required to have some over-head costs, leading to banks starting at larger scale than they would otherwise. Having a strict positive variance service the same purpose: banks’ size will not be arbitrarily small.
Figure 4: The drift $m(z,k) = Rz - \rho(z,k)$

Figure 4 shows the equilibrium net drift, that is the difference between expected bank revenue and gross cost of bank debt, with parameter values $\alpha = 0.1$, $\beta = 0.08$, $r = 0.045$ and $\mu = 0.03$. That is, the bank’s margin between risky rate of return and the risk free interest rate is 1.5 percentage points, which is not an unrealistic margin. With the parameters set, the drift is relatively insensitive to equity at low levels of lending. This is due to the fact that at low levels of lending the variance is low and most losses can be covered by the margin on borrowers that fulfills their contracts. Along the “ridge” with maximum drift lending, the effect from equity successively gets rather substantial.
4.4.2 Base-line Solution, First-Best Case

In order to illustrate the solution in the first-best case we calculate $z(k)$ and $z^*(k)$, where the latter is the solution to $m_z(z(k), k) = 0$ for different $k$’s. Hence, $z^*(k)$ represents bank profit maximizing lending, were it only interested in current period profit and not forward looking. The base-line parameters are: $\alpha = 0.1$, $\beta = 0.08$, $r = 0.045$ and $\mu = 0.03$.

![Optimal Lending Graph](image)

Figure 5. Optimal lending in first-best situation and under maximization of current profit.

Figure 5 illustrates the desired level of bank capital $b^*$ and that when the bank is under-capitalized it lends less than were it only to maximize current period profit. Hence, although bank value is the risk-neutral present value of dividends, the bank behaves as if it were risk-averse. The reason is that it factors-in the risk of losing future dividends if it takes-on too much risk.
4.4.3 Sensitivity Analysis of First-Best Case

To shed light on the dynamics of a bank maximization we here perform a series of sensitivity analyses: changing the bank’s net expected return on its lending \(r\) i.e. we vary the relationship banking premium, changing the level of risk in the relationship banking market \(\beta\), and changing the marginal value of banks’ private information \(\alpha\).

**Expected return on lending**

Increased \(r\), not only increases banks expected return on its lending but also lowers its borrowing cost since larger losses can be covered by current profit on performing loans. Varying \(r\) from 3.25% to 5.75% to analyze banks optimal controls yields Figure 6.

![Images of graphs showing sensitivity analysis](image)

**Figure 6:** Sensitivity of bank investment, optimal bank capital, bank profit and profit per risk unit when changing banks expected lending rate \(r\) from 3.25% to 5.75%.
When the expected return on lending increases, the bank increases its risk by increasing its lending. This is optimal since the increased return from the expected lending rate now covers larger potential credit losses. The implication is that less bank capital is needed as collateral at a given level of borrowing. The bank’s response to that is to increase its lending and borrowing. The increase in lending is at a decreasing rate since risk is strictly convex in the size of bank lending. Optimal bank capital also increases at a decreasing rate when $r$ increases. When the expected current profit, the drift, increases it raises the marginal value of bank capital and therefore an increase occurs until $V' = l$, again.

The lower left panel shows how, expected current profit, the drift, increases at an increasing rate with higher expected lending rate. That is due to the second order effect from cheaper bank borrowing. In the lower right panel the expected current profit, standardized with risk is pictured. It is increasing at a decreasing rate.

**Level of risk in the relationship banking-market ($\beta$)**

Changes in $\beta$ shifts risk in a multiplicative fashion, at all levels of $z$. That has two effects: shocks to bank capital through credit losses are scaled up as the variance of returns to bank lending increases, bank investors then require a higher risk-premium. That lowers banks’ margin on their lending. Banks’ optimal response to that is to lower risk through operating on a lower scale.

Figure 7 shows changes in beta from 0.06 to 0.10. The upper left panel shows that bank investment decreases at slightly decreasing rate. A result of an increase of $\beta$ is that banks need to keep more bank capital for all levels of their own investment to absorb shocks in order not to jeopardize the promised repayment to its creditors and through that its future dividends. Optimal bank capital is therefore increasing linearly with increasing beta which is shown in the right upper panel of Figure 7.
Figure 7: Sensitivity of bank investment, optimal bank capital, bank profit and profit per risk unit when changing the level of risk in relationship banking market, beta, from 0.06 to 0.10.

The expected current profit is falling in a slightly decreasing rate as beta increases, shown in the lower left panel of Figure 7. This is due to the decrease in the bank’s margin on lending to which the optimal response is to reduce risk through operating at a lower scale, i.e. they reduce their lending. The lower right panel in Figure 7 shows that the risk-adjusted expected current profit also falls when beta increases. That is, the reduction of banks’ investment does not reduce risk enough to compensate for the shift of risk. The margin on the bank’s lending falls both through the lower margin on its investment as well as the fact that it invests less.

Value of private information ($\alpha$)

Changing alpha corresponds to altering the quality, i.e., the value of the private information the “local” banks have about their clients. A low $\alpha$ corresponds to a low value of private
information, risk increase little when adding more and more clients although the bank has less information about them. Altering the sensitivity to private information reveals an interesting picture of the dynamic responses when comparing with changing the risk level above.

Figure 8 the upper left panel shows a similar picture as that in Figure above, only that the effect of changing $\alpha$ is larger in magnitude. When the marginal risk increases the bank reduces its investments more than with a reduction on the level of risk that is not related to the size of banks’ lending. In the upper right panel of Figure 8, a completely different picture than when altering $\beta$ is shown: optimal bank capital falls at a decreasing rate. When the value of the bank’s private information is low, it pays off to operate at a large scale but when alpha increases risk increases rapidly and banks optimally counter that with large reductions of their exposure to risk through reducing their investment.

**Figure 8:** Sensitivity of bank investment, optimal bank capital, bank profit and profit per risk unit when changing the sensitivity of risk to size in relationship banking market, $\alpha$, from 0.08 to 0.12.
In the lower left and right panels of Figure 8 a similar picture to that of Figure 5 is shown. The difference is that the drift and risk-adjusted drift falls more rapidly when the marginal risk on size of investment increases rather than a parallel shift of risk.

**Solutions in Second-Best Cases**

We have characterized solutions to the bank’s value maximization problem for two cases of regulation. The first is when there is a restriction on dividends and bonuses. The second is when there is a capital adequacy regulation.

The upper panel of Figure 9 illustrates bank lending for different levels of bank capital, with the base-line parameters and a cap on dividends on 0.83 which is 150 percent of maximum current profit (or drift). Graphs are constructed with a maximum drift at $k=b$ of roughly 0.55. Optimal bank capital is reduced by more than 50 percent compared to the unconstrained case.
Figure 9: Optimal controls and the resulting value-functions plotted against level of bank capital for First Best and Constrained Dividends. Dashed lines are, from left, constrained optimum, first best optimum and the end-point condition \( V_2(k) = k \).

The dynamic paths of bank lending are depicted by \( z_1(k) \) and \( z_2(k) \) where the first represents bank lending in the first-best case and the second, bank lending with a cap on dividends. In the first-best scenario the bank will hoard capital until the expected drift for the optimal dynamic choice is equal to the maximum drift in the one period case: the bank acts as if it were risk-neutral at the optimal level of bank capital. In Figure 9 it is also revealed that optimal bank
capital (i.e., the level of capital at which the bank pays dividends) shrinks considerably when a constraint on maximum dividend payments is imposed (from $b$ to $b^c$).

From the upper panel it can also be seen that leverage $(k/z)$ and that $v(z)/k$ above $b^c$ both are lower in the constrained case than in the first-best case. The latter probably is what policy makers want to achieve with constraints on dividend payments. However, an undesired effect that comes from the lowering of the threshold level of capital, at which dividends are paid out, is that it will take longer time to increase the volume of credit to borrowers after a negative shock to bank capital has occurred.

The lower panel of Figure 9 shows the value functions for the first-best case, the solid curve, and the second-best case, the dashed curve. The figure shows that the bank’s value in the second-best case is for all levels of bank capital lower than that in the first-best case. The figure also shows that at the level of bank capital where $z_2(k) = z^*(k)$ the value of the bank is in fact lower than bank capital. In principle, it is therefore possible to increase $b^c$ slightly to obtain a case where $z_2(k) = z^*(k)$ and $V_2(k) = k$. However, the location of $z_2(k)$ is very sensitive to the initial condition and we have therefore chosen to only demonstrate the feasibility of an admissible optimal second-best investment curve. One could, of course, also question our end-point condition since the second-best level of bank capital where the bank starts to pay dividends is around 4 and the end-point is at bank capital around 24. It seems as if there is a very low probability that bank capital will ever grow up to that level.

A pure capital adequacy regulation stipulates a lower bound on the ratio of bank capital, measured as common equity tier 1 capital, and risk weighted assets. As our model is a bare-bone model with no other distinction between bank liability than that between equity capital and retained earnings, and debt, and with only one type of asset, the constraint takes the following form.
where $\gamma$ is the capital adequacy requirement. Perfect maturity matching between assets and debt implies that banks will never violate the capital adequacy constraint since cutting down on lending and investment is feasible when bank capital has suffered a negative shock.

\[
\frac{k}{z} \geq \gamma, \quad (17)
\]

**Figure 10:** Optimal controls when capital adequacy rule of 10 percent is imposed.

Figure 10 shows three different bank investment strategies. Bank investment under maximization of expected current period profit without regard to limited liability is illustrated by $z^*(k)$. That is, it is the solution to $m_z(z^*(k),k) = 0$. The dynamic strategy which satisfies equation (9) is $z(k)$. The straight line with the slope $1/\gamma$ shows the border of 10 percent capital adequacy regulation. The bank’s capital ratio is required to be to the right of that line. Hence, under the capital adequacy regulation the dynamic value-maximizing
strategy follows the straight line from \( k = 0 \) up to the point \( k = ad \) where the capital adequacy constraint crosses \( z(k) \). For larger \( k \) it coincides with \( z(k) \). The implication is that when the capital adequacy regulation does not bind it does not affect the size of bank investment. Therefore, when not binding, the regulation has no effect on bank risk-taking and bank choice of capital buffer. When the constraint binds, it results in lower bank investment than under the value-maximizing dynamic strategy. The result is that the regulation leads to a more forceful credit squeeze than without regulation. Lower level of bank investment implies lower bank revenue. It also implies lower cost of bank borrowing. However, the former effect dominates the latter. That, in turn, implies that the expected time to build bank capital up to the desired level will increase.

5. Numerical procedure in time domain

Bank value under capital adequacy regulation is not possible to characterize in a transparent way in the continuous time model. Therefore, we perform numerical procedures in time domain to shed light on that result. In addition to that, the time domain simulations also deliver results on the expected time to first dividend after a negative shock to bank capital and on expected lifetime of banks starting at different levels of bank capital and under the different regulatory regimes. The time to first dividend represents the time to optimal capitalization.

Our numerical setup in the simulations in time domain is to use the optimal control for size of bank investment given common equity \( k \), \( z(k) \), from the continuous time solution but with compounding depending on the length of the time period. This means that we use the diffusion (4) with the optimal control for size but where \( dt \) becomes 0.25 and so forth and \( dk \) and \( w \) are the result of a simulation.
We simulate the paths for many banks starting at various specific current common equity levels.\footnote{Since the simulation is highly asymmetric due to the absorbing barrier, we draw 15 million paths for each initial level of common equity and average the resulting values.} The value function is traced out by repeating the simulation for current common equity levels between 0 and \( b \). Each bank lives at most 550 years. At the assumed discount rate of 3 percent, the discount factor in the 550\(^{th}\) period is 0.000000087. Hence, dividends after that time have insignificant impacts on bank value: at optimal capitalization where variance is at its highest the discounted value of the typically largest number drawn is 0.0000017.

Figure 11 shows bank values in the no policy case and the case with a capital adequacy constraint at 10 percent. In both cases dividends are paid yearly. The no policy value function is above the value function with a capital adequacy constraint. The diagram also shows the continuous time, no policy, desired level of capital buffer, \( b \), the discrete time level of desired capital buffer, \( b^d \) and the level of bank capital where the capital adequacy constraint binds. An important result is that the capital adequacy constraint lowers bank value also for levels of bank capital where it is not binding.
Figure 11: Bank value with and without capital adequacy constraint and yearly dividends

It is obvious in Figure 11 that the value functions are in fact parallel for levels of bank capital, $k \geq ad$. Note that for levels of bank capital above $b^d$, $V'(k) = 1$ by construction as all excess bank capital above this level is distributed as dividends. The reduction of bank value in the presence of the constraint is 2.25 percent compared to the first best value at the optimal level of bank capital $b^d$. This implies that return on equity decreases with almost 6 percent with the inclusion of a capital adequacy constraint at 10 percent. The reason is that when capital adequacy binds, marginal value of bank capital is lower since the constraint reduces the drift of bank capital when leverage is lowered.

An unexpected result in the numerical simulations is discovered, namely that optimal bank capital is depending on the length of time period in the simulations and in the model frequency for dividend distribution. The longer the time between distributing dividends, the lower is the optimal bank capital $b$. 
The length of time periods give a flavor of effects of maturity mismatch since at the start of the time period, the bank’s borrowing cost and lending are determined. The borrowing cost remains constant over the time period while the stochastic process that determines repayment to the bank evolves over the period. A longer period implies more uncertainty about repayment. That affects both the bank’s borrowing cost and the level of desired capital buffer. The flavor of mismatch comes from the bank’s inability to adjust during the time period which is one aspect of mismatch since during the time period, the bank has outstanding loans which cannot be withdrawn or easily scaled down although the situation has developed in an unexpected way. However, the need for the bank to refinance outstanding loans is not present in our analysis.

The intuition for lower bank value in discrete time is that going from continuous time to discrete time is similar to imposing a binding constraint on the timing on payments of dividends. Such a binding constraint lowers bank value for two reasons. The first is discounting; owners would have to wait for dividends. The second is that during a time period there may be occasions at which bank capital exceeds the threshold level for dividend payment but over the full period those occasions may be balanced by occasions with below threshold levels of capital.

Figure 12: Optimal bank capital when the length of time between the points where dividend is distributed change from continuous up to annual.

Figure 12, left panel, shows how the optimal bank capital shrinks when banks distribute dividends more seldom. The time-periods start with the continuous time bank
capital and then bank capital for \( t = \{0.25, 0.5, 1.0\} \). The result of these discrete simulations is that there is a considerable fall in optimal bank capital when time is long between the times where the bank can distribute dividends. An implication that might be drawn from this is that a regulatory pressure on banks to adopt a shorter time span between the decisions to distribute dividends may improve banks’ capital buffers. Notably, \( b \) is 21 percent lower with yearly dividends than continuous dividend payments. In the right panel of Figure 13, the effect on banks’ leverage is displayed. The resulting leverage increases with 18 percent with annual dividends as opposed to continuous.

An important issue during the current financial crisis is whether entrepreneurs that need financing are declined loans by banks although loans would have been granted under normal circumstances. If that is the case the reason may be that many banks have not fully recovered after the Financial Crisis and the Euro Crisis and that they still are undercapitalized. Undercapitalized banks hold back lending and build capital buffers. Figure 14 the expected time to first dividend following shocks of various sizes to bank capital and with \( t = 1 \). The interpretation of expected time to first dividend is the time it takes for the bank to build the desired capital buffer at which level the volume of credit is normal.
**Figure 13:** Time to first dividend given different levels of current bank capital. Solid line is the first best solution with a time period between dividend distributions of 1 year, dash dotted is the waiting time with a regulation stipulating 10 percent minimum capital adequacy that starts binding at dashed line.

It is clear that it takes quite a while for banks that have suffered a significant negative shock to recover and reach their optimal bank capital where supply of credit is normal. During the adjustment period the credit volumes are below the equilibrium level; investment and economic growth is slowed down. That is an important factor for stabilization policy and macro prudential regulation since policymakers want banks to increase, or at least not decrease, lending when systematic shocks have occurred. Obvious in Figure 13 is that although a capital adequacy may increase banks expected lifetime, it has a counter-productive effect on economic growth as it slows down the recovery of banks’ balance sheets following significant credit losses.

| Table 1 |
| Simulated values for First best and Capital adequacy case |
|----------|-----------|----------|-----------|-----------|----------|-----------|
| K = 2.575 | k = 4.81  | k = 6.021 | K = 2.575 | k = 4.81  | k = 6.021 | K = 2.575 | k = 4.81  |
| No. of Div.| 41.159    | 40.694    | 42.497    | 41.972    | 42.954    | 42.412    |
| Longevity | 180.292   | 185.921   | 179.796   | 184.977   | 179.727   | 184.833   |

Table 1 summarizes simulated values for bank value, number of dividends, waiting time to first dividend and longevity of banks for different levels of bank capital. The results are that, compared to the first-best case, a capital adequacy constraint lowers bank value, it decreases the number of dividends, it increases the waiting time to first dividend and it increases banks expected lifetime.
6. Concluding Comments

In this paper we have developed a novel dynamic model of banking aiming at exploring the relationships among bank capital, bank lending, dividend payouts to bank owners, bank risk-taking and time to build capital after significant credit losses. Those issues are at the center of micro- and macro prudential policy.

Two results of the basic model seem to be particularly interesting. One is that the bank’s dynamic value maximization significantly reduces incentives for excessive risk-taking induced by bank limited liability. The other is, given risk neutral bank owners, an optimally capitalized bank invests so as to maximize current period expected net return on investment without taking account of its limited liability but if the bank is undercapitalized, it behaves as if it were risk-averse.

We put the model to work by analyzing the functioning of two different components in the Basle III accord. Some results obtained are that both capital adequacy regulation and constraints on dividends and bonuses increase expected survival of banks. However, capital adequacy requirement will not improve equilibrium bank capitalization when the regulation does not bind but it would lead to a more forceful credit contraction when it binds and it will take time to rebuild bank capital. That runs counter to a primary policy goal, namely to keep up credit in severe downturns. Constraints on dividend and bonuses payments will induce banks to pay dividends and bonuses when they are less well capitalized than were there no such constraints. Hence, while such a constraint prevents owners and management from eroding bank capital in bad times, the possibility that such a regulation may be binding, lowers the level of capitalization at which banks enter bad times. That is, of course, an unwanted side-effect of the regulation.

The analysis highlights a conflict between micro- and macro prudential policies where the former aims at making banks risk-taking more prudent and the latter to keep up the
volume of credit in significant downturns and financial crises. Reconciling micro- and macro prudential banking regulation is a very topical research field.
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