When do Firms Break the Law in Order to Reduce Marginal Cost? - An Application to the Problem of Environmental Inspection*

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Abstract

This study attempts to identify firm characteristics that are important in determining whether or not a specific firm has strong incentives for non-compliance with environmental laws. In particular, we analyze how these incintives are related to the size of the cost reductions associated with non-compliance, business cycle conditions, the degree of product differentiation, market structure, and price versus quantity competition. When cost reductions are non-dramatic, in the sense that they do not lead to monopoly, the following rules of thumb are suggested. 1) Inspection should be intensified during booms, 2) firms that face high costs of compliance should be inspected more intensely and 3) firms that are insulated from competition by product differentiation or by lack of competitors should be inspected more intensely. Although our prime focus is environmental inspection, the theoretical findings readily extends to other similar applications such as VAT fraud and violations against import restrictions. They can also have some bearing on the monitoring of financial markets that are subject to regulation.

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1 Introduction

Firms often have incentives to try to reduce marginal costs by violating various types of legislation. The focus of this study is on environmental inspections and the incentives to violate environmental law, but the results extend readily to similar examples such as VAT fraud and violations of various types of import restrictions.They can also have some bearing on the monitoring of financial markets that are subject to regulation.

Environmental concerns play an increasingly important role in modern societies. Individual consumers affect emissions through their consumption patterns. Firms' production choices have an important impact on the environment both globally, e.g. through emissions of greenhouse gases, and locally, by polluting the environment surrounding the production facilities. Since the core problem is a standard economic externality, the market solution will fail to internalize the environmental impact of consumption and production decisions. Hence, there is scope for government intervention. Indeed, most countries try to affect the behavior of economic agents by implementing various laws protecting the environment, and in order to make sure that firms, in particular, live up to the standards it is also common to introduce some kind of environmental inspection and enforcement regime.

There is a fairly large economics literature on environmental inspection. Following Cohen (1998), it can be divided into a positive strand, that aims at understanding the objectives of firms and inspection agencies, and a normative strand focusing on optimal regulation.

The most important positive question is perhaps the "Harrington paradox" which essentially asks why firms tend to comply with environmental laws to such a great extent despite the fact that monitoring is limited and fines are low. The explanation offered by Harrington (1988) is that non-complying firms face the threat of being re-categorized into a group of firms facing tougher regulation and higher fines. See e.g., Heyes & Rickman (1999), Heyes (1996) for complementary explanations. Other important positive questions relate to the objective function applied by the inspection agency. Is the agency budget maximizing (Lee, 1983), compliance maximizing (Garvie & Keeler, 1994) or perhaps governed by a median voter process (Selden & Terrones, 1993)?

Normative studies typically look at the interplay between a representative firm and an inspection agency, given various assumptions regarding e.g. informational structures. While firms are often modeled quite simplistically, the inspection and enforcement technology tends to be very carefully described. Important tradeoffs studied include those between inspection and self-reporting (See e.g., Kaplow & Shavell (1994) and Bontems & Rotillon (2000), between ex ante and ex post monitoring (Cohen (1987)) and between inspection intensity and fines in the tradition of Becker (1968) (See e.g., (Cohen (1987)).

Our study has a normative perspective too, but the focus is more or less reversed in relation to most previous studies. We focus on firm heterogeneity and take an agnostic position regarding the exact inspection technology. Specifically, we try to model carefully characteristics that we believe are important in determining whether or not a firm has strong incentives for non-compliance.

The situation we have in mind is the following. There is an environmental inspection agency. Its task is to make sure that all firms in the region comply with the law. Although the exact technology available to the agency is not explicitly modeled, we assume that more resources must be devoted to firms facing stronger incentives to break the law. A central assumption is that compliance with the law increases marginal production costs.[1]

Firms belong to markets that may, or may not, coincide with the jurisdiction of the agency. Some firms belong to concentrated markets, others to competitive markets. Some firms produce differentiated products, while others do not. Some firms may cut costs a lot by deviating while others can comply at a low cost. Some firms compete à la Cournot while others compete à la Bertrand. Finally, the temptation to deviate might be affected by the business cycle. The objective of this study is simply to try to identify firms that are especially likely to break the law. Evidently, those are firms that the environmental inspection agency should devote special attention to. Note that we assume that the agency allocates its resources in a way that is successful in the sense that it actually prevents all firms from breaking the law. Hence, from a technical point of view, the experiment is basically to compare symmetric profits under compliance with the profit obtained by a firm that unilaterally deviates from compliance.[2]

From a theoretical point of view, it can of course be argued that an inspection agency that has perfect information about market mechanisms should be able to identify non-compliance immediately just by observing changes in quantities and prices. From a real life perspective,

[1]It is conceivable that clean production processes are associated with increases in both fixed and variable costs, but here we focus entirely on the latter effect. Obviously, adding fixed costs would introduce a strong link between the level of these costs and market structure.

[2]This makes our analysis different from studies on cost reducing innovations. There the focus is on cost reductions as an *equilibrium* outcome. See d'Aspremont & Jacquemin (1988) for an early reference.

however, changes in prices and quantities may have hundreds of explanations. Hence, it is virtually impossible for an inspection agency (or a competing firm) to attribute them to compliance specifically.

Our main findings are the following. Not surprisingly, the incentives to deviate from compliance are stronger the larger the cost saving and the higher the overall level of demand in the economy.

There is a u-shaped relationship between the degree of product differentiation and the incentives to deviate, i.e., incentives are strongest when differentiation is very small and very large. Basically, when differentiation is small a deviating firm may increase demand significantly by reducing price by a small amount. Hence, the increase in profits will be large. When differentiation is large the deviator does not have to bother much about the strategic responses of the competitors. This too results in a large increase in profits.

The relationship between deviation incentives and market structure is the following. For small cost savings, incentives are weakened as the number of firms increases. If the cost reduction is large, the opposite relation holds. When the market is competitive, prices are low initially. Hence a deviator gaining a small cost advantage will find it difficult to exploit this in a way that increases profits significantly. When cost savings are large however, they will permit the deviator to earn significant profits. Hence, given the competitive point of departure, the deviator will experience a large increase in profits.

Finally, when the cost saving is small, quantity competition implies stronger deviation incentives than price competition. If the cost reduction is large, the opposite relation holds. The intuition is the following. Suppose that a firm is considering a deviation from compliance and that firms compete in prices. Typically, price competition is more intense than quantity competition. Hence, price competition will lead to a situation where initial prices are low. If, in that case, a deviation leads to a small reduction in cost, the deviator will not be in a position where he/she can use this advantage in a way that increases profits to a large extent. Under quantity competition, initial prices are higher, which makes it easier to benefit from the cost advantage. If the cost reduction is large the situation is different. Then, it is relatively easy to increase profits by capturing a large fraction of the competitors' demand. However, attracting a large fraction of industry demand is less costly under Bertrand than under Cournot. The reason is that a deviator competing in quantities will have to expand output a lot in order to achieve this, thereby putting a significant downward pressure on price. Under Bertrand competition, a relatively small reduction in price is generally sufficient to shift demand, and especially so when products are close substitutes.

The paper is organized as follows. In section 2 the basic model is presented and equilibria are characterized. Section 3 presents the main results. The paper concludes with some final remarks in section 4.

2 The Model

The basic model is taken from Häckner (2000) where the utility function is of the following type:

$$U(q, I) = \sum_{i=1}^{n} q_i \alpha_i - \frac{1}{2}(\sum_{i=1}^{n} q_i^2 + 2\gamma \sum_{i=1}^{n} \sum_{j \neq i}^{n-1} q_i q_j) + I. \tag{1}$$

Thus, utility is quadratic in the consumption of q-goods and linear in the consumption of other goods I. The parameter $\gamma \in [0, 1]$ measures the substitutability between the products. If $\gamma = 0$, each firm has monopolistic market power, while if $\gamma = 1$, the products are perfect substitutes.[3] Finally, α_i is a firm specific demand shifting parameter. However, we will assume that $\alpha_i = \alpha$ for all i. Hence, α can be can be treated as a parameter reflecting the business cycle. Consumers maximize utility subject to the budget constraint $\sum p_i q_i + I \leq m$, where m denotes income and the price of the composite good is normalized to one. The first-order condition determining the inverse demand of good k is

$$\frac{\partial U}{\partial q_k} = \alpha - q_k - \gamma \sum_{j \neq k} q_j - p_k = 0. \tag{2}$$

Now, let c_k be the constant marginal cost of a firm violating environmental legislation by using a low-cost, high-pollution technology. Also, assume that all other firms comply with the law and that they face a common marginal cost c such that $c_k < c$. To simplify notation, let us define the parameters $z_k \equiv \alpha - c_k$ and $z \equiv \alpha - c$.

There are basically two ways to proceed when calculating the level of profits for a deviating firm. Either, one can assume that competing firms observe true cost levels and react strategically to changes in these, or it can be assumed that changes in cost come as a surprise, in which case there is no strategic response. In the latter scenario, competitors' will of course realize that they are not maximizing profits and hence they will want to adjust their behavior accordingly. Considering that real world firms learn over time, we believe that the former path is more realistic.

[3] A negative γ implies that the goods are complementary, a scenario that will not be further analyzed in this study.

However, as will be discussed in section 3, most results still hold if the assumption of strategic responses by compliant firms is relaxed.

If firm k unilaterally deviates from compliance with environmental legislation, the other firms will adapt by reducing production. There are two possible outcomes, either an interior solution with all firms continuing to produce some quantity, or a corner solution, where it is optimal for the other firms to cease production.[4]

Given that firms $i \neq k$ do not cease production, the profit maximizing quantity and price of firm k under Cournot competition equal (see the Appendix, A1.1.1.1 and A1.2.1.1, for derivation):

$$q_k^C = p_k^C - c_k = \frac{1}{2+(n-1)\gamma}\left[z + \frac{2+(n-2)\gamma}{2-\gamma}(z_k - z)\right], \tag{3}$$

while under Bertrand competition:

$$p_k^B = c_k + \frac{\left\{\begin{array}{c}(1-\gamma)[2+(2n-3)\gamma]z \\ +[2+3(n-2)\gamma+(n^2-5n+5)\gamma^2](z_k - z)\end{array}\right\}}{[2+(n-3)\gamma][2+(2n-3)\gamma]} \tag{4}$$

and

$$q_k^B = [1+(n-2)\gamma]\frac{\left\{\begin{array}{c}(1-\gamma)[2+(2n-3)\gamma]z \\ +[2+3(n-2)\gamma+(n^2-5n+5)\gamma^2](z_k - z)\end{array}\right\}}{(1-\gamma)[1+\gamma(n-1)][2+(n-3)\gamma][2+(2n-3)\gamma]}. \tag{5}$$

Letting $M \in \{B, C\}$ denote the mode of competition, the symmetric equilibrium profit under compliance is given by

$$\pi^M = \Phi^M z^2,$$

where $\Phi^M \equiv \Phi^M(\gamma, n) > 0$ (unless $\gamma = 1$ and $n \geq 2$ when firms compete in prices, in which case $\Phi^B(1, n) = 0$) captures the effects of the degrees of product differentiation and market concentration on profits. More specifically (see the Appendix, A1.1.1.1 and A1.2.1.1, for derivation),

$$\Phi^C(\gamma, n) = \frac{1}{[2+(n-1)\gamma]^2},$$
$$\Phi^B(\gamma, n) = \frac{(1-\gamma)[1+(n-2)\gamma]}{[1+(n-1)\gamma][2+(n-3)\gamma]^2}.$$

[4] It is assumed that a firm's capacity for production is not eliminated instantly and hence, firms remain present in the market even when no production is optimal. If, however, firm k is able to maintain its cost advantage, it is likely that a monopoly will eventually emerge as competitors leave the market.

It is easily verified that $\frac{\partial \Phi^M}{\partial \gamma} < 0$ for $n \geq 2$ ($\Phi^M(\gamma, 1) = \frac{1}{4}$) and that $\frac{\partial \Phi^M}{\partial n} < 0$ (unless $\gamma = 1$ and $n \geq 2$ when firms compete in prices). Thus, Φ^M can be interpreted as an inverse measure for the degree of market competition. A higher number of firms or a lower degree of product differentiation decrease the value of Φ^M and hence, profits fall as the degree of competition increases.

Under Cournot competition the profit maximizing price and quantity of firms $i \neq k$ equal (see the Appendix, A1.1.1.2 and A1.2.1.2, for derivation):

$$q_i^C = p_i^C - c = \frac{1}{2 + (n-1)\gamma} \left[z - \frac{\gamma}{2 - \gamma}(z_k - z) \right],$$

while under Bertrand competition:

$$p_i^B = c + \frac{(1-\gamma)[2 + (2n-3)\gamma]z - \gamma[1 + (n-2)\gamma](z_k - z)}{[2 + (n-3)\gamma][2 + (2n-3)\gamma]}$$

and

$$q_i^B = [1 + (n-2)\gamma] \frac{(1-\gamma)[2 + (2n-3)\gamma]z - \gamma[1 + (n-2)\gamma](z_k - z)}{(1-\gamma)[1 + (n-1)\gamma][2 + (n-3)\gamma][2 + (2n-3)\gamma]}.$$

For notational convenience, we define the variable $t = \frac{z}{z_k} = \frac{\alpha - c}{\alpha - c_k} \in (0, 1]$. Basically, t is a measure of the cost saving obtained by a non-complying firm. When the cost saving is small, t is close to one, but the lower is c_k, the lower is t.

There will be an interior solution, such that $q_i^M > 0$, if and only if $t > \bar{t}^M$, where (see the Appendix, A1.1.1.2 and A1.2.1.2, for derivation)

$$\bar{t}^C = \frac{\gamma}{2} \text{ and } \bar{t}^B = \frac{\gamma[1 + (n-2)\gamma]}{2 + 2(n-2)\gamma - (n-1)\gamma^2}.$$

Hence, to avoid a monopoly-like outcome, the degree of cost savings must not be too large. It is easily established that the threshold value $\bar{t}^M$ increases in γ, taking on values in the interval $[0, \frac{1}{2}]$ in the Cournot case, implying that the above condition is satisfied for all degrees of product differentiation when cost savings are sufficiently small ($t \geq \frac{1}{2}$), and taking on values in the interval $[0, 1]$ in the Bertrand case. Thus, the above condition is not satisfied for any degree of cost savings when products are undifferentiated and firms compete in prices. Since $\bar{t}^C < \bar{t}^B$ for $\gamma > 0$, a corner solution is more likely in the Bertrand case, which is to be expected, given that the scope for eliminating competitors is larger when firms compete in prices, especially at low degrees of product differentiation. The threshold value $\bar{t}^C$ is independent of the degree of market concentration, as can be expected when firms compete in quantities; if the degree if product differentiation is not too low or the degree of cost savings is not too large, there is still scope for compliant firms to produce some small quantity when facing a violating competitor. The threshold value $\bar{t}^B$, however, also depends on the degree of market concentration. It is easily established that $\bar{t}^B$ increases in n when $\gamma < 1$ ($\bar{t}^B = 1$ for $\gamma = 1$), taking on values in the interval $[\frac{\gamma}{2}, \frac{\gamma}{2-\gamma})$. When firms compete in prices the effect on profits of being exposed to a violation by a competitor is stronger the fiercer is competition and hence, an interior solution is less likely the larger is the number of firms.

Given that the degree of cost savings is not too large (i.e. $t > \bar{t}^M$), such that firm k's violation does not lead to zero production by compliant firms, its profits will be given by

$$\pi_k^{DM} = \Phi^M[z + (z_k - z)\Psi^M]^2,$$

where $\Psi^M \equiv \Psi^M(\gamma, n) > 0$ captures the effects of the degrees of product differentiation and market concentration on the extent to which cost savings impact on profits under unilateral deviation, taking strategic responses by competitors into account. More specifically (see the Appendix, A1.1.1.1 and A1.2.1.1, for derivation),

$$\Psi^C(\gamma, n) = \frac{2 + (n-2)\gamma}{2-\gamma},$$
$$\Psi^B(\gamma, n) = \frac{2 + 3(n-2)\gamma + (n^2 - 5n + 5)\gamma^2}{(1-\gamma)[2 + (2n-3)\gamma]}.$$

It is easily verified that $\frac{\partial \Psi^M}{\partial \gamma} > 0$ for $n \geq 2$ ($\Psi^M(\gamma, 1) = 1$) and that $\frac{\partial \Psi^M}{\partial n} > 0$ (unless $\gamma = 1$ when firms compete in prices, in which case, however $\bar{t}^B = 1$, such that there exist no interior solutions for $t < 1$). Thus, Ψ^M can be interpreted as a measure for the impact of the degree of market competition on cost savings. A higher number of firms or a lower degree of product differentiation increase the value of Ψ^M; a more competitive market increases the scope for taking advantage of unilaterally lowering costs by violating legislation. When $t > \bar{t}^M$, the profitability of violating the law is denoted by Δ^M and given by

$$\Delta^M \equiv \pi^{DM} - \pi^M = (z_k - z)\Phi^M\Psi^M[2z + (z_k - z)\Psi^M].$$

If the degree of cost savings is sufficiently large (i.e. $t \leq \bar{t}^M$), firms $i \neq k$ cease to produce. In this case the optimal quantities and prices of firm k are given by (see the Appendix, A1.1.2 and A1.2.2, for derivation)

$$\overline{q}_k^C = \overline{p}_k^C - c_k = \frac{1}{2} z_k$$

in the Cournot case, while under Bertrand

$$\overline{p}_k^B = c_k + \frac{(1-\gamma)z + [1+(n-2)\gamma](z_k - z)}{2[1+(n-2)\gamma]},$$
$$\overline{q}_k^B = \frac{(1-\gamma)z + [1+(n-2)\gamma](z_k - z)}{2(1-\gamma)[1+(n-1)\gamma]}.$$

Hence, firm k's profits are given by

$$\overline{\pi}_k^{DC} = \frac{1}{4} z_k^2$$

when firms compete in quantities, while under price competition (see the Appendix, A1.2.2, for derivation)

$$\overline{\pi}^{DB} = \overline{\Phi}^B [z + \overline{\Psi}^B (z_k - z)]^2,$$

where

$$\overline{\Phi}^B = \frac{1-\gamma}{4[1+(n-1)\gamma][1+(n-2)\gamma]},$$
$$\overline{\Psi}^B = \frac{1+(n-2)\gamma}{1-\gamma}.$$

The gain from breaching legislation when the degree of cost savings is sufficiently large is denoted by $\overline{\Delta}^M$ and given by

$$\overline{\Delta}^C \equiv \overline{\pi}_k^{DC} - \pi^C = \frac{1}{4} z_k^2 - \Phi^C z^2,$$
$$\overline{\Delta}^B \equiv \overline{\pi}_k^{DB} - \pi^B = \overline{\Phi}^B [z + \overline{\Psi}^B (z_k - z)]^2 - \Phi^B z^2$$

in the Cournot and Bertrand case, respectively.

3 Results

We will now conduct a number of experiments in order to analyze how various market characteristics affect the incentives to violate environmental legislation. Specifically, we study how these incentives are related to

1) the size of the cost reduction
2) business cycle conditions
3) the degree of product differentiation
4) market structure
5) Bertrand versus Cournot competition

Our first result is intuitively straightforward.

Proposition 1 *The incentive to violate the environmental law is stronger the larger the reduction in cost.*

Proof. First, consider the case when $t > \bar{t}^M$, i.e., when z_k is small. It is easily verified that $\frac{\partial \Delta^M}{\partial z_k} = 2\Phi^M \Psi^M [z + (z_k - z)\Psi^M]$. Since $\Phi^M \Psi^M > 0$ and $\Psi^M > 0$, it immediately follows that $\frac{\partial \Delta^M}{\partial z_k} > 0$, regardless of whether firms compete in prices or in quantities. Next, consider the case when $t \leq \bar{t}^M$, i.e. when z_k is large. It is easily verified that $\frac{\partial \overline{\Delta}^C}{\partial z_k} = \frac{z_k}{2} > 0$ and that $\frac{\partial \overline{\Delta}^B}{\partial z_k} = 2\overline{\Phi}^B \overline{\Psi}^B [z + (z_k - z)\overline{\Psi}^B]$, which is strictly positive, because $\overline{\Phi}^M \overline{\Psi}^M > 0$ and $\overline{\Psi}^M > 0$. Thus, the gain from violating legislation increases in cost savings, regardless of whether competing firms cease to produce in response to the violation. See also *Figure 1.* ■

Figure 1 about here

The next proposition is also quite intuitive. The profitability of a cost reduction is simply larger the higher the overall level of demand.

Proposition 2 *The incentive to violate the environmental law is stronger in booms than in recessions.*

Proof. For the case when $t > \bar{t}^M$, it is easily estanlished that $\frac{\partial \Delta^M}{\partial \alpha} = 2(z_k - z)\Phi^M \Psi^M$. Since $\Phi^M \Psi^M > 0$, it immediately follows that $\frac{\partial \Delta^M}{\partial \alpha} > 0$ when $z_k > z$, regardless of whether firms compete in prices or in quantities. For the case when $t \leq \bar{t}^M$ it can be shown that $\frac{\partial \overline{\Delta}^M}{\partial \alpha} > 0$, both under Bertrand and Cournot competition (see the Appendix, A2.1 and A2.2). See also *Figure 2.* ■

Figure 2 about here

The effect of the degree of product differentiation on the gains from violating legislation is not straightforward. A higher γ implies a higher degree of competition on the one hand, but more scope for taking advantage of cost savings by breaching legislation on the other hand. It turns out that the former effect dominates for high degrees of product differentiation, while the latter effect dominates for low degrees of product differentiation. Hence, as illustrated by *Figure 3*, the relationship between Δ and γ is U-shaped. This means that the incentives for non-compliance are strong for remote and for close substitutes while they are weak for intermediate levels of product differentiation.

Figure 3 about here

Proposition 3 *The relationship between the incentive to violate the environmental law and the degree of product differentiation is U-shaped. Hence, incentives are strongest when differentiation is very small and very large.*

Proof. When $t > \bar{t}^M$ the total effect of changes in the degree of product differentiation can be assessed as follows:

$$\frac{\partial\Delta^M/\partial\gamma}{\Delta^M} = \frac{\partial\Phi^M/\partial\gamma}{\Phi^M} + \frac{\partial\Psi^M/\partial\gamma}{\Psi^M} + \frac{\partial\Psi^M/\partial\gamma(z_k - z)}{2z + \Psi^M(z_k - z)}.$$

Hence,

$$\frac{\partial\Delta^M/\partial\gamma}{\Delta^M} > 0 \Leftrightarrow 1 + \frac{\Psi^M(1-t)}{2t + \Psi^M(1-t)} > -\frac{\frac{\partial\Phi^M/\partial\gamma}{\Phi^M}}{\frac{\partial\Psi^M/\partial\gamma}{\Psi^M}} \equiv \Omega_\gamma^M.$$

It can be established that Ω_γ^M decreases in γ (see the Appendix, A3.1). In particular, $\Omega_0^C = \Omega_0^B = 2$, $\Omega_1^C = \frac{n}{n+1}$, and $\lim_{\gamma\to 1}\Omega_\gamma^B = 1$. The left-hand side equals $\frac{2}{1+t} \in (1,2)$ when $\gamma = 0$ and increases in γ (unless $t = 1$, in which case it equals 1 for all γ). Hence, for every t there exists a unique $\widehat{\gamma}_t \in (0,1)$ such that $\frac{\partial\Delta^M}{\partial\gamma} > 0$ if and only if $\gamma > \widehat{\gamma}_t$, i.e. Δ^M is U-shaped in γ. Furthermore, since the left-hand side of the above condition decreases in t, it immediately folllows that $\widehat{\gamma}_t$ will be closer to zero the higher are cost savings. The above condition can also be rearranged in the following way:

$$\frac{\partial\Delta^M/\partial\gamma}{\Delta^M} > 0 \Leftrightarrow 2t + 2\Psi^M(1-t) > [2t + \Psi^M(1-t)]\Omega_\gamma^M$$

$$\Leftrightarrow t < \frac{(2-\Omega_\gamma^M)\Psi^M}{2(\Omega_\gamma^M - 1) + (2-\Omega_\gamma^M)\Psi^M} = \frac{1}{1 + \frac{2(\Omega_\gamma^M - 1)}{(2-\Omega_\gamma^M)\Psi^M}} \equiv \widehat{t}_\gamma^M.$$

Since $\frac{\partial\bar{\Delta}^M}{\partial\gamma} > 0$ for all $t \leq \bar{t}^M$ (see the Appendix, A3.2) and $\widehat{t}_\gamma^M > \bar{t}^M$ (see the Appendix, A3.3), it can be concluded that the gain from violating legislation increases in γ if the degree of cost savings is sufficiently high ($\frac{\partial\bar{\Delta}^M}{\partial\gamma} > 0$ for $t \leq \bar{t}^M$ and $\frac{\partial\Delta^M}{\partial\gamma} > 0$ for $\bar{t}^M \leq t < \widehat{t}_\gamma^M$). Moreover, since Ω_γ^M decreases and Ψ^M increases in γ, it immediately follows that $\widehat{t}_\gamma^M$ increases in γ; $\widehat{t}_0^C = \widehat{t}_0^B = 0$, $\widehat{t}_1^C = \frac{n(n+2)}{n(n+2)-2} > 1$ and $\widehat{t}_1^B = 1$. See also *Figure 4.* ■

Figure 4 about here

The intuition for Proposition (3) is the following. When products are close substitutes, even a small reduction in price will lead to a large increase in demand. Hence, a cut in costs will translate into a large increase in profits. When products are remote substitutes, firms are essentially insulated from competition. Hence, a firm that cuts prices will not have to bother much about other firms responding strategically by cutting their prices too. A given reduction in cost can therefore be exploited in a way that increases demand and profits significantly.

The relationship between Δ and the market structure is also complex. While a higher n implies fiercer competition, it also increases the scope for taking advantage of cost savings by violating legislation. For any given level of product differentiation, there is a threshold level of t such that Δ decreases in the number of firms whenever t is larger than this threshold, and vice versa. Hence, if the cost reduction is small enough, the temptation to violate environmental legislation is largest when the market is concentrated. If the cost reduction is large, the opposite relation holds. Proposition (4) summarizes the results illustrated by *Figure 5*.

Figure 5 about here

Proposition 4 *When cost savings are small/large the incentive to violate the environmental law becomes stronger/weaker the more concentrated the market structure. The set of parameters for which there is a positive relationship between incentives to deviate and the number of firms is larger the smaller the degree of product differentiation.*

Proof. When $t > \bar{t}^M$, $\frac{\partial \Phi^M}{\partial n} = 0$ and $\frac{\partial \Psi^M}{\partial n} = 0$ and hence, $\frac{\partial \Delta^M}{\partial n} = 0$ for all z_k and n if $\gamma = 0$, while $\frac{\partial \Phi^M}{\partial n} < 0$ and $\frac{\partial \Psi^M}{\partial n} > 0$ if $\gamma > 0$. Analogously to the proof of Proposition 3, we obtain the following condition:

$$\frac{\partial \Delta^M / \partial n}{\Delta^M} > 0 \Leftrightarrow 1 + \frac{\Psi^M(1-t)}{2t + \Psi^M(1-t)} > -\frac{\frac{\partial \Phi^M/\partial n}{\Phi^M}}{\frac{\partial \Psi^M/\partial n}{\Psi^M}} \equiv \Omega_n^M.$$

It can be established that Ω_n^M decreases in γ (see the Appendix, A4.1). In particular, $\Omega_n^C = \Omega_n^B = 2$ for $\gamma = 0$, $\Omega_n^C = \frac{2n}{n+1} \in [1, 2)$ for $\gamma = 1$ and $\lim_{\gamma \to 1} \Omega_n^B = \frac{2(n-\frac{1}{2})^2}{n^2} \in [\frac{1}{2}, 2)$. The left-hand side equals 1 for all γ when $t = 1$. For $t < 1$, it increases in γ, taking on values in the interval $[\frac{2}{1+t}, 2]$ in the Bertrand case, and taking on values in the interval $(\frac{2}{1+t}, \frac{2n}{n+1}]$ in the Cournot case (for $n > 1$). Hence, given n, there exists a unique $\widetilde{\gamma}_t \in (0,1)$ for every t such that $\frac{\partial \Delta^M}{\partial n} > 0$ if and only if $\gamma > \widetilde{\gamma}_t$. Moreover, since the left-hand side of the above condition decreases in t, $\widetilde{\gamma}_t$ increases in t. Rearrangement analogously to the proof of Proposition 3 renders the following condition:

$$\frac{\partial \Delta^M / \partial n}{\Delta^M} > 0 \Leftrightarrow t < \frac{1}{1 + \frac{2(\Omega_n^M - 1)}{(2-\Omega_n^M)\Psi^M}} \equiv \widetilde{t}_n^M.$$

Since $\frac{\partial \overline{\Delta}^M}{\partial n} > 0$ when $n > 1$ and $\frac{\partial \overline{\Delta}^M}{\partial n} = 0$ when $n = 1$ for all $t \leq \overline{t}^M$ (see the Appendix, A4.2), and $\widetilde{t}_n^M > \overline{t}^M$ (see the Appendix, A4.3), it can be concluded that the gain from violating legislation increases in n if the degree of cost savings is sufficiently high ($\frac{\partial \overline{\Delta}^M}{\partial n} > 0$ for $t \leq \overline{t}^M$ and $\frac{\partial \Delta^M}{\partial n} > 0$ for $\overline{t}^M \leq t < \widetilde{t}_n^M$). Moreover, since Ω_n^M decreases and Ψ^M increases in γ, it immediately follows that $\widetilde{t}_n^M$ increases in γ; $\widetilde{t}_n^C = \widetilde{t}_n^B = 0$ for $\gamma = 0$, $\widetilde{t}_n^C = \frac{n}{2n-1} \in (\frac{1}{2}, 1]$ and $\widetilde{t}_n^B = 1$ for $\gamma = 1$. ■

The intuition for Proposition (4) is the following. Suppose that the number of firms is large and that a firm is considering not to comply with the law. Obviously, the competitive pressure will be high initially which leads to low prices. If, in that case, a deviation leads to a small reduction in cost, the deviator will not be in a position to use this advantage in a way that increases profits to a large extent. If the cost reduction is large the situation is different. Since the number of firms is large, the potential deviator will initially have a small market share and earn low profits. However, for large cost reductions, the deviator is in a position to capture a large fraction of industry demand and profits. Hence, the increase in profits will tend to be large.

It should be noted that the threshold depicted in *Figure 5* is itself a function of the degree of market concentration. However, as illustrated by *Figures 6* and *7* the threshold is surprisingly robust, and especially so under price competition.

Figures 6 and 7 about here

The final proposition deals with Cournot versus Bertrand competition. The results are illustrated in *Figure 8*. For any given level of product differentiation, there is a threshold level of t such that quantity competition yields stronger incentives to violate the law than price competition whenever t is larger than this threshold, and vice versa. Hence, if the cost reduction is small enough, the temptation to deviate from compliance is largest under Cournot competition. If the cost reduction is large, the opposite relation holds. Proposition (5) summarizes these results.

Figure 8 about here

Proposition 5 *When cost savings are small/large quantity competition leads to stronger/weaker incentives to violate the environmental law than price competition. The set of parameters for which quantity competition leads to stronger incentives increases in the degree of product differentiation.*

Proof. First consider the case when $t \in (\bar{t}^B, 1)$, i.e. we have interior solutions under both Bertrand and Cournot competition. If $n = 1$ or $\gamma = 0$ it is easily verified that $\Phi^C = \Phi^B = \frac{1}{4}$ and $\Psi^C = \Psi^B = 1$ and hence, $\Delta^C = \Delta^B$ for any t. When $n > 1$ and $\gamma > 0$, $\Delta^C > \Delta^B$ if and only if

$$(z_k - z)\Phi^C\Psi^C[2z + (z_k - z)\Psi^C] > (z_k - z)\Phi^B\Psi^B[2z + (z_k - z)\Psi^B]$$
$$\Leftrightarrow \frac{\Phi^C\Psi^C}{\Phi^B\Psi^B}[\frac{2t}{1-t} + \Psi^C] > \frac{2t}{1-t} + \Psi^B$$

Let $\sigma \equiv \frac{\Phi^C\Psi^C}{\Phi^B\Psi^B}$ and $T \equiv \frac{2t}{1-t}$. Thus, for $n > 1$ and $\gamma > 0$,

$$\Delta^C > \Delta^B \Leftrightarrow \sigma(T + \Psi^C) > T + \Psi^B \Leftrightarrow T > \frac{\Psi^B - \sigma\Psi^C}{\sigma - 1} \equiv T^*.$$

Since T^* increases in γ (see the Appendix, A5.1), taking on values in the interval $[0, \infty)$, it immediately follows that $\Delta^C > \Delta^B$ if and only if $t > \frac{T^*}{2+T^*} \equiv t^*$, where t^* increases in γ, taking on values in the interval $[0, 1)$. Moreover it can be shown that $t^* \geq \bar{t}^B$ (see the Appendix, A5.2).

When $t \in (\bar{t}^C, \bar{t}^B]$, such that compliant firms cease to produce under Bertrand, but not under Cournot competition, it is the case that $\Delta^B \leq \overline{\Delta}^B$ and hence, $\Delta^C \leq \overline{\Delta}^B$ (see the Appendix, A5.3).

When $t \leq \bar{t}^C$, such that compliant firms cease to produce under both Bertrand and Cournot competition, it is the case that $\overline{\Delta}^C \leq \Delta^C$ and hence, $\overline{\Delta}^C < \overline{\Delta}^B$ (see the Appendix, A5.4). Thus, the gain from violating legislation is larger under Cournot than under Bertrand competition if and only if the degree of cost savings is sufficiently small. ■

Suppose that a firm is considering a deviation from compliance and that firms compete in prices. Typically, price competition is more intense than quantity competition.[5] Hence, price competition will lead to

[5] For a discussion of situations where this is not the case, see Häckner (2000).

a situation where initial prices are low. If, in that case, a deviation leads to a small reduction in cost, the deviator will not be in a position to use this advantage in a way that increases profits to a large extent. Under quantity competition, initial prices are higher, which makes it easier to increase profits. If the cost reduction is large the situation is different. Then, it is relatively easy to increase profits by capturing a large fraction of the competitors' customers. However, attracting a large fraction of industry demand is less costly under Bertrand than under Cournot competition. The reason is that a deviator competing in quantities will have to expand output a lot in order to achieve this, thereby putting a significant downward pressure on price. Under Bertrand, a relatively small reduction in price is generally sufficient to shift demand, and especially so when products are close substitutes.

In section 2, we made the assumption that changes in cost are observable from the point of view of competing firms. Hence, competitors act strategically by adjusting prices or quantities. It is straightforward to go through the same exercise as above assuming non-strategic responses.[6] Fortunately, most results go through unchanged (see the Appendix, A6). An exception is Proposition (5) which would then state that the incentives for non-compliance are always stronger under Bertrand than under Cournot. The reason is simply that competitors no longer match price reductions. Hence, it becomes easier than before to attract new customers. Propositions (1)-(4) go through unchanged assuming Bertrand competition. Under Cournot competition, Propositions (1) and (2) go through unchanged. Proposition (3), however, would indicate a positive relationship between deviation incentives and the degree of product differentiation. Likewise, Proposition (4) would indicate a positive relationship between deviation incentives and the degree of market concentration.

4 Concluding remarks

We may conclude that there are few general findings that the environmental inspection agency can use without having access to detailed firm level data. However, given that the agency has some idea about the firms' costs of compliance and business cycle conditions, the following rules of thumb apply.

1) Inspection should be intensified during booms.

[6] Note that there will be no corner solutions when compliant firms do not react strategically to violations. It is therefore possible that compliant firms incur losses if one firm violates legislation, in contrast to the case of strategic responses to violations, where it is possible for firms to cease production.

2) Firms that face high costs of compliance should be inspected more intensely.

If restricting attention to non-extreme levels of cost savings, further guidelines can be established. Specifically, when non-complience does not lead to a monopoly situation it is clear from *Figure 5* that the set of parameters that give rise to a *positive* relationship between deviation incentives and the number of firms is very small. Indeed, as illustrated in *Figures 9* and *10,* this area is always smaller than 10 percent of the permissable parameter space. Hence, a reasonable policy would be to intensify inspection in concentrated markets.

Figures 9 an 10 about here

Although Proposition (3) indicates that the incentives for non-compliance are strongest for very differentiated *and* very homogenous products, one might suspect that the problem is particularly severe in the former case, where market profits are high. Combining these conjectures we arrive at a third rule of thumb.

3) Firms that are insulated from competition by product differentiation or by lack of competitors should be inspected more intensely.

It should be noted that these results are independent of whether firms compete in prices or quantities. The choice of strategic variable obviously plays a role of its own (Proposition (5)), but it does not affect other mechanisms from a qualitative point of view. This is reassuring since information on the mode of competition is likely to be difficult for the inspection agency to obtain. For the same reason, we refrain from using the results in Proposition (5) to construct a fourth rule of thumb.

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A Appendix

A1 Model solutions

A1.1 Cournot competition

A1.1.1 Interior solutions

A1.1.1.1 Firm k Given that all firms $i \neq k$ comply, the reaction functions of firm k and firms $i \neq k$ are given by (see Häckner, 2000)

$$q_k(\mathbf{q}_{-k}) = \frac{1}{2}[z_k - \gamma \sum_{j \neq k} q_j], \tag{6}$$

$$q_i(\mathbf{q}_{-i}) = \frac{1}{2}[z - \gamma \sum_{j \neq k, j \neq i} q_j - \gamma q_k]. \tag{7}$$

Hence,

$$\sum q_j = \frac{1}{2}\left[(n-1)z + z_k - (n-1)\gamma \sum q_j\right]$$
$$\Leftrightarrow \sum q_j = \frac{(n-1)z + z_k}{2 + (n-1)\gamma}. \tag{8}$$

Combining (6) and (8) yields

$$q_k^C = \frac{1}{2-\gamma}\left[z_k - \frac{\gamma[(n-1)z + z_k]}{2 + (n-1)\gamma}\right]$$
$$= \frac{1}{2 + (n-1)\gamma}\left[z + \frac{2 + (n-2)\gamma}{2-\gamma}(z_k - z)\right]. \tag{9}$$

Plugging (6) into (2) we obtain

$$p_k^C = \alpha - q_k^C - \gamma \sum_{j \neq k} q_j = \alpha - q_k^C + 2q_k^C - z_k = c_k + q_k^C. \tag{10}$$

Hence, firm k's profit is given by

$$\pi_k^C = \frac{1}{[2 + (n-1)\gamma]^2}\left[z + \frac{2 + (n-2)\gamma}{2-\gamma}(z_k - z)\right]^2.$$

Defining

$$\Phi^C(\gamma, n) \equiv \frac{1}{[2 + (n-1)\gamma]^2},$$
$$\Psi^C(\gamma, n) \equiv \frac{2 + (n-2)\gamma}{2-\gamma},$$

profits under compliance and non-compliance are thus given by $\pi_k^C = \Phi^C z^2$ and $\pi_k^{DC} = \Phi^C[z + \Psi^C(z_k - z)]^2$.

A1.1.1.2 Firms $i \neq k$ The quantities of the other firms ($i \neq k$) are given by (7):

$$q_i^C = \frac{1}{2}[z - \gamma \sum_{j \neq k, j \neq i} q_j^C - \gamma q_k^C] = \frac{1}{2}[z - \gamma \sum q_j^C + \gamma q_i^C].$$

By plugging (8) into this expression and rearranging terms we obtain

$$(2-\gamma)q_i^C = z - \gamma \frac{(n-1)z + z_k}{2+(n-1)\gamma} = \frac{2z - \gamma z_k}{2+(n-1)\gamma} = \frac{(2-\gamma)z - \gamma(z_k - z)}{2+(n-1)\gamma}.$$

Hence,

$$q_i^C = p_i^C - c = \frac{1}{2+(n-1)\gamma}\left[z - \frac{\gamma}{2-\gamma}(z_k - z)\right].$$

Plugging (7) into (2) yields

$$p_i^C = \alpha - q_i^C - \gamma \sum_{j \neq i} q_j = \alpha - q_i^C + 2q_i^C - z = c + q_i^C.$$

Thus, to obtain an interior solution, it has to be that

$$z - \frac{\gamma}{2-\gamma}(z_k - z) > 0 \Leftrightarrow t > \frac{\gamma}{2} \equiv \bar{t}^C.$$

A1.1.2 Corner solutions

When $t \leq \bar{t}^C$, firms $i \neq k$ cease to produce. Using (6) and (2), firm k's quantity and price are then given by

$$\overline{q}_k^C = \overline{p}_k^C - c_k = \frac{1}{2}z_k.$$

Hence, its profit is given by

$$\overline{\pi}_k^{DC} = \frac{1}{4}z_k^2.$$

A1.2 Bertrand competition

A1.2.1 Interior solutions

A1.2.1.1 Firm k Given that all firms $i \neq k$ comply, the reaction functions of firm k and firms $i \neq k$ are given by (see Häckner, 2000)

$$p_k(\mathbf{p}_{-k}) = \frac{c_k}{2} + \frac{(1-\gamma)\alpha}{2[1+(n-2)\gamma]} + \frac{\gamma \sum_{i \neq k} p_i}{2[1+(n-2)\gamma]}, \quad (11)$$

$$p_i(\mathbf{p}_{-i}) = \frac{c}{2} + \frac{(1-\gamma)\alpha}{2[1+(n-2)\gamma]} + \frac{\gamma \sum_{j \neq i} p_j}{2[1+(n-2)\gamma]} \qquad (i \neq k). \quad (12)$$

Hence,

$$\sum p_j = \frac{(n-1)c + c_k}{2} + \frac{(1-\gamma)n\alpha}{2[1+(n-2)\gamma]} + \frac{(n-1)\gamma}{2[1+(n-2)\gamma]} \sum p_j$$
$$\Leftrightarrow [2+(n-3)\gamma] \sum p_j = [1+(n-2)\gamma][(n-1)c + c_k] + (1-\gamma)n\alpha. \tag{13}$$

Plugging this expression into (11) yields

$$p_k = \frac{c_k}{2} + \frac{(1-\gamma)\alpha}{2[1+(n-2)\gamma]} + \frac{\gamma\left\{\frac{[1+(n-2)\gamma][(n-1)c+c_k]+(1-\gamma)n\alpha}{[2+(n-3)\gamma]} - p_k\right\}}{2[1+(n-2)\gamma]}$$

Rearranging terms we obtain

$$\begin{aligned} p_k^B &= \frac{\begin{array}{c}[1+(n-2)\gamma][2+(n-3)\gamma]c_k + (1-\gamma)[2+(n-3)\gamma]\alpha \\ +\gamma\{[1+(n-2)\gamma][(n-1)c+c_k] + (1-\gamma)n\alpha\}\end{array}}{[2+(n-3)\gamma][2+(2n-3)\gamma]} \\ &= c_k + \frac{(1-\gamma)[2+(2n-3)\gamma]z + [2+3(n-2)\gamma + (n^2-5n+5)\gamma^2](z_k - z)}{[2+(n-3)\gamma][2+(2n-3)\gamma]}. \end{aligned}$$

Plugging the expression for $\gamma \sum_{i \neq k} p_i$, obtained by rearrangement of (11), into the expression for demand, derived in Häckner (2000), we obtain

$$\begin{aligned} q_k^B &= \frac{(1-\gamma)\alpha - [1+(n-2)\gamma]p_k^B + \gamma \sum_{i \neq k} p_i^B}{(1-\gamma)[1+(n-1)\gamma]} \\ &= \frac{1+(n-2)\gamma}{(1-\gamma)[1+(n-1)\gamma]}(p_k^B - c_k). \end{aligned}$$

Hence, profits are given by

$$\pi_k^B = \frac{(1-\gamma)[1+(n-2)\gamma]}{[1+\gamma(n-1)][2+(n-3)\gamma]^2}\left[z + \frac{2+3(n-2)\gamma+(n^2-5n+5)\gamma^2}{(1-\gamma)[2+(2n-3)\gamma]}(z_k - z)\right]^2.$$

Defining

$$\begin{aligned} \Phi^B(\gamma, n) &\equiv \frac{(1-\gamma)[1+\gamma(n-2)]}{[1+\gamma(n-1)][2+\gamma(n-3)]^2}, \\ \Psi^B(\gamma, n) &\equiv \frac{2+3\gamma(n-2)+\gamma^2(n^2-5n+5)}{(1-\gamma)[2+\gamma(2n-3)]}, \end{aligned}$$

profits under compliance and non-compliance are thus given by $\pi_k^B = \Phi^B z^2$ and $\pi_k^{DB} = \Phi^B[z + \Psi^B(z_k - z)]^2$.

A1.2.1.2 Firms $i \neq k$ The prices of the compliant firms ($i \neq k$) are given by (12):

$$\begin{aligned} p_i^B &= \frac{c}{2} + \frac{(1-\gamma)\alpha}{2[1+(n-2)\gamma]} + \frac{\gamma \sum_{j\neq i} p_j^B}{2[1+(n-2)\gamma]} \\ &= \frac{c}{2} + \frac{(1-\gamma)\alpha}{2[1+(n-2)\gamma]} + \frac{\gamma \sum p_j^B}{2[1+(n-2)\gamma]} - \frac{\gamma p_i^B}{2[1+(n-2)\gamma]}. \end{aligned}$$

Using (13) and rearrangement of terms yields

$$\begin{aligned} &[2+(2n-3)\gamma]p_i^B \\ &= [1+(n-2)\gamma]c + (1-\gamma)\alpha + \gamma\frac{[1+(n-2)\gamma][(n-1)c+c_k] + (1-\gamma)n\alpha}{2+(n-3)\gamma} \\ &= [2+(2n-3)\gamma]c - [1+(n-1)\gamma]c \\ &\quad + \frac{\gamma[1+(n-2)\gamma][(n-1)c+c_k] + (1-\gamma)[2+(2n-3)\gamma]\alpha}{2+(n-3)\gamma} \\ &= [2+(2n-3)\gamma]c + \frac{\begin{array}{c}[-2-2(n-2)\gamma+(n-1)\gamma^2]c \\ +\gamma[1+(n-2)\gamma]c_k + (1-\gamma)[2+(2n-3)\gamma]\alpha\end{array}}{2+(n-3)\gamma}. \end{aligned}$$

Thus,

$$[2+(n-3)\gamma][2+(2n-3)\gamma](p_i^B - c) = (1-\gamma)[2+(2n-3)\gamma]z - \gamma[1+(n-2)\gamma](z_k - z).$$

Hence,

$$p_i^B = c + \frac{(1-\gamma)[2+(2n-3)\gamma]z - \gamma[1+(n-2)\gamma](z_k - z)}{[2+(n-3)\gamma][2+(2n-3)\gamma]}$$

Plugging the expression for $\gamma \sum_{j\neq i} p_j$, obtained by rearrangement of (12), into the expression for demand, derived in Häckner (2000), we obtain

$$\begin{aligned} q_i^B &= \frac{(1-\gamma)\alpha - [1+(n-2)\gamma]p_i^B + \gamma \sum_{j\neq i} p_j^B}{(1-\gamma)[1+(n-1)\gamma]} \\ &= \frac{1+(n-2)\gamma}{(1-\gamma)[1+(n-1)\gamma]}(p_i^B - c). \end{aligned}$$

Thus, to obtain an interior solution, it has to be that

$$\begin{aligned} &(1-\gamma)[2+(2n-3)\gamma]z - \gamma[1+(n-2)\gamma](z_k - z) > 0 \\ \Leftrightarrow &(1-\gamma)[2+(2n-3)\gamma]t > \gamma[1+(n-2)\gamma](1-t) \\ \Leftrightarrow &t > \frac{\gamma[1+(n-2)\gamma]}{2+2(n-2)\gamma-(n-1)\gamma^2} \equiv \bar{t}^B. \end{aligned}$$

A1.2.2 Corner solutions

When $t \leq \bar{t}^B$, firms $i \neq k$ cease to produce. Using (11), firm k's price and quantity are then given by

$$\begin{aligned}
\overline{p}_k^B &= \frac{c_k}{2} + \frac{(1-\gamma)\alpha}{2[1+(n-2)\gamma]} + \frac{(n-1)\gamma c}{2[1+(n-2)\gamma]} \\
&= c_k + \frac{(1-\gamma)z + [1+(n-2)\gamma](z_k - z)}{2[1+(n-2)\gamma]}, \\
\overline{q}_k^B &= \frac{[1+(n-2)\gamma]}{(1-\gamma)[1+(n-1)\gamma]}(\overline{p}_k^B - c_k).
\end{aligned}$$

Defining

$$\begin{aligned}
\overline{\Phi}^B &= \frac{1-\gamma}{4[1+(n-1)\gamma][1+(n-2)\gamma]}, \\
\overline{\Psi}^B &= \frac{1+(n-2)\gamma}{1-\gamma},
\end{aligned}$$

the profit when violating legislation is given by $\pi^{DB} = \overline{\Phi}^B [z + \overline{\Psi}^B (z_k - z)]^2$.

A2 Proof proposition 2

A2.1 $\overline{\Delta}^C$ increases in α

It is easily verified that

$$\frac{\partial \overline{\Delta}^C}{\partial \alpha} = \frac{z_k}{2} - 2\Phi^C z.$$

Since $\Phi^C \leq \frac{1}{4}$ and $z_k > z$, it immediately follows that $\frac{\partial \overline{\Delta}^C}{\partial \alpha} > 0$.

A2.2 $\overline{\Delta}^B$ increases in α

It is easily established that

$$\begin{aligned}
\frac{\partial \overline{\Delta}^B}{\partial \alpha} &= 2\overline{\Phi}^B [z + (z_k - z)\overline{\Psi}^B] - 2\Phi^B z \\
&= 2z_k\{\overline{\Phi}^B \overline{\Psi}^B - [\Phi^B - \overline{\Phi}^B + \overline{\Phi}^B \overline{\Psi}^B] t\}.
\end{aligned}$$

When $\gamma = 1$, $\Phi^B = \overline{\Phi}^B = 0$ and $\overline{\Phi}^B \overline{\Psi}^B = \frac{1}{4n}$; since $t < 1$, it immediately follows that $\frac{\partial \overline{\Delta}^B}{\partial \alpha} > 0$. When $\gamma < 1$, it can be concluded that, because $\Phi^B \geq \overline{\Phi}^B$ and $t \leq \bar{t}^B$,

$$\frac{\partial \overline{\Delta}^B}{\partial \alpha} \geq 2z_k\{\overline{\Phi}^B \overline{\Psi}^B - [\Phi^B - \overline{\Phi}^B + \overline{\Phi}^B \overline{\Psi}^B] \bar{t}^B\}.$$

Rearrangement of the term inside the bracket yields

$$
\begin{aligned}
&\overline{\Phi}^B\overline{\Psi}^B - [\Phi^B - \overline{\Phi}^B + \overline{\Phi}^B\overline{\Psi}^B]\overline{t}^B \\
&= \frac{1}{4[1+(n-1)\gamma]} - \frac{\gamma[1+(n-2)\gamma]}{2+2(n-2)\gamma-(n-1)\gamma^2}\left\{\begin{array}{c} \frac{(1-\gamma)[1+(n-2)\gamma]}{[1+(n-1)\gamma][2+(n-3)\gamma]^2} \\ -\frac{1-\gamma}{4[1+(n-1)\gamma][1+(n-2)\gamma]} + \frac{1}{4[1+(n-1)\gamma]} \end{array}\right\} \\
&= \frac{2+2(n-2)\gamma-2(n-1)\gamma^2 - \frac{4\gamma(1-\gamma)[1+(n-2)\gamma]^2}{[2+(n-3)\gamma]^2}}{4[1+(n-1)\gamma][2+2(n-2)\gamma-(n-1)\gamma^2]} \\
&= (1-\gamma)\frac{[2+2(n-1)\gamma][2+(n-3)\gamma]^2 - 4\gamma[1+(n-2)\gamma]^2}{4[1+(n-1)\gamma][2+(n-3)\gamma]^2[2+2(n-2)\gamma-(n-1)\gamma^2]} \\
&= (1-\gamma)\frac{8+(16n-36)\gamma+(10n^2-52n+58)\gamma^2+(2n^3-18n^2+46n-34)\gamma^3}{4[1+(n-1)\gamma][2+(n-3)\gamma]^2[2+2(n-2)\gamma-(n-1)\gamma^2]} \\
&= (1-\gamma)\frac{(1-\gamma)\{4(1-\gamma)[2+(4n-5)\gamma]+10(n-2)^2\}+2(n-1)^2(n-2)}{4[1+(n-1)\gamma][2+(n-3)\gamma]^2[2+2(n-2)\gamma-(n-1)\gamma^2]}.
\end{aligned}
$$

This term is strictly positive for any n and hence, $\frac{\partial\overline{\Delta}^B}{\partial\alpha} > 0$ if $\gamma < 1$.

A3 Proof proposition 3

A3.1 Ω_γ^M decreases in γ

A3.1.1 Cournot

For the case when $t > \overline{t}^C$ it is easily established that

$$
\begin{aligned}
\frac{\partial\Phi^C/\partial\gamma}{\Phi^C} &= -\frac{2(n-1)}{2+(n-1)\gamma}, \\
\frac{\partial\Psi^C/\partial\gamma}{\Psi^C} &= \frac{2(n-1)}{(2-\gamma)[2+(n-2)\gamma]}.
\end{aligned}
$$

Hence,

$$
\Omega_\gamma^C = -\frac{\frac{\partial\Phi^M/\partial\gamma}{\Phi^M}}{\frac{\partial\Psi^M/\partial\gamma}{\Psi^M}} = \frac{(2-\gamma)[2+(n-2)\gamma]}{2+(n-1)\gamma}.
$$

It is easily verified that Ω_γ^C decreases in γ for all n.

A3.1.2 Bertrand

For the case when $t > \overline{t}^B$ it can be established that

$$
\begin{aligned}
\frac{\partial\Phi^B/\partial\gamma}{\Phi^B} &= -(n-1)\frac{2+2(2n-5)\gamma+2(n^2-5n+7)\gamma^2-(n^2-5n+6)\gamma^3}{(1-\gamma)[1+(n-1)\gamma][1+(n-2)\gamma][2+(n-3)\gamma]}, \\
\frac{\partial\Psi^B/\partial\gamma}{\Psi^B} &= (n-1)\frac{2+4(n-2)\gamma+(2n^2-7n+7)\gamma^2}{(1-\gamma)[2+(2n-3)\gamma][2+3(n-2)\gamma+(n^2-5n+5)\gamma^2]}.
\end{aligned}
$$

Hence,

$$\Omega_\gamma^B = -\frac{\frac{\partial \Phi^B/\partial\gamma}{\Phi^B}}{\frac{\partial \Psi^B/\partial\gamma}{\Psi^B}} = \frac{\begin{array}{c}[2+(2n-3)\gamma][2+3(n-2)\gamma+(n^2-5n+5)\gamma^2] \\ *[2+2(2n-5)\gamma+2(n^2-5n+7)\gamma^2-(n^2-5n+6)\gamma^3]\end{array}}{\begin{array}{c}[1+(n-1)\gamma][1+(n-2)\gamma][2+(n-3)\gamma] \\ *[2+4(n-2)\gamma+(2n^2-7n+7)\gamma^2]\end{array}}.$$

It is easily verified that Ω_γ^B decreases in γ for $n = 1$ and $n = 2$. For $n \geq 3$, the impact of γ on Ω_γ^B is assessed as follows:

$$\frac{\partial \Omega_\gamma^B/\partial\gamma}{\Omega_\gamma^B} = \frac{2n-3}{2+(2n-3)\gamma} + \frac{3(n-2)+2(n^2-5n+5)\gamma}{2+3(n-2)\gamma+(n^2-5n+5)\gamma^2}$$
$$+\frac{2(2n-5)+4(n^2-5n+7)\gamma-3(n^2-5n+6)\gamma^2}{2+2(2n-5)\gamma+2(n^2-5n+7)\gamma^2-(n^2-5n+6)\gamma^3} - \frac{n-1}{1+(n-1)\gamma}$$
$$-\frac{n-2}{1+(n-2)\gamma} - \frac{n-3}{2+(n-3)\gamma} - \frac{4(n-2)+2(2n^2-7n+7)\gamma}{2+4(n-2)\gamma+(2n^2-7n+7)\gamma^2}$$

$$= -\frac{1}{[1+(n-1)\gamma][2+(2n-3)\gamma]}$$
$$+\frac{2-4\gamma-(n^2-2n-1)\gamma^2}{[1+(n-2)\gamma][2+(n-3)\gamma][2+3(n-2)\gamma+(n^2-5n+5)\gamma^2]}$$
$$+\frac{2(2n-5)+4(n^2-5n+7)\gamma-3(n^2-5n+6)\gamma^2}{2+2(2n-5)\gamma+2(n^2-5n+7)\gamma^2-(n^2-5n+6)\gamma^3}$$
$$-\frac{4(n-2)+2(2n^2-7n+7)\gamma}{2+4(n-2)\gamma+(2n^2-7n+7)\gamma^2}$$
$$= -\frac{\left\{\begin{array}{c}4(n-2)\gamma+(11n^2-37n+36)\gamma^2+(13n^3-48n^2+84n-54)\gamma^3 \\ +(3n^4-19n^3+47n^2-51n+27)\gamma^4\end{array}\right\}}{\left\{\begin{array}{c}[1+(n-1)\gamma][1+(n-2)\gamma][2+(n-3)\gamma] \\ *[2+(2n-3)\gamma][2+3(n-2)\gamma+(n^2-5n+5)\gamma^2]\end{array}\right\}}$$
$$-\frac{\left\{\begin{array}{c}4+4(3n-7)\gamma+(14n^2-68n+78)\gamma^2 \\ +8(n-2)^2(n-3)\gamma^3+(n-2)(n-3)(2n^2-7n+7)\gamma^4\end{array}\right\}}{\left\{\begin{array}{c}[2+4(n-2)\gamma+(2n^2-7n+7)\gamma^2] \\ *[2+2(2n-5)\gamma+2(n^2-5n+7)\gamma^2-(n^2-5n+6)\gamma^3]\end{array}\right\}}.$$

Since the first term is negative for $n \geq 2$ and the second term is negative for $n \geq 3$, it can be concluded that Ω_γ^B decreases in γ for all n.

A3.2 $\overline{\Delta}^M$ increases in γ

A3.2.1 Cournot

For the case when $t \leq \overline{t}^C$ it is easily verified that $\frac{\partial \overline{\Delta}^C}{\partial\gamma} = -\frac{\partial \Phi^C}{\partial\gamma} z^2 > 0$ for $n > 1$ and $\frac{\partial \overline{\Delta}^C}{\partial\gamma} = 0$ for $n = 1$.

A3.2.2 Bertrand

For the case when $t \leq \bar{t}^B$ the impact of a change in γ is assessed as follows:

$$
\begin{aligned}
\frac{\partial \overline{\Delta}^B}{\partial \gamma} &= [z + \overline{\Psi}^B(z_k - z)]^2 \frac{\partial \overline{\Phi}^B}{\partial \gamma} + 2(z_k - z)[z + \overline{\Psi}^B(z_k - z)]\overline{\Phi}^B \frac{\partial \overline{\Psi}^B}{\partial \gamma} - z^2 \frac{\partial \Phi^B}{\partial \gamma} \\
&= [z + \overline{\Psi}^B(z_k - z)]^2 \overline{\Phi}^B \left[\frac{\partial \overline{\Phi}^B / \partial \gamma}{\overline{\Phi}^B} + \frac{2(\frac{1}{t} - 1)}{1 + \overline{\Psi}^B(\frac{1}{t} - 1)} \frac{\partial \overline{\Psi}^B}{\partial \gamma} \right] - z^2 \frac{\partial \Phi^B}{\partial \gamma}.
\end{aligned}
$$

It can be established that

$$
\begin{aligned}
\frac{\partial \overline{\Phi}^B / \partial \gamma}{\overline{\Phi}^B} &= -(n-1) \frac{2 + 2(n-2)\gamma - (n-2)\gamma^2}{(1-\gamma)[1 + (n-1)\gamma][1 + (n-2)\gamma]}, \\
\frac{\partial \overline{\Psi}^B}{\partial \gamma} &= \frac{n-1}{(1-\gamma)^2}.
\end{aligned}
$$

Since $\frac{\partial \Phi^B}{\partial \gamma} = 0$ for $n = 1$, it immediately follows that $\frac{\partial \overline{\Delta}^B}{\partial \gamma} = 0$ for $n = 1$. When $n > 1$,

$$
\begin{aligned}
\frac{2(\frac{1}{t} - 1)}{1 + \overline{\Psi}^B(\frac{1}{t} - 1)} \frac{\partial \overline{\Psi}^B}{\partial \gamma} &\geq \frac{2(\frac{1}{\bar{t}^B} - 1)}{1 + \overline{\Psi}^B(\frac{1}{\bar{t}^B} - 1)} \frac{\partial \overline{\Psi}^B}{\partial \gamma} \\
&= \frac{2 \frac{2 + (2n-5)\gamma - (2n-3)\gamma^2}{\gamma[1 + (n-2)\gamma]}}{1 + \frac{1 + (n-2)\gamma}{1-\gamma} \frac{2 + (2n-5)\gamma - (2n-3)\gamma^2}{\gamma[1 + (n-2)\gamma]}} \frac{n-1}{(1-\gamma)^2} \\
&= (n-1) \frac{2 + (2n-3)\gamma}{(1-\gamma)[1 + (n-1)\gamma][1 + (n-2)\gamma]},
\end{aligned}
$$

and hence,

$$
\begin{aligned}
\frac{\partial \overline{\Phi}^B / \partial \gamma}{\overline{\Phi}^B} + \frac{2(\frac{1}{t} - 1)}{1 + \overline{\Psi}^B(\frac{1}{t} - 1)} \frac{\partial \overline{\Psi}^B}{\partial \gamma} &\geq -(n-1) \frac{2 + 2(n-2)\gamma - (n-2)\gamma^2}{(1-\gamma)[1 + (n-1)\gamma][1 + (n-2)\gamma]} \\
&\quad + (n-1) \frac{2 + (2n-3)\gamma}{(1-\gamma)[1 + (n-1)\gamma][1 + (n-2)\gamma]} \\
&= \frac{(n-1)\gamma}{(1-\gamma)[1 + (n-1)\gamma]^2},
\end{aligned}
$$

Since $\frac{\partial \Phi^B}{\partial \gamma} < 0$ for $n > 1$, it immediately follows that $\frac{\partial \overline{\Delta}^B}{\partial \gamma} > 0$ for $n > 1$.

A3.3 $\widehat{t}_\gamma^M > \bar{t}^M$

It is easily established that

$$\widehat{t}_\gamma^M - \bar{t}^M = \frac{(2-\Omega_\gamma^M)\Psi^M(1-\bar{t}^M) - 2(\Omega_\gamma^M - 1)\bar{t}^M}{2(\Omega_\gamma^M - 1) + (2-\Omega_\gamma^M)\Psi^M}.$$

A3.3.1 Cournot

In the Cournot case the numerator is given by

$$\begin{aligned}
&(2-\Omega_\gamma^C)\Psi^C(1-\bar{t}^C) - 2(\Omega_\gamma^C - 1)\bar{t}^C \\
=&\gamma\frac{4+(n-2)\gamma}{2+(n-1)\gamma}\frac{2+(n-2)\gamma}{2-\gamma}(1-\frac{\gamma}{2}) - 2\frac{2+(n-5)\gamma-(n-2)\gamma^2}{2+(n-1)\gamma}\frac{\gamma}{2} \\
=&\gamma\frac{4+(4n-2)\gamma+(n^2-2n)\gamma^2}{2[2+(n-1)\gamma]} \geq 0.
\end{aligned}$$

Hence, $\widehat{t}_\gamma^C \geq \bar{t}^C$ for all n.

A3.3.1 Bertrand

In the Bertrand case the numerator is given by

$$\begin{aligned}
&(2-\Omega_\gamma^B)\Psi^B(1-\bar{t}^B) - 2(\Omega_\gamma^B - 1)\bar{t}^B \\
=&(2-\Omega_\gamma^B)\frac{2+3(n-2)\gamma+(n^2-5n+5)\gamma^2}{2+2(n-2)\gamma-(n-1)\gamma^2} - 2(\Omega_\gamma^B - 1)\frac{\gamma[1+(n-2)\gamma]}{2+2(n-2)\gamma-(n-1)\gamma^2} \\
=&\frac{2[2+(3n-5)\gamma+(n^2-4n+3)\gamma^2] - [2+(3n-4)\gamma+(n^2-3n+1)\gamma^2]\Omega_\gamma^B}{2+2(n-2)\gamma-(n-1)\gamma^2}.
\end{aligned}$$

When $n=1$ this term equals $\frac{\gamma(4+2\gamma-\gamma^2)}{4}$, which is positive. For $n>1$ it the case that

$$\begin{aligned}
&2[2+(3n-5)\gamma+(n^2-4n+3)\gamma^2] \geq [2+(3n-4)\gamma+(n^2-3n+1)\gamma^2]\Omega_\gamma^B \\
\Leftrightarrow&\left\{\begin{array}{c} 2[1+(n-1)\gamma][1+(n-2)\gamma][2+(n-3)\gamma] \\ *[2+(3n-5)\gamma+(n^2-4n+3)\gamma^2][2+4(n-2)\gamma+(2n^2-7n+7)\gamma^2] \end{array}\right\} \\
\geq&\left\{\begin{array}{c} [2+(2n-3)\gamma][2+3(n-2)\gamma+(n^2-5n+5)\gamma^2][2+(3n-4)\gamma+(n^2-3n+1)\gamma^2] \\ *[2+2(2n-5)\gamma+2(n^2-5n+7)\gamma^2-(n^2-5n+6)\gamma^3] \end{array}\right\} \\
\Leftrightarrow&\left\{\begin{array}{c} [2+(2n-2)\gamma][2+(3n-7)\gamma+(n^2-5n+6)\gamma^2] \\ *[2+(3n-5)\gamma+(n^2-4n+3)\gamma^2][2+4(n-2)\gamma+(2n^2-7n+7)\gamma^2] \end{array}\right\} \\
\geq&\left\{\begin{array}{c} [2+(2n-2)\gamma-\gamma][2+(3n-7)\gamma+(n^2-5n+6)\gamma^2+\gamma(1-\gamma)] \\ *\{2+(3n-5)\gamma+(n^2-4n+3)\gamma^2+\gamma[1+(n-2)\gamma]\} \\ *\{2+4(n-2)\gamma+(2n^2-7n+7)\gamma^2-\gamma[1+(n-2)\gamma][2+(n-3)\gamma]\} \end{array}\right\} \\
=&\{[2+(2n-2)\gamma][2+(3n-7)\gamma+(n^2-5n+6)\gamma^2]-\gamma^2[(n-2)+(n^2-3n+3)\gamma]\} \\
&*\left\{\begin{array}{c} [2+(3n-5)\gamma+(n^2-4n+3)\gamma^2][2+4(n-2)\gamma+(2n^2-7n+7)\gamma^2] \\ -\gamma[1+(n-2)\gamma][2+(4n-6)\gamma+(3n^2-12n+7)\gamma^2+(n^3-6n^2+10n-3)\gamma^3] \end{array}\right\}
\end{aligned}$$

It is easy to see that this inequality holds for all $n > 1$. Thus, $\widehat{t}_{\gamma}^{B} \geq \bar{t}^{B}$ for all n.

A4 Proof proposition 4

A4.1 Ω_n^M decreases in γ

A4.1.1 Cournot

For the case when $t > \bar{t}^C$ it is easily established that

$$\frac{\partial \Phi^C / \partial n}{\Phi^C} = -\frac{\gamma}{2 + (n-1)\gamma},$$
$$\frac{\partial \Psi^C / \partial n}{\Psi^C} = \frac{\gamma}{2 + (n-2)\gamma}.$$

Hence,

$$\Omega_n^C = -\frac{\frac{\partial \Phi^C / \partial n}{\Phi^C}}{\frac{\partial \Psi^C / \partial n}{\Psi^C}} = \frac{2[2 + (n-2)\gamma]}{2 + (n-1)\gamma}.$$

It is straightforward that Ω_n^C decreases in γ for all n.

A4.1.2 Bertrand

For the case when $t > \bar{t}^B$ it can be established that

$$\frac{\partial \Phi^B / \partial n}{\Phi^B} = -\gamma \frac{2 + 4(n-2)\gamma + (2n^2 - 7n + 7)\gamma^2}{[1 + (n-1)\gamma][1 + (n-2)\gamma][2 + (n-3)\gamma]},$$
$$\frac{\partial \Psi^B / \partial n}{\Psi^B} = \gamma \frac{2 + (4n-7)\gamma + (2n^2 - 6n + 5)\gamma^2}{[2 + (2n-3)\gamma][2 + 3(n-2)\gamma + (n^2 - 5n + 5)\gamma^2]}.$$

Hence,

$$\Omega_n^B = -\frac{\frac{\partial \Phi^B / \partial n}{\Phi^B}}{\frac{\partial \Psi^B / \partial n}{\Psi^B}} = \frac{\begin{array}{c}[2 + (2n-3)\gamma][2 + 3(n-2)\gamma + (n^2 - 5n + 5)\gamma^2] \\ *[2 + 4(n-2)\gamma + (2n^2 - 7n + 7)\gamma^2]\end{array}}{\begin{array}{c}[1 + (n-1)\gamma][1 + (n-2)\gamma][2 + (n-3)\gamma] \\ *[2 + (4n-7)\gamma + (2n^2 - 6n + 5)\gamma^2]\end{array}}.$$

It can be established that Ω_n^B decreases in γ for $n = 1$. For $n \geq 2$, the impact of γ on Ω_n^B is assessed as follows:

$$\frac{\partial\Omega_n^B/\partial\gamma}{\Omega_n^B} = \frac{2n-3}{2+(2n-3)\gamma} + \frac{3(n-2)+2(n^2-5n+5)\gamma}{2+3(n-2)\gamma+(n^2-5n+5)\gamma^2}$$
$$+\frac{4(n-2)+2(2n^2-7n+7)\gamma}{2+4(n-2)\gamma+(2n^2-7n+7)\gamma^2} - \frac{n-1}{1+(n-1)\gamma}$$
$$-\frac{n-2}{1+(n-2)\gamma} - \frac{n-3}{2+(n-3)\gamma} - \frac{(4n-7)+2(2n^2-6n+5)\gamma}{2+(4n-7)\gamma+(2n^2-6n+5)\gamma^2}$$
$$= -\frac{\begin{array}{c}4(n-2)\gamma+(11n^2-37n+36)\gamma^2+(13n^3-48n^2+84n-54)\gamma^3\\ +(3n^4-19n^3+47n^2-51n+27)\gamma^4\end{array}}{\begin{array}{c}[1+(n-1)\gamma][1+(n-2)\gamma][2+(n-3)\gamma]\\ *[2+(2n-3)\gamma][2+3(n-2)\gamma+(n^2-5n+5)\gamma^2]\end{array}}$$
$$-\frac{2+4(n-2)+(2n^2-9n+9)\gamma^2}{[2+4(n-2)\gamma+(2n^2-7n+7)\gamma^2][2+(4n-7)\gamma+(2n^2-6n+5)\gamma^2]}.$$

The first term is negative for $n \geq 2$ and the second term is negative for $n \geq 1$. Hence, Ω_n^B decreases in γ for all n.

A4.2 $\overline{\Delta}^M$ increases in n

A4.2.1 Cournot

For the case when $t \leq \bar{t}^C$ it is easily verified that $\frac{\partial\overline{\Delta}^C}{\partial n} = -\frac{\partial\Phi^C}{\partial n}z^2 > 0$ for $\gamma > 0$ and $\frac{\partial\overline{\Delta}^C}{\partial n} = 0$ for $\gamma = 0$.

A4.2.2 Bertrand

For the case when $t \leq \bar{t}^B$ it is easily established that

$$\frac{\partial\overline{\Delta}^B}{\partial n} = [z+\overline{\Psi}^B(z_k-z)]^2\frac{\partial\overline{\Phi}^B}{\partial n} + 2(z_k-z)[z+\overline{\Psi}^B(z_k-z)]\overline{\Phi}^B\frac{\partial\overline{\Psi}^B}{\partial n} - z^2\frac{\partial\Phi^B}{\partial n}$$
$$= [z+\overline{\Psi}^B(z_k-z)]^2\overline{\Phi}^B\left[\frac{\partial\overline{\Phi}^B/\partial n}{\overline{\Phi}^B} + \frac{2(\frac{1}{t}-1)}{1+\overline{\Psi}^B(\frac{1}{t}-1)}\frac{\partial\overline{\Psi}^B}{\partial n}\right] - z^2\frac{\partial\Phi^B}{\partial n}.$$

It can be established that

$$\frac{\partial\overline{\Phi}^B/\partial n}{\overline{\Phi}^B} = -\gamma\frac{2+(2n-3)\gamma}{[1+(n-1)\gamma][1+(n-2)\gamma]},$$
$$\frac{\partial\overline{\Psi}^B}{\partial n} = \frac{\gamma}{1-\gamma}.$$

Since $\frac{\partial\Phi^B}{\partial n} = 0$ for $\gamma = 0$, it immediately follows that $\frac{\partial\overline{\Delta}^B}{\partial n} = 0$ for $\gamma = 0$.
When $\gamma > 0$,

$$\frac{2(\frac{1}{t}-1)}{1+\overline{\Psi}^B(\frac{1}{t}-1)}\frac{\partial\overline{\Psi}^B}{\partial n} \geq \frac{2(\frac{1}{\overline{t}^B}-1)}{1+\overline{\Psi}^B(\frac{1}{\overline{t}^B}-1)}\frac{\partial\overline{\Psi}^B}{\partial n} = \frac{2\frac{2+(2n-5)\gamma-(2n-3)\gamma^2}{\gamma[1+(n-2)\gamma]}}{1+\frac{1+(n-2)\gamma}{1-\gamma}\frac{2+(2n-5)\gamma-(2n-3)\gamma^2}{\gamma[1+(n-2)\gamma]}}\frac{\gamma}{1-\gamma}$$
$$= \frac{2\frac{2+(2n-5)\gamma-(2n-3)\gamma^2}{[1+(n-2)\gamma]}}{1-\gamma+\frac{2+(2n-5)\gamma-(2n-3)\gamma^2}{\gamma}} = \gamma\frac{2+(2n-3)\gamma}{[1+(n-1)\gamma][1+(n-2)\gamma]},$$

and hence,

$$\frac{\partial\overline{\Phi}^B/\partial n}{\overline{\Phi}^B} + \frac{2(\frac{1}{t}-1)\partial\overline{\Psi}^B/\partial n}{1+\overline{\Psi}^B(\frac{1}{t}-1)} \geq 0.$$

Since $\frac{\partial\Phi^B}{\partial n} < 0$ for $\gamma > 0$, it is straightforward that $\frac{\partial\overline{\Delta}^B}{\partial n} > 0$.

A4.3 $\widetilde{t}_n^M > \overline{t}^M$

It is easily established that

$$\widetilde{t}_n^M - \overline{t}^M = \frac{(2-\Omega_n^M)\Psi^M(1-\overline{t}^M) - 2(\Omega_n^M-1)\overline{t}^M}{2(\Omega_n^M-1)+(2-\Omega_n^M)\Psi^M}.$$

A4.3.1 Cournot

In the Cournot case the numerator is given by

$$(2-\Omega_n^C)\Psi^C(1-\overline{t}^C) - 2(\Omega_n^C-1)\overline{t}^C$$
$$= \frac{2\gamma}{2+(n-1)\gamma}\frac{2+(n-2)\gamma}{2-\gamma}(1-\frac{\gamma}{2}) - 2\frac{2+(n-3)\gamma}{2+(n-1)\gamma}\frac{\gamma}{2} = \frac{\gamma^2}{2+(n-1)\gamma} \geq 0.$$

Hence, $\widetilde{t}_n^C \geq \overline{t}^C$ for all n and all γ.

A4.3.1 Bertrand

In the Bertrand case the numerator is given by

$$(2-\Omega_n^B)\Psi^B(1-\overline{t}^B) - 2(\Omega_n^B-1)\overline{t}^B$$
$$= (2-\Omega_n^B)\frac{2+3(n-2)\gamma+(n^2-5n+5)\gamma^2}{2+2(n-2)\gamma-(n-1)\gamma^2} - 2(\Omega_n^B-1)\frac{\gamma[1+(n-2)\gamma]}{2+2(n-2)\gamma-(n-1)\gamma^2}$$
$$= \frac{2[2+(3n-5)\gamma+(n^2-4n+3)\gamma^2] - [2+(3n-4)\gamma+(n^2-3n+1)\gamma^2]\Omega_n^B}{2+2(n-2)\gamma-(n-1)\gamma^2}.$$

For this term to be positive, it has to be that

$$2[2+(3n-5)\gamma+(n^2-4n+3)\gamma^2] \geq [2+(3n-4)\gamma+(n^2-3n+1)\gamma^2]\Omega_n^B$$

$$\Leftrightarrow \left\{ \begin{array}{c} 2[1+(n-2)\gamma][2+(4n-7)\gamma+(2n^2-6n+5)\gamma^2] \\ *[2+(3n-5)\gamma+(n^2-4n+3)\gamma^2][1+(n-1)\gamma][2+(n-3)\gamma] \end{array} \right\}$$

$$\geq \left\{ \begin{array}{c} [2+(2n-3)\gamma][2+4(n-2)\gamma+(2n^2-7n+7)\gamma^2] \\ *[2+3(n-2)\gamma+(n^2-5n+5)\gamma^2][2+(3n-4)\gamma+(n^2-3n+1)\gamma^2] \end{array} \right\}$$

$$\Leftrightarrow [2+2(n-2)\gamma][2+(4n-7)\gamma+(2n^2-6n+5)\gamma^2][2+(3n-5)\gamma+(n^2-4n+3)\gamma^2]^2$$

$$\geq \left\{ \begin{array}{c} [2+2(n-2)\gamma+\gamma]\{2+(4n-7)\gamma+(2n^2-6n+5)\gamma^2-\gamma[1+(n-2)\gamma]\} \\ *\{[2+(3n-5)\gamma+(n^2-4n+3)\gamma^2]-\gamma[1+(n-2)\gamma]\} \\ *\{[2+(3n-5)\gamma+(n^2-4n+3)\gamma^2]+\gamma[1+(n-2)\gamma]\} \end{array} \right\}$$

$$= \{[2+2(n-2)\gamma][2+(4n-7)\gamma+(2n^2-6n+5)\gamma^2]+(n-1)\gamma^3\}$$
$$*\{[2+(3n-5)\gamma+(n^2-4n+3)\gamma^2]^2-\gamma^2[1+(n-2)\gamma]^2\}$$
$$= [2+2(n-2)\gamma][2+(4n-7)\gamma+(2n^2-6n+5)\gamma^2][2+(3n-5)\gamma+(n^2-4n+3)\gamma^2]^2$$
$$+(n-1)\gamma^3[2+(3n-5)\gamma+(n^2-4n+3)\gamma^2]^2$$
$$-2\gamma^2[1+(n-2)\gamma]^3[2+(4n-7)\gamma+(2n^2-6n+5)\gamma^2].$$

Since

$$(n-1)\gamma[2+(3n-5)\gamma+(n^2-4n+3)\gamma^2]^2$$
$$-2[1+(n-2)\gamma]^3[2+(4n-7)\gamma+(2n^2-6n+5)\gamma^2]$$
$$=4(n-1)\gamma+4(3n^2-8n+5)\gamma^2+(13n^3-59n^2+83n-37)\gamma^3$$
$$+2(3n^4-20n^3+46n^2-44n+15)\gamma^4+(n^5-9n^4+30n^3-46n^2+33n-9)\gamma^5$$
$$-4+-2(10n-19)\gamma-2(20n^2-75n+71)\gamma^2-2(20n^3-111n^2+207n-130)\gamma^3$$
$$-2(10n^4-73n^3+201n^2-248n+116)\gamma^4-2(2n^5-18n^4+65n^3-118n^2+108n-40)\gamma^5$$
$$=-4-(16n-34)\gamma-(28n^2-118n+122)\gamma^2-(27n^3-163n^2+331n-223)\gamma^3$$
$$-(14n^4-106n^3+310n^2-408n+202)\gamma^4-(3n^5-27n^4+100n^3-190n^2+183n-71)\gamma^5,$$

which is strictly negative for all n, it immediately follows that the above inequality holds for all n. Thus, $\widetilde{t}_n^B \geq \bar{t}^B$ for all n and all γ.

A5 Proof proposition 5

A5.1 T^* increases in γ

To assess the impact of changes in γ on T^*, rearrange terms in the following way:

$$T^* = \frac{\Psi^B - \sigma\Psi^C}{\sigma - 1} = \frac{\Psi^B - \Psi^C}{\sigma - 1} - \Psi^C = \Psi^C[\frac{\Psi^B - \Psi^C}{\Psi^C(\sigma - 1)} - 1].$$

It is easily established that

$$\Psi^B - \Psi^C = \frac{(n-1)^2\gamma^3}{(1-\gamma)(2-\gamma)[2+(2n-3)\gamma]}.$$

Rearrangement of σ yields

$$\sigma = \frac{\Phi^C\Psi^C}{\Phi^B\Psi^B} = \frac{[1+(n-1)\gamma][2+(n-2)\gamma][2+(n-3)\gamma]^2[2+(2n-3)\gamma]}{(2-\gamma)[1+(n-2)\gamma][2+(n-1)\gamma]^2[2+3(n-2)\gamma+(n^2-5n+5)\gamma^2]} \tag{14}$$

$$= \frac{[2+(3n-4)\gamma+(n^2-3n+2)\gamma^2][4+4(n-3)\gamma+(n^2-6n+9)\gamma^2][2+(2n-3)\gamma]}{\left\{\begin{array}{c}[2+(2n-5)\gamma-(n-2)\gamma^2][4+4(n-1)\gamma+(n^2-2n+1)\gamma^2]\\ *[2+3(n-2)\gamma+(n^2-5n+5)\gamma^2]\end{array}\right\}}$$

$$= \frac{\left[\begin{array}{c}8+20(n-2)\gamma+(18n^2-76n+74)\gamma^2\\ +(7n^3-46n^2+95n-60)\gamma^3+(n^4-9n^3+29n^2-39n+18)\gamma^4\end{array}\right][2+(2n-3)\gamma]}{\left[\begin{array}{c}8+(16n-28)\gamma+(10n^2-36n+30)\gamma^2\\ +(2n^3-13n^2+24n-13)\gamma^3-(n^3-4n^2+5n-2)\gamma^4\end{array}\right][2+3(n-2)\gamma+(n^2-5n+5)\gamma^2]}$$

$$= \frac{\left\{\begin{array}{c}16+(56n-104)\gamma+(76n^2-292n+268)\gamma^2+(50n^3-298n^2+566n-342)\gamma^3\\ +(16n^4-131n^3+386n^2-483n+216)\gamma^4+(2n^5-21n^4+85n^3-165n^2+153n-54)\gamma^5\end{array}\right\}}{\left\{\begin{array}{c}16+(56n-104)\gamma+(76n^2-292n+268)\gamma^2+(50n^3-302n^2+574n-346)\gamma^3\\ +(16n^4-139n^3+418n^2-523n+232)\gamma^4+(2n^5-26n^4+117n^3-237n^2+221n-77)\gamma^5\\ -(n^5-9n^4+30n^3-47n^2+35n-10)\gamma^6\end{array}\right\}}$$

$$= 1 + \frac{\left\{\begin{array}{c}(4n^2-8n+4)\gamma^3+(8n^3-32n^2+40n-16)\gamma^4\\ +(5n^4-32n^3+72n^2-68n+23)\gamma^5+(n^5-9n^4+30n^3-47n^2+35n-10)\gamma^6\end{array}\right\}}{(2-\gamma)[1+(n-2)\gamma][2+(n-1)\gamma]^2[2+3(n-2)\gamma+(n^2-5n+5)\gamma^2]}$$

$$= 1 + (n-1)^2\gamma^3\frac{4+8(n-2)\gamma+(5n^2-22n+23)\gamma^2+(n-2)(n^2-5n+5)\gamma^3}{(2-\gamma)[1+(n-2)\gamma][2+(n-1)\gamma]^2[2+3(n-2)\gamma+(n^2-5n+5)\gamma^2]}$$

$$= 1 + (n-1)^2\gamma^3\frac{[2+(n-2)\gamma][2+3(n-2)\gamma+(n^2-5n+5)\gamma^2]+\gamma^2}{(2-\gamma)[1+(n-2)\gamma][2+(n-1)\gamma]^2[2+3(n-2)\gamma+(n^2-5n+5)\gamma^2]}. \tag{15}$$

It is easy to see that $\sigma > 1$ for $n > 1$ and $\gamma > 0$. Let

$$\tau \equiv \frac{\Psi^B - \Psi^C}{\Psi^C(\sigma - 1)}$$

$$= \frac{(2-\gamma)[1+(n-2)\gamma][2+(n-1)\gamma]^2[2+3(n-2)\gamma+(n^2-5n+5)\gamma^2]}{(1-\gamma)[2+(n-2)\gamma][2+(2n-3)\gamma]\{[2+(n-2)\gamma][2+3(n-2)\gamma+(n^2-5n+5)\gamma^2]+\gamma^2\}}.$$

The impact of a change in γ on τ is assessed as follows:

$$\frac{\partial\tau/\partial\gamma}{\tau} = -\frac{1}{2-\gamma} + \frac{n-2}{1+(n-2)\gamma} + \frac{2(n-1)}{2+(n-1)\gamma} + \frac{3(n-2)+2(n^2-5n+5)\gamma}{2+3(n-2)\gamma+(n^2-5n+5)\gamma^2}$$

$$+\frac{1}{1-\gamma} - \frac{n-2}{2+(n-2)\gamma} - \frac{2n-3}{2+(2n-3)\gamma}$$

$$-\frac{\left\{\begin{array}{c}(n-2)[2+3(n-2)\gamma+(n^2-5n+5)\gamma^2]\\ +[2+(n-2)\gamma][3(n-2)+2(n^2-5n+5)\gamma]+2\gamma\end{array}\right\}}{[2+(n-2)\gamma][2+3(n-2)\gamma+(n^2-5n+5)\gamma^2]+\gamma^2}.$$

It is easily established that

$$-\frac{1}{2-\gamma} + \frac{1}{1-\gamma} = \frac{1}{(1-\gamma)(2-\gamma)} > 0,$$

and that

$$\frac{n-2}{1+(n-2)\gamma} + \frac{n-1}{2+(n-1)\gamma} - \frac{n-2}{2+(n-2)\gamma} - \frac{2n-3}{2+(2n-3)\gamma}$$

$$= \frac{n-2}{[1+(n-2)\gamma][2+(n-2)\gamma]} - \frac{2(n-2)}{[2+(n-1)\gamma][2+(2n-3)\gamma]}$$

$$= \frac{(n-2)\gamma[4+(3n-5)\gamma]}{[1+(n-2)\gamma][2+(n-1)\gamma][2+(n-2)\gamma][2+(2n-3)\gamma]},$$

which is positive for $n \geq 2$. Moreover,

$$\frac{n-1}{2+(n-1)\gamma}+\frac{3(n-2)+2(n^2-5n+5)\gamma}{2+3(n-2)\gamma+(n^2-5n+5)\gamma^2}$$
$$-\frac{\left\{\begin{array}{c}(n-2)[2+3(n-2)\gamma+(n^2-5n+5)\gamma^2]\\ +[2+(n-2)\gamma][3(n-2)+2(n^2-5n+5)\gamma]+2\gamma\end{array}\right\}}{[2+(n-2)\gamma][2+3(n-2)\gamma+(n^2-5n+5)\gamma^2]+\gamma^2}$$
$$=\frac{(n-1)[2+3(n-2)\gamma+(n^2-5n+5)\gamma^2]+[2+(n-1)\gamma][3(n-2)+2(n^2-5n+5)\gamma]}{[2+(n-1)\gamma][2+3(n-2)\gamma+(n^2-5n+5)\gamma^2]}$$
$$-\frac{\left\{\begin{array}{c}(n-2)[2+3(n-2)\gamma+(n^2-5n+5)\gamma^2]\\ +[2+(n-2)\gamma][3(n-2)+2(n^2-5n+5)\gamma]+2\gamma\end{array}\right\}}{[2+(n-2)\gamma][2+3(n-2)\gamma+(n^2-5n+5)\gamma^2]+\gamma^2}$$
$$=\frac{\left\{\begin{array}{c}2[2+3(n-2)\gamma+(n^2-5n+5)\gamma^2]^2+(n-1)\gamma^2[2+3(n-2)\gamma+(n^2-5n+5)\gamma^2]\\ +\gamma^2[2+(n-1)\gamma][3(n-2)+2(n^2-5n+5)\gamma]\\ -2\gamma[2+(n-1)\gamma][2+3(n-2)\gamma+(n^2-5n+5)\gamma^2]\end{array}\right\}}{[2+3(n-2)\gamma+(n^2-5n+5)\gamma^2]\{[2+(n-2)\gamma][2+3(n-2)\gamma+(n^2-5n+5)\gamma^2]+\gamma^2\}}$$
$$=\frac{\left\{\begin{array}{c}2[2+3(n-2)\gamma+(n^2-5n+5)\gamma^2]^2-\gamma[4+(n-1)\gamma][2+3(n-2)\gamma+(n^2-5n+5)\gamma^2]\\ +\gamma^2[2+(n-1)\gamma][3(n-2)+2(n^2-5n+5)\gamma]\end{array}\right\}}{[2+3(n-2)\gamma+(n^2-5n+5)\gamma^2]\{[2+(n-2)\gamma][2+3(n-2)\gamma+(n^2-5n+5)\gamma^2]+\gamma^2\}}$$
$$=\frac{\left\{\begin{array}{c}[2+3(n-2)\gamma+(n^2-5n+5)\gamma^2][4+(6n-16)\gamma+(2n^2-11n+11)\gamma^2]\\ +\gamma^2[2+(n-1)\gamma][3(n-2)+2(n^2-5n+5)\gamma]\end{array}\right\}}{[2+3(n-2)\gamma+(n^2-5n+5)\gamma^2]\{[2+(n-2)\gamma][2+3(n-2)\gamma+(n^2-5n+5)\gamma^2]+\gamma^2\}},$$

which is positive for $n \geq 3$. Since $\frac{\partial\tau/\partial\gamma}{\tau}=\frac{2(1-\gamma)+\gamma^2}{(1-\gamma)(2-\gamma^2)}>0$ for $n=2$, it can be concluded that $\frac{\partial\tau/\partial\gamma}{\tau}>0$ for $n \geq 2$. Since Ψ^C also increases in γ, it immediately follows that T^* increases in γ when $n \geq 2$.

A5.2 $t^* \geq \bar{t}^B$

To show that $t^* \geq \bar{t}^B$, the following rearrangements can be made:

$$t^*=\frac{T^*}{2+T^*}\geq \bar{t}^B \Leftrightarrow T^*\geq\frac{2\bar{t}^B}{1-\bar{t}^B}\Leftrightarrow\frac{\Psi^B-\sigma\Psi^C}{\sigma-1}\geq\frac{2\bar{t}^B}{1-\bar{t}^B}$$
$$\Leftrightarrow\sigma(\Psi^B-\Psi^C)\geq(\sigma-1)(\frac{2\bar{t}^B}{1-\bar{t}^B}+\Psi^B).$$

Since

$$\bar{t}^B=1-\frac{(1-\gamma)[2+(2n-3)\gamma]}{2+2(n-2)\gamma-(n-1)\gamma^2},$$

it immediately follows that

$$\frac{2\bar{t}^B}{1-\bar{t}^B}=\frac{2\gamma[1+(n-2)\gamma]}{(1-\gamma)[2+(2n-3)\gamma]}.$$

It is easily verified that

$$\Psi^B - \Psi^C = \frac{(n-1)^2\gamma^3}{(1-\gamma)(2-\gamma)[2+(2n-3)\gamma]},$$
$$\frac{2\bar{t}^B}{1-\bar{t}^B} + \Psi^B = \frac{2+(3n-4)\gamma+(n^2-3n+1)\gamma^2}{(1-\gamma)[2+(2n-3)\gamma]}.$$

Using expressions (14) and (15), it follows that

$$\begin{aligned}
&\sigma(\Psi^B - \Psi^C) \geq (\sigma-1)(\frac{2\bar{t}^B}{1-\bar{t}^B} + \Psi^B) \\
\Leftrightarrow &[1+(n-1)\gamma][2+(n-2)\gamma][2+(n-3)\gamma]^2[2+(2n-3)\gamma] \\
\geq &(2-\gamma)\{[2+(n-2)\gamma][2+3(n-2)\gamma+(n^2-5n+5)\gamma^2]+\gamma^2\} \\
&*[2+(3n-4)\gamma+(n^2-3n+1)\gamma^2].
\end{aligned}$$

Since

$$[1+(n-1)\gamma][2+(n-2)\gamma] = 2+(3n-4)\gamma+(n^2-3n+2)\gamma^2 \geq 2+(3n-4)\gamma+(n^2-3n+1)\gamma^2,$$

and

$$\begin{aligned}
&(2-\gamma)\{[2+(n-2)\gamma][2+3(n-2)\gamma+(n^2-5n+5)\gamma^2]+\gamma^2\} \\
= &8+(16n-36)\gamma+(10n^2-48n+54)\gamma^2+(2n^3-15n^2+36n-27)\gamma^3 \\
&-4(n-2)\gamma^2-4(n-2)^2\gamma^2-(n-2)(n^2-5n+5)\gamma^4 \\
\leq &8+(16n-36)\gamma+(10n^2-48n+54)\gamma^2+(2n^3-15n^2+36n-27)\gamma^3 \\
= &[2+(n-3)\gamma]^2[2+(2n-3)\gamma],
\end{aligned}$$

it immediately follows that $t^* \geq \bar{t}^B$ when $n \geq 2$.

A5.3 $\overline{\Delta}^B > \Delta^C$ when $t \in (\bar{t}^C, \bar{t}^B]$

Let

$$\delta^B \equiv \frac{\overline{\Phi}^B}{\Phi^B}\frac{[1+(\frac{1}{t}-1)\overline{\Psi}^B]^2}{[1+(\frac{1}{t}-1)\Psi^B]^2}.$$

The impact of t on δ^B is assessed as follows:

$$\frac{\partial\delta^B/\partial t}{\delta^B} = -\frac{2\overline{\Psi}^B}{1+(\frac{1}{t}-1)\overline{\Psi}^B}\frac{1}{t^2}+\frac{2\Psi^B}{1+(\frac{1}{t}-1)\Psi^B}\frac{1}{t^2} = \frac{\Psi^B-\overline{\Psi}^B}{[1+(\frac{1}{t}-1)\Psi^B][1+(\frac{1}{t}-1)\overline{\Psi}^B]}\frac{2}{t^2},$$

It is easily verified that $\Psi^B \leq \overline{\Psi}^B$. Hence, $\frac{\partial \delta^B}{\partial t} \leq 0$ and

$$\begin{aligned}
\delta^B &\geq \frac{\overline{\Phi}^B}{\Phi^B} \frac{[1+(\frac{1}{\bar{t}^B}-1)\overline{\Psi}^B]^2}{[1+(\frac{1}{\bar{t}^B}-1)\Psi^B]^2} \\
&= \frac{[1+(n-1)\gamma][2+(n-3)\gamma]^2}{4[1+(n-1)\gamma][1+(n-2)\gamma]^2} \frac{[1+\frac{(1-\gamma)[2+(2n-3)\gamma]}{\gamma[1+(n-2)\gamma]}\frac{1+(n-2)\gamma}{1-\gamma}]^2}{[1+\frac{(1-\gamma)[2+(2n-3)\gamma]}{\gamma[1+(n-2)\gamma]}\frac{2+3(n-2)\gamma+(n^2-5n+5)\gamma^2}{(1-\gamma)[2+(2n-3)\gamma]}]^2} \\
&= \frac{[1+(n-1)\gamma][2+(n-3)\gamma]^2}{4[1+(n-1)\gamma][1+(n-2)\gamma]^2} \left[\frac{1+\frac{[2+(2n-3)\gamma]}{\gamma}}{1+\frac{2+3(n-2)\gamma+(n^2-5n+5)\gamma^2}{\gamma[1+(n-2)\gamma]}}\right]^2 \\
&= \frac{[2+(n-3)\gamma]^2}{4[1+(n-2)\gamma]^2} \left[\frac{2[1+(n-1)\gamma][1+(n-2)\gamma]}{2+(3n-5)\gamma+(n^2-4n+3)\gamma^2}\right]^2 = 1.
\end{aligned}$$

Thus, $\overline{\Delta}^B \geq \Delta^B$ for $t \leq \bar{t}^B$. Above it was established that $\overline{\Delta}^B > \Delta^C$ for $t < t^*$ and that $t^* \geq \bar{t}^B$. Hence, it can be concluded that $\overline{\Delta}^B > \Delta^C$ for $t \leq \bar{t}^B$.

A5.4 $\overline{\Delta}^C \leq \Delta^C$ when $t \leq \bar{t}^C$

Let

$$\delta^C \equiv \frac{\overline{\Phi}^C}{\Phi^C} \frac{[1+(\frac{1}{t}-1)\overline{\Psi}^C]^2}{[1+(\frac{1}{t}-1)\Psi^C]^2}.$$

The impact of t on δ^C is assessed as follows:

$$\frac{\partial \delta^C/\partial t}{\delta^C} = -\frac{2\overline{\Psi}^C}{1+(\frac{1}{t}-1)\overline{\Psi}^C}\frac{1}{t^2} + \frac{2\Psi^C}{1+(\frac{1}{t}-1)\Psi^C}\frac{1}{t^2} = \frac{\Psi^C-\overline{\Psi}^C}{[1+(\frac{1}{t}-1)\Psi^C][1+(\frac{1}{t}-1)\overline{\Psi}^C]}\frac{2}{t^2},$$

It is easily verified that $\Psi^C \geq \overline{\Psi}^C$. Hence, $\frac{\partial \delta^C}{\partial t} \geq 0$ and

$$\delta^C \leq \frac{\overline{\Phi}^C}{\Phi^C} \frac{[1+(\frac{1}{\bar{t}^C}-1)\overline{\Psi}^C]^2}{[1+(\frac{1}{\bar{t}^C}-1)\Psi^C]^2} = \frac{[2+(n-1)\gamma]^2}{4} \frac{\frac{4}{\gamma^2}}{[1+\frac{2-\gamma}{\gamma}\frac{2+(n-2)\gamma}{2-\gamma}]^2} = 1.$$

Thus, $\overline{\Delta}^C \leq \Delta^C$ for $t \leq \bar{t}^C$. Since $\overline{\Delta}^B > \Delta^C$ for $t < t^*$, it immediately follows that $\overline{\Delta}^B > \overline{\Delta}^C$ for $t \leq \bar{t}^C$.

A6 No strategic responses

A6.1 Cournot

In this case all other firms choose quantities as if firm k were in compliance, i.e. $q_j = \frac{z}{2+\gamma(n-1)}$ for $j \neq k$ (see Häckner, 2000). Firm k chooses quantity q_k^{CN} according to its best response function (6):

$$q_k^{CN} = \frac{1}{2}[z_k - \gamma(n-1)\frac{z}{2+\gamma(n-1)}] = \frac{z}{2+(n-1)\gamma} + \frac{z_k - z}{2}.$$

As before, the price is given by

$$p_k^{CN} = q_k^{CN} + c_k$$

Hence, firm k's profit is given by

$$\pi_k^{CN} = \frac{1}{[2+(n-1)\gamma]^2}\left[z + \frac{2+(n-1)\gamma}{2}(z_k - z)\right]^2.$$

Naturally profits under compliance are the same as before. Defining $\Psi^{CN}(\gamma, n) \equiv \frac{2+(n-1)\gamma}{2}$, profits under non-compliance can be expressed as

$$\pi_k^{DCN} = \Phi^C[z + \Psi^{CN}(z_k - z)]^2.$$

The gain from breaching legislation is thus given by

$$\Delta^{CN} \equiv \pi^{DCN} - \pi^{*C} = (z_k - z)\Phi^C\Psi^{CN}[2z + (z_k - z)\Psi^{CN}].$$

It is easily established that $\Psi^{CN} < \Psi^C$ and hence, $\pi_k^{DCN} < \pi_k^C$ (unless $n = 1$ or $\gamma = 0$, in which case $\Psi^{CN} = \Psi^C$ and $\pi_k^{DCN} = \pi_k^C$). Hence, the gain from violating legislation will always be smaller compared to the case when competitors react strategically to firm k's violation of legislation. In the latter case the other firms adapt by reducing their produced quantities, thus making it more profitable for firm k to increase its production.

It is straightforward that Propositions 1 and 2 still hold, i.e. Δ^{CN} increases in both z_k and α, although the impact of changes will be smaller now. Since $\Omega_\gamma^{CN} = \Omega_n^{CN} = 2$ and $1 + \frac{\Psi^{CN}(1-t)}{2t+\Psi^{CN}(1-t)} < 2$ for all γ, it immediately follows that $\frac{\partial\Delta^{CN}}{\partial\gamma} < 0$ and $\frac{\partial\Delta^{CN}}{\partial n} < 0$, i.e. the gain from breaching legislation increases unambiguously both in the degree of product differentiation and in the degree of market concentration. Thus, Proposition 3 no longer holds for low degrees of product differentiation, and Proposition 4 no longer holds for high degrees of cost savings.

A6.2 Bertrand

Since all other firms choose prices as if firm k were in compliance, prices will be given by $p_j = \frac{[1+\gamma(n-2)]c+(1-\gamma)\alpha}{2+\gamma(n-3)}$ for $j \neq k$. Firm k chooses price

p_k^{BN} according to its best response function (11):

$$p_k^{BN} = \frac{c_k}{2} + \frac{(1-\gamma)\alpha}{2[1+(n-2)\gamma]} + \frac{(n-1)\gamma\frac{[1+(n-2)\gamma]c+(1-\gamma)\alpha}{2+(n-3)\gamma}}{2[1+(n-2)\gamma]}$$
$$= c_k + \frac{\left\{\begin{array}{c} -[1+(n-2)\gamma][2+(n-3)\gamma]c_k \\ +2(1-\gamma)[1+(n-2)\gamma]\alpha+(n-1)\gamma[1+(n-2)\gamma]c \end{array}\right\}}{2[1+(n-2)\gamma][2+(n-3)\gamma]}$$
$$= c_k + \frac{2(1-\gamma)z+[2+(n-3)\gamma][z_k-z]}{2[2+(n-3)\gamma]}.$$

Analogously to the case where other firms react strategically, we obtain

$$q_k^{BN} = \frac{1+(n-2)\gamma}{(1-\gamma)[1+(n-1)\gamma]}(p_k^{BN}-c_k).$$

Hence, profits are given by

$$\pi_k^{BN} = \frac{(1-\gamma)[1+(n-2)\gamma]}{[1+(n-1)\gamma][2+(n-3)\gamma]^2}\left[z+\frac{2+(n-3)\gamma}{2(1-\gamma)}(z_k-z)\right]^2.$$

Naturally profits under compliance are the same as before. Defining $\Psi^{BN}(\gamma,n) \equiv \frac{2+(n-3)\gamma}{2(1-\gamma)}$, profits under non-compliance can be expressed as

$$\pi_k^{DBN} = \Phi^B[z+\Psi^{BN}(z_k-z)]^2.$$

The gain from breaching legislation is thus given by

$$\Delta^{BN} \equiv \pi^{DBN} - \pi^{*B} = (z_k-z)\Phi^B\Psi^{BN}[2z+(z_k-z)\Psi^{BN}].$$

It is easily established that $\Psi^{BN} > \Psi^B$ and hence, $\pi_k^{DBN} > \pi_k^B$ (unless $n=1$ or $\gamma=0$, in which case $\Psi^{BN}=\Psi^B$ and $\pi_k^{DBN}=\pi_k^B$). Hence, the gain from violating legislation will now always be larger compared to the case when competitors react strategically to firm k's violation of legislation. In the latter case the other firms adapt by reducing their prices, thus reducing the gains from breaching legislation.

It is easy to see that Propositions 1 and 2 still hold, i.e. Δ^{BN} increases in both z_k and α, and the impact of changes will be larger in the absence of strategic responses.

The impact of changes in the degree of product differentiation turns out to be qualitatively similar to the case with strategic responses. Since $\frac{\partial\Psi^{BN}/\partial\gamma}{\Psi^{BN}} = \frac{n-1}{(1-\gamma)[2+(n-3)\gamma]}$, it follows that

$$\Omega_\gamma^{BN} = -\frac{\frac{\partial\Phi^B/\partial\gamma}{\Phi^B}}{\frac{\partial\Psi^{BN}/\partial\gamma}{\Psi^{BN}}} = \frac{2+2(2n-5)\gamma+2(n^2-5n+7)\gamma^2-(n-2)(n-3)\gamma^2}{[1+(n-1)\gamma][1+(n-2\gamma]}.$$

Just as in the case of strategic responses by competitors, Ω_γ^{BN} decreases monotonously in γ for all n, taking on values in the interval $[1, 2]$ (when $n = 2$, Ω_γ^{BN} falls below one and then increases; however, since $\Omega_0^{BN} = 2$ and $\Omega_1^{BN} = 1$, results are not affected). Hence, Proposition 3 still holds: the incentive to deviate is U-shaped in the degree of product differentiation. However, since $\Psi^{BN} > \Psi^B$ and $\Omega_\gamma^{BN} < \Omega_\gamma^B$, it follows that $\widehat{t}_\gamma^{BN} > \widehat{t}_\gamma^B$, i.e. the range of degrees of cost savings, for which the gain from violating legislation increases in γ, will be wider.

Changes in the degree of market concentration yield similar results as when firms react strategically. Since $\frac{\partial\Psi^{BN}/\partial n}{\Psi^{BN}} = \frac{\gamma}{2+(n-3)\gamma}$ it follows that

$$\Omega_n^{BN} = -\frac{\frac{\partial\Phi^B/\partial n}{\Phi^B}}{\frac{\partial\Psi^{BN}/\partial n}{\Psi^{BN}}} = \frac{2 + 4(n-2)\gamma + (2n^2 - 7n + 7)\gamma^2}{[1 + (n-1)\gamma][1 + (n-2)\gamma]}.$$

Just as in the case of strategic responses by competitors, Ω_n^{BN} decreases monotonously in γ for all n, taking on values in the interval $[2 - \frac{1}{n}, 2]$. Hence, we obtain a result that is qualitatively similar to Proposition 4.

A6.3 Proposition 5

Analoguously to the proof of Proposition 5 the following condition is obtained

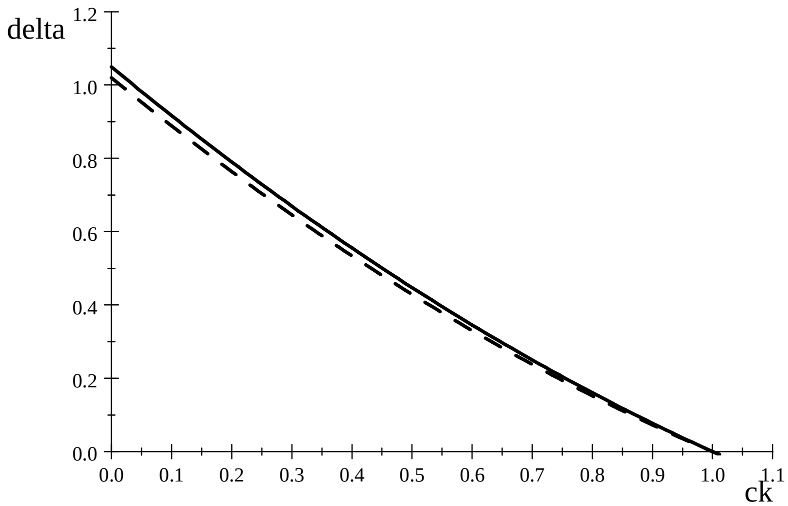
$$\Delta^{CN} > \Delta^{BN} \Leftrightarrow \frac{\Phi^C\Psi^{CN}}{\Phi^B\Psi^{BN}}[T + \Psi^C] > T + \Psi^B.$$

Since

$$\frac{\Phi^C\Psi^{CN}}{\Phi^B\Psi^{BN}} = \frac{[1 + (n-1)\gamma][2 + (n-3)\gamma]}{[1 + (n-2)\gamma][2 + (n-1)\gamma]} = 1 - \frac{(n-1)\gamma^2}{[1 + (n-2)\gamma][2 + (n-1)\gamma]},$$

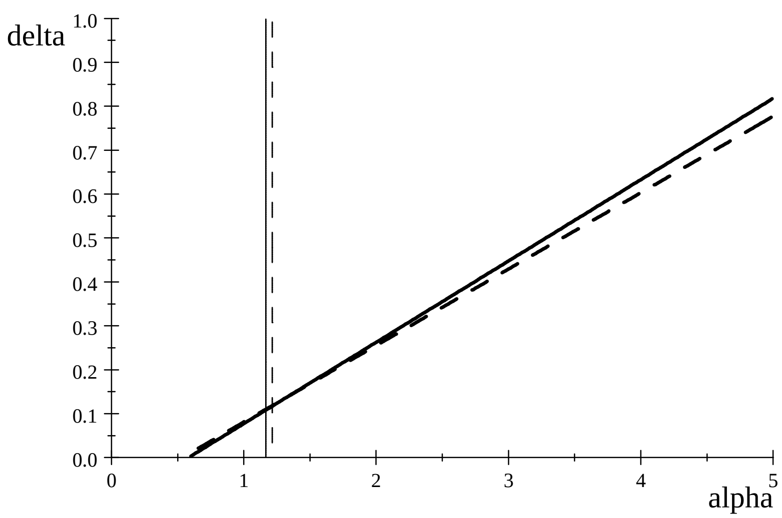
which is strictly smaller than unity for $n > 1$ and $\gamma > 0$, and $\Psi^{CN} < \Psi^{BN}$ for $n > 1$ and $\gamma > 0$, it immediately follows that $\frac{\Phi^C\Psi^{CN}}{\Phi^B\Psi^{BN}}[T+\Psi^C] < T+\Psi^B$ and hence, $\Delta^{CN} < \Delta^{BN}$ for $n > 1$ and $\gamma > 0$.

Figure 1



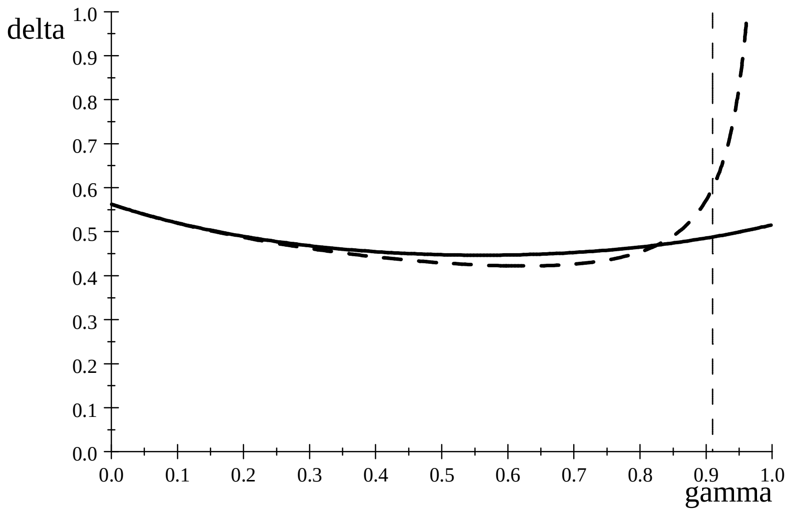
The gain from deviation under Cournot (solid line) and Bertrand (dashed line) as a function of the cost reduction, illustrating Proposition (1). α=3, *c*=1, γ=0.5, *n*=3

Figure 2



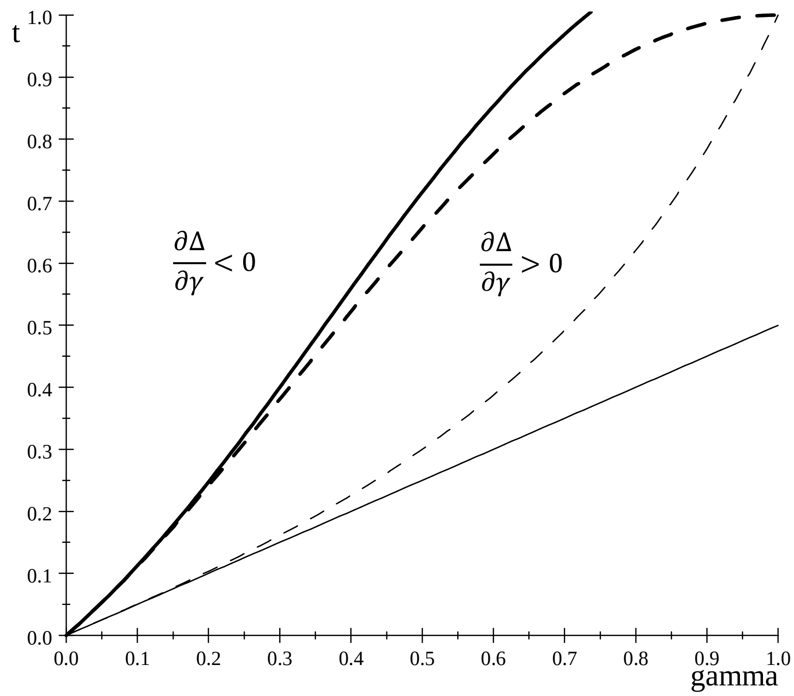
The gain from deviation under Cournot (solid line) and Bertrand (dashed line) as a function of the business cycle parameter, illustrating Proposition (2). Corresponding thresholds for interior solution equilibria are indicated by the thin vertical lines. *c*=1, c_k=*0.5,* γ=0.5, *n*=3

Figure 3



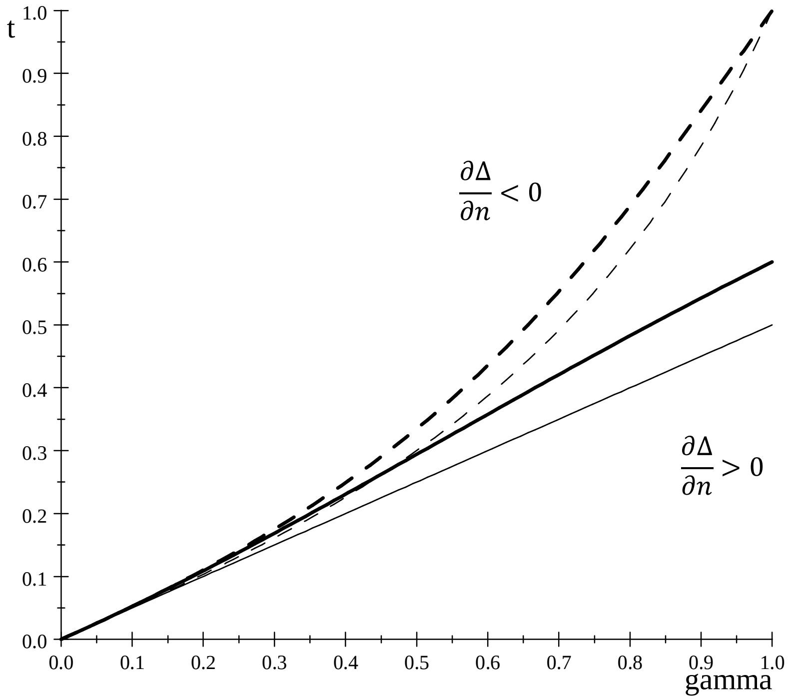
The gain from deviation under Cournot (solid line) and Bertrand (dashed line) as a function of the degree of product differentiation, illustrating Proposition (3). The threshold for interior solution equilibria under Bertrand is indicated by the thin vertical dashed line. α=3, c=1, c_k=*0.5, n*=3

Figure 4



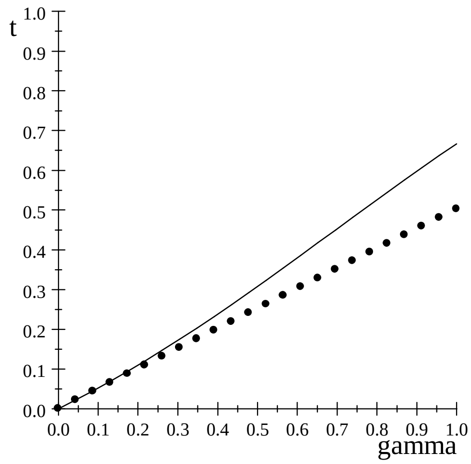
Thresholds for the impact of product differentiation under Cournot (solid thick line) and Bertrand (dashed thick line), illustrating Proposition (3). Corresponding thresholds for interior solution equilibria are indicated by the thin lines. n=3

Figure 5



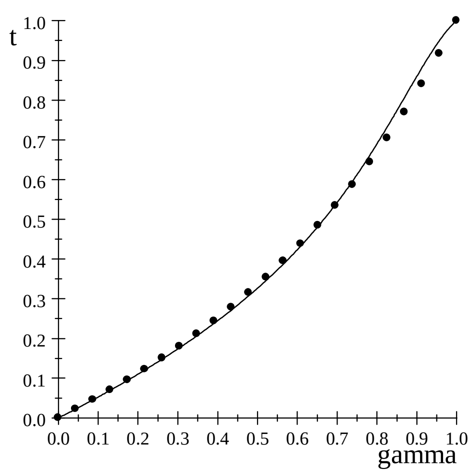
Thresholds for the impact of market concentration under Cournot (solid thick line) and Bertrand (dashed thick line), illustrating Proposition (4). Corresponding thresholds for interior solution equilibria are indicated by the thin lines. n=3

Figure 6



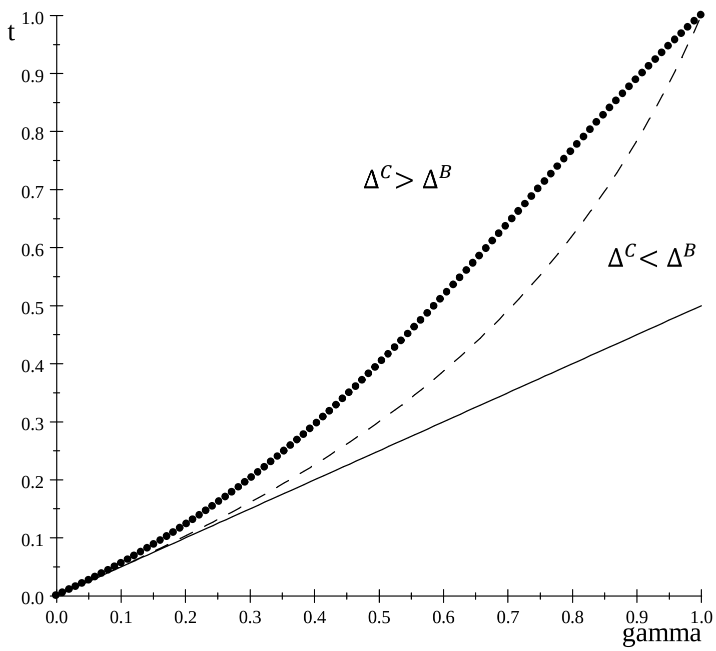
Thresholds for the impact of market concentration under Cournot for *n*=2 (solid line) and *n*=100 (dotted line).

Figure 7



Thresholds for the impact of market concentration under Bertrand for *n*=2 (solid line) and *n*=100 (dotted line).

Figure 8



The threshold determining whether Cournot or Bertrand competition leads to the strongest incentives to pollute (dotted line), illustrating Proposition (5). Corresponding thresholds for interior solution equilibria are indicated by the thin lines. *n*=3

Figure 9

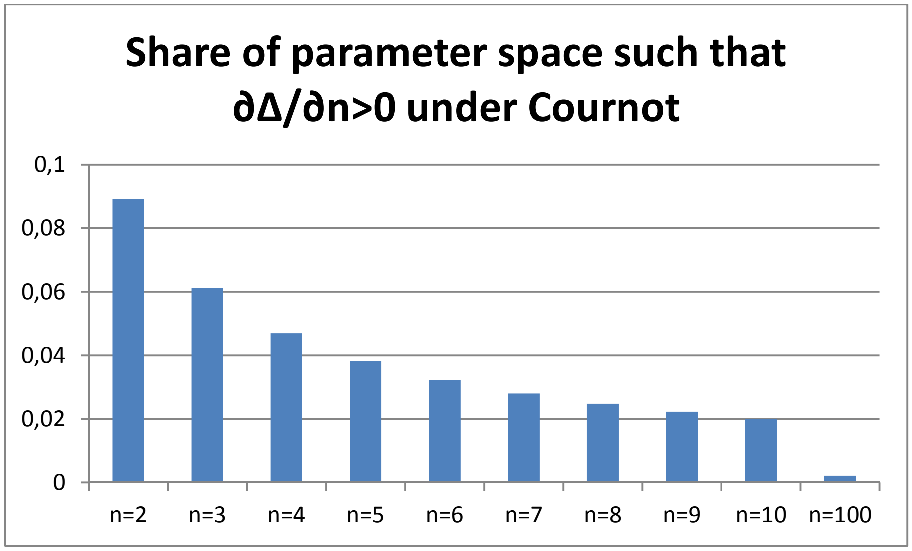


Figure 10