Banking on Regulations?

by

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Abstract
The financial crisis that erupted 2007-2008 has reinforced demand for regulation of banks. The Basle III accord which is to be implemented January first 2013 encompasses two types of regulations with the goal to enforce more prudence among banks. One is capital adequacy regulation which stipulates a lowest ratio between bank capital and bank assets. The other is constraints on dividends and bonuses payments. Banking on these regulations to raise prudence regarding risk taking among banks may lead to disappointment. Within a dynamic model of a value maximizing bank we find that both regulations lower bank value, also in situations where regulations do not bind. None of the regulations leads to increased optimal ratio between common equity and lending. Capital adequacy regulation reinforces credit squeeze when binding. More frequent dividend payouts leads to higher equilibrium bank capital.
1. Introduction

The financial crisis 2007-2008 very clearly revealed a most serious fragility of the financial system and that disturbances on financial markets can hurt the real economy severely. As the crisis started with significant credit-losses in the sub-prime lending and considerable disturbances in the interbank credit market it is clear that banks did not have liquidity and capital enough to cushion such large disturbances. Maturity mismatch, that is, long term lending financed by short term borrowing, exacerbated the problems of banks’ too low liquidity and capital levels. Acharya, Gujral, Kulkarni and Shin (2011) report that bank leverage, the ratio between assets and common equity, increased by more than 30 percent between the first quarter 2000 until the fourth quarter 2007. The increase was lower among government sponsored enterprises, GSEs, such as Fannie Mae and Freddy Mac, only by about 6 percent but notably from an already high leverage level of approximately 40. The high leverage, of course, increased banks’ vulnerability to credit losses and to the disturbances on the interbank credit-market. Uncertainty about individual banks’ holdings of troubled assets virtually brought the interbank market to a halt. Hence, the somewhat paradoxical result: Financial markets that were supposed to lubricate the real economy and to mitigate disturbances to it by absorbing shocks were in themselves the cause of disturbances and a very costly slow-down of the real economy.

The crisis on financial markets has, of course, created a strong demand for policies that can prevent or, at least, reduce the risk for costly growth reductions that either ultimately originates on financial markets or real disturbances that are transmitted through and possibly also propagated on financial markets. The Basle III accord which is due to be implemented the first of January 2013 encompasses two essential elements of regulatory

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1 Assets/common equity ratio increased from 15.0 to 22.51 in commercial banks and from 26.90 to 35.85 in investment banks. In government sponsored enterprises (GSEs) the increase was from 39.59 to 41.85. Achaya et al., page 11.
measures, capital requirement, that is, the requirement of common equity tier 1 capital\(^2\) in relation to risk weighted assets to exceed certain threshold values, and constraints on dividends and bonuses if the capital requirement falls below certain level. The required capital level in the Basle III accord is 7 percent on top of which individual countries can add up to 2.5 percent of a counter-cyclical buffer. The counter-cyclical component can be deactivated during bad times. When the common equity tier 1 capital requirement level in a bank falls below the 7 percent but still above 4.5 percent, the bank is not allowed to pay bonuses to management or dividends.\(^3\) Should the total capital level\(^4\), i.e. with other type of capital rather than common equity tier 1 capital included, fall below 8 percent\(^5\) the bank is judged insolvent and it has to be either liquidated or recapitalized.\(^6\) The intent with this accord, of course, is to force banks to be better capitalized than they would be without the regulation so that they can continue to fund sound investment projects also in turbulent times\(^7\).

It is obvious that a binding capital adequacy regulation may affect lending since reducing lending would be a way to avoid violating the rules of the regulation when bank capital is eroded by credit losses, difficulties to refinance loans or excessive dividends and bonus payments. The regulation may therefore cause a more forceful credit squeeze compared to the unregulated situation. Binding constraints on dividends and bonuses reduce erosion of bank capital in bad times which is surely beneficial to stability of banks. However, as owners and managers know that such a situation may occur, an issue is whether regulation will affect

\(^2\)Tier 1 capital consists of equity capital, retained earnings and some forms of hybrid capital.

\(^3\)In practice there will always be a delay between the current financial situation which is probably, in many cases, fairly well understood by the management and board of directors and the accounting. In cases where the real capital adequacy situation is slightly above 7 percent and on the way down but that is not as yet shown by the accounting, it must be very tempting to pay out generous dividends and bonuses before that is prevented by the regulation.

\(^4\) The capital allowed in this ratio includes not only common equity tier 1 capital, but also other types of capital, for example, hybrid capital and subordinated debt instruments.

\(^5\)Of these percentage units 4.5 must be common equity Tier 1 capital.

\(^6\)Capital adequacy regulation was first probably considered as a means to safeguard small uninformed depositors from excessively leveraged banks that might go bust and not have enough assets to repay depositors. Small depositors are nowadays safeguarded through deposit insurance.

\(^7\)See Aaron S. Edlin and Dwight M. Jafee (2009) on reserves and lending during the 2007-2009 crisis.
banks’ choices of leverage levels, target levels of bank capital and lending also in periods when the regulations are not binding.

This paper investigates the dynamics between bank capital, that is equity capital and retained earnings, bank borrowing from depositors, bank lending and investment, and dividend payouts. A novelty in the paper is that we develop a dynamic model of a bank which maximizes its fundamental market value. We solve for optimal decision variables, which in our model are the scale of the bank in terms of lending/investing and dividend payouts.\(^8\)

Within the model we analyze, one by one, the functioning of the two essential elements of the Basle III regulation; capital adequacy requirements and constraints on dividends and bonuses. Bank capital, is central to the analysis, as it serves as a cushion between bank borrowers and its depositors.\(^9\) Whenever those who have borrowed from the bank default on their loans the bank has to reimburse its depositors from the margins on current loans and, if necessary, from bank capital. The bank’s borrowing cost is endogenous and depends on the size of lending and investment, volatility of asset return and on bank capital (i.e. equity capital). The analysis is based on the assumption that bank lending and investment are financed through deposits by informed agents.\(^10\)

We make no attempt to explain as to why deposits are provided on standard debt contracts. We also assume perfect maturity matching between bank borrowing and its asset side. That assumption technically facilitates the analysis considerably and possibly also affects the relevance of the results obtained somewhat since it implies no distinction between bank solidity and liquidity. However, we believe that it is still highly relevant to explore issues such as credit squeeze or credit tightening and effects of different regulations of banks within a dynamic framework of value maximizing banks.

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\(^8\) An alternative objective would be to maximize the bank’s expected life-time, Borch (1968).

\(^9\) Bank capital to cushion between bank lenders and borrowers is somewhat similar to corporate cash holding to manage risky cash flow, Opler, Pinkowitz, Stultz and Williamson (1999) and Kim, Mauer and Sherman (1998).

\(^10\) Informed agents require compensation for the risk they take. An alternative assumption would be uninformed but insured agents and the bank is charged actuarial fair insurance premiums by the insurer.
We obtain the following results. When risk-neutral banks are less than optimally capitalized, that is, bank capital is below the target level, they behave as if they were risk averse and they hold back lending as compared to the level of lending that would be optimal were they maximizing only expected current period profit. When they are optimally capitalized they do not consider risk in their lending. At this optimal level of bank capital, the optimally chosen size of lending is that which maximizes the expected single period profit. Capital adequacy regulation does not affect optimal capitalization or size of lending and other investment when it is not binding; though the value of banks are lesser in the presence of capital adequacy rules, binding or not. In cases where capital adequacy regulation binds an adverse shock to bank capital results in a severe credit squeeze or credit crunch. The reason is that the volume of credit is proportional to bank capital when the capital adequacy requirement binds but lending typically decreases less than proportional without the regulation. On the other hand expected bank survival, or expected time to bankruptcy, increases. That is an important result because it questions the efficiency of the regulation as a means to mitigate the impact of disturbances on financial markets on the real economy. Increased survival of banks may or may not, be a primary policy objective but harsher credit contractions in down-turns is certainly not generally desirable.

Constraints on dividend and bonuses payouts from banks reduce the fundamental values of banks since the expected present value of dividends will be reduced. That has the effect that optimal capitalization is reduced as compared to the unconstrained case. Intuitively, dividends will be paid out at lower levels of capitalization than in the unconstrained case since owners and managers realize that if they don’t pay out at those lower levels there is an increased risk that the bank will be over-capitalized in the future. In equilibrium, bank lending will be lower with the constraint than without it and the bank will be less well-capitalized. That is clearly an adverse effect of the regulation.
Our theoretical analysis is in continuous time with bank capital as the central state variable that develops over time. Retained earnings are the only way to increase bank capital. We exclude the possibility for the bank to issue new stock. An assumption that may motivate this is that outsiders may observe the current situation in banks but they are badly informed about future options. Dividend payments and size of lending and investment are instantaneous decisions and we see them as controls. However, several results, such as target level of capital under constraints on dividend and bonuses, cannot be theoretically characterized in sensible ways and we therefore use numerical procedures. In that process we by necessity employ discrete time simulations. From that we obtain a quite surprising result, in addition to the results we were aiming for, namely that banks which at more frequent occasions decide about –and pay out- dividends would be better capitalized than banks that pay dividends more infrequently. The intuition for that result is that with infrequent occasions for dividend payments stock owners would have to wait longer for dividend payments. Therefore with given dividend payment, the marginal value of capital needs to be higher and that can only be obtained through reducing bank capital below the optimal level in the continuous time case.

A similar framework to the one in this paper is adopted by Milne and Whalley (1998), Milne (2002) and Peura and Keppo (2006) for analyzing the relation between bank capital and risk taking, the role of regulation of bank capital for risk taking and the optimal level of bank capital when new equity capital is costly. The relation between bank capital and size of bank assets, which is central to the analysis in this paper, is not explicitly considered in those analyses. Important technical contributions on optimization of the flow of dividends with stochastic profit and when there is a risk of bankruptcy are provided by Jeanblanc-Piqué and Shiryaev (1995) and Radner (1998). Jeanblanc-Piqué and Shiryaev explore the case
where firm scale is given and dividend payment is the only control. That is the technical approach in Milne and Whalley (1998), Milne (2002) and Peura and Keppo (2006). The framework in Radner (1998) is richer in that also the scale is a choice variable. Hence, he considers a producing firm in which the production capacity is the state variable and dividends is a decision variable which has the effect that it erodes capacity. The scale is the firm’s capacity utilization. Capacity utilization affects the variance of the production result. In our analysis bank capital bears some resemblance to production capacity and bank lending and other investment technically corresponds to Radner’s capacity utilization.

Other literature on financial intermediation or banks has mostly been geared towards information asymmetries. Diamond (1984) explores how costly state verification, CSV, Townsend (1979), leads to the existence of banks or financial intermediation and standard debt loan contracts. The framework entails many investors with savings that are small as compared to investment projects. Intermediaries economize with monitoring resources since when investments are channeled through the bank; it does the monitoring of investment projects, and there is no need for the individual savers to monitor them or, as it turns out, the bank. Others deriving endogenous credit contracts and intermediation are Gale and Hellwig (1985) who use slightly more plausible assumptions than Diamond (1984) as they do not have non-pecuniary auditing costs, Williamson (1986, 1987) takes the CSV models even closer to the market as all auditing costs and bankruptcy costs are borne by the investor. Williamson’s model also generates endogenous credit rationing in addition to delegated monitoring. Credit rationing is also analyzed by Stiglitz and Weiss (1981) in a model with borrower limited liability which implies that the rate of interest affects risk-taking as well as demand for loans. Those analyses are focusing on one time period. In such a context bank limited liability and financing costs which do not reflect risk-taking perfectly

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11 See also Radner and Shepp (1996) and Dutta and Radner (1999) for similar analyses of producing firms with fixed scale. There is also a related biology oriented literature dealing with harvesting represented by Gleit (1978) and Lungu and Øksendal (1997) which obtain similar results in similar models.
gives incentives to excessive risk-taking. The analysis in this paper is dynamic to its character which weakens the incentives for excessive risk taking considerably since banks facture in the risk of loosing future profits.

Diamond and Dybvig (1983) deal with another problem namely the role of banks to fund illiquid investment projects with liquid deposits. The information asymmetry in the model is that depositors have private information about when they need to withdraw funds from the bank. That may give rise to two equilibriums: one equilibrium, in which depositors do not believe that other depositors will withdraw funds to the extent that the bank is forced into a fire-sale of assets, the other is when confidence is lost and depositors believe that the bank cannot honor its obligations. In the latter case each depositor has better to be first in line to withdraw funds since no funds may remain for latecomers. Deposit insurance may for retail deposits mitigate such a coordination failure. However, for wholesale depositors such guarantees may not be ubiquitous. The Diamond and Dybvig analysis seems to have some bearing on the 2007-2008 financial crises although excessive withdrawals were not the cause of the crisis rather it was caused by a series of other factors such as too low a lending standards among which the subprime housing credits are, bad maturity matching and of course too low a reserve levels in banks which is the focus of this paper. The issue of systemic risk has recently been addressed by Allen and Carletti (2011) in the context of inflated real estate prices and earlier by Allen and Gale (2000) and Rochet and Tirole (1996) in the context of liquidity pooling through the interbank market.

The paper is organized as follows. Section 2 contains a description of the model and analyses of bank behavior under different conditions. First, the core dynamics of a bank’s value maximization problem is described and analyzed without regulation of capital adequacy and dividend payouts. Second, the interest rate the bank has to pay on its lending is

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12 The model in this paper is an adaptation of the analysis is in Radner (1998) to banking.
analyzed. That analysis is of first-best type. Third, the model with capital adequacy regulation is analyzed. Fourth, bank behavior under dividend regulation is analyzed. In Section 3 we describe how we implement the model numerically and we present results from simulations in time domain. Finally, concluding comments are in Section 4.

2. The Model

2.1 Value maximization –first-best case

Consider a bank that borrows funds and uses the borrowed funds for lending and trading which we refer to as bank investment. The bank has bank capital, that is, common equity and retained earnings which it uses as collateral for bank debt. We assume that bank capital is not part of bank investment and it can only be held as cash\textsuperscript{13}. The return to bank investment is proportional to the size of investment, but uncertain. The degree of uncertainty is increasing with the size of investment and the stochastic disturbances are normally distributed. The bank’s lenders have a correct picture of the uncertainty of the returns to bank investment and they realize that the bank cannot repay its lenders from current cash-flow in all states of the world, and that bank capital may not be enough to honor the bank’s obligations. Therefore they demand a risk premium for lending to the bank. The size of the risk premium depends positively on the size of the bank’s investment or its borrowing and on volatility of return to investment. It depends negatively on bank capital and expected cash-flow.

An alternative modeling strategy to assuming that return to bank investment is proportional to the size of bank investment and to assuming that the volatility of return is increasing in the size of bank investment is to assume that volatility of return to bank investment is proportional, or may be even less than proportional, to bank investment but that

\textsuperscript{13} Allowing for bank capital to be invested in risk-free assets would alter the analysis slightly but the essence of the results would be unchanged.

\textsuperscript{14} A potential alternative strategy for a bank would be to use bank capital for its lending. However, that reduces the benefits from leverage. We show in Appendix that the expected profit with that strategy is in fact dominated by the levered strategy using equity as collateral.
the demand curve for bank investment is decreasing. It is not obvious which one of the two strategies is the more realistic. However, in the strategy chosen uncertainty of bank return plays a more important role. We believe that is an important component in the short- and medium range.

At some moment of time which we refer to as time $0$, the bank has equity capital, $k_0 \geq 0$. Owners maximize the expected present value of all future dividends $w_t$, from the bank, up to the point where there remains no equity capital, i.e., the point at which the bank goes bankrupt, if that ever occurs. We write the value of the bank, $V$, as follows.

$$V(k_0) = E_0 \int_0^T w_t e^{-ct} \, dt,$$  \hspace{1cm} (1)

where $E_0$ is the expectation operator and $c$ is the risk-free rate of discount and $T = \inf\{t | k_t \leq 0\}$. \hspace{1cm} (2)

That is, $T$ is the first time $k_t \leq 0$. We also impose the conditions that

$$V(0) = 0.$$ \hspace{1cm} (3)

The evolution of $k_t$ is described by a controlled Brownian motion where $z_t$ is the size of bank investment, $r$ is the proportionality factor giving the gross return to bank investment, $rp(z_t, k_t)$ is the risk premium above the risk-free rate of interest, $v(z_t)$ is the variance of return to bank investment and $d\epsilon_t$ is the stochastic disturbance to the return to bank investment. We assume that the disturbance is a Wiener process resulting in normally distributed errors with an expected value of zero and a standard deviation of one.

Disturbances are scaled by the standard deviation of bank investment, $\sqrt{v(z_t)}$. We assume $v'(z_t) > 0$ and $v''(z_t) > 0$, i.e., the variance of the return on investment is increasing at an increasing rate with the scale of bank investment.

Defining $m(z_t, k_t) = z_t (r - (c + rp(z_t, k_t)))$, that is $m$ equals current period revenue minus
current period bank borrowing cost which is the size of bank investment times the risk-free rate of interest plus the risk premium. (For the moment we assume that \( m(z_t, k_t) \) is an exogenous concave function. Below we endogenize the bank’s borrowing cost.) Hence we write the evolution of bank capital as follows.\(^{15}\)

\[
dk_t = (m(z_t, k_t) - w_t) dt + \sqrt{v(z_t)} \times d\varepsilon_t
\]

Only non-negative \( w_t \) are considered since we assume that the bank cannot receive capital injections, that is, \( 0 \leq w_t \). Note that \( z_t \) and \( w_t \) are controlled by the bank and they represent its tools for maximizing the value on currently invested capital \( k_t \).

The Bellman principle gives the following necessary condition for the value of the bank, the scale of the bank’s assets and dividends.

\[
cV(k) = \max_{z, w} \{ (1 - V'(k))w_t + mV'(k_t) + \frac{1}{2} v(z_t) V''(k_t) \}.
\]

It follows that the optimal choices of \( z_t \) and \( w_t \) only depend on \( k_t \) and that the actual time elapsed since the beginning at time 0 lacks significance. In the following we therefore omit the time index.

In order to characterize the solution to the maximization problem we first note that \( V(k) \geq 0 \) and that \( V'(k) \geq 0 \). We continue by exploring the optimal dividend policy. Note that if \( V'(k) > 1 \) then \( w = 0 \) is the optimal choice. If the stochastic disturbance instantaneously increases bank capital up to a point where \( V'(k) < 1 \), the optimal dividend policy is to instantaneously pay a large dividend so that \( V'(k) = 1 \) after the dividend has been paid out. That implies that bank capital never gets larger than the level at which \( V'(k) = 1 \). We refer to that level of bank capital as \( b \). Jeanblanc-Piqué and Shiryaev (1995) and Radner (1998) refer to such a dividend policy as an overflow policy. The intuition is simple. If the value of the uncertainties

\(^{15}\) Technically there is nothing that binds the uncertainty to bank revenue. It is therefore possible to interpret the uncertainty as interest rate uncertainty which is a relevant issue for inter-bank markets.
bank increases by less than the increase in equity capital it is better to give the capital back to
the owners. Below we return to further characterizing $b$.

Having established that when $k \leq b$, $w = 0$ we continue by characterizing the
optimal sizes of bank investment. Hence, for $k \leq b$, the optimal choice of size, that is how
large investments to make, needs to satisfy the following condition.

$$ m z V'(k) + \frac{1}{2} v' (z)V''(k) = 0 \quad (6) $$

Using (6) in (5) we obtain,

$$ cV = mV' - \frac{m v}{v'} V', \quad (7) $$

where $z$ satisfies (6) or more conveniently written,

$$ cV = \{ h(z(k), k)\} V', \quad (7') $$

where $h(z(k), k) = m(z(k), k) - \frac{m (z(k), k)v(z(k))}{v'(z(k))}$. 

Note that (7) and (7') holds for all $k < b$. We therefore have,

$$ cV' = hV'' + h_k V' + h z V' z'(k). \quad (8) $$

By using (6) one more time, we obtain.

$$ z'(k) = \frac{2 h m z}{v} - h_k + c \quad (9) $$

which does not involve the unknown function $V(k)$. The solution to (9) is obviously key to
solving for the optimal $V(k)$. Note that, $V'(0) > 1$. It therefore follows from (7') that the
initial condition $V(0) = 0$ is satisfied only if $h(z(0), 0) = 0$. That in turn implies that the
optimal $z(0)$ should be the one that maximizes $\frac{m}{v}$, that is the value of $z$ that maximizes the
risk standardized current period profit. That runs opposite to the optimum choice in a static
model with a limited liability constraint.
The optimal bank value, which follows from the optimal dividend policy and the optimal sizes of bank investment is characterized as follows. From (6) we obtain,

$$\frac{V''(k)}{V'(k)} = -\frac{2m_\nu}{\nu'} \equiv H(z(k), k),$$

(10)

which has the solution

$$V'(k) = e^{-\int_k^b H(z(s), s)ds}$$

for $k \leq b$,

(11)

which satisfies the initial condition $V'(b) = 1$.

The part of the solution that remains to characterize is the optimal $b$. Note that $V'(k), k < b$ increases up to the value of $b$ where $H(z(k), k)$ changes sign from negative to positive. Hence, $b$ should satisfy the following equation: $H(z(b), b) = 0$ to maximize $V'(k), 0 \leq k \leq b$.

An observation: $H(z(k), k)=0$, only if $m_\nu = 0$, which is that the current period profit, the drift term in (5) is, maximized. That in turn from (6) implies that $V'(b)=0$. The interpretation is that at the level of capital where the bank pays dividends it is -and it behaves as if it is- risk-neutral. However, at lower levels of equity capital it behaves as if it were risk averse although it actually is risk neutral. Hence, a result is that the incentive to excessive risk taking created by limited liability is not present in the dynamic model in this paper.

To summarize, we have found that $z(0)$ maximizes $\frac{m}{v}$ and that $z$ evolves according to (9). The optimal dividend policy is to pay no dividend when $k < b$ and to pay the excess of $k$ over $b$ whenever $k$ exceeds $b$. The optimal bank capital, $b$, is the level of bank capital for which the optimal size of bank investment, $z(b)$, maximizes current period profit. Bank value is given by (7) in which all components on the right hand side are
characterized.\textsuperscript{16} So far, in the analysis the bank’s borrowing cost and the volatility of return to bank investment are exogenous functions. Below we specify the function for volatility of bank investment and we make bank borrowing cost endogenous.

\subsection*{2.2 Interest Rate on Borrowed Funds}

The interest rate the bank pays its depositors compensate for the risk of bank default. We assume that there is no information asymmetry between the bank and its lenders as regards the current period situation. Hence, lenders can observe the amount of bank capital. They can also observe the scale of bank investment and they have the same view as the bank itself about expected repayment to the bank and about the variability of the repayment. Lenders require a risk premium on their charged return that equalizes the expected repayment by the bank to the return on risk-free lending.

We explore the statutory gross repayment, $\rho$, for a loan, $z$, taken at time $0$ and repaid at time $t$. At time $t$ the bank receives the expected return $ze^{rt}$. The return on investments is uncertain and the deviation from the expected value is normally distributed, $N(0, v)$, where $v = v(z)t$ is the variance. The amount of equity capital is $k$.

In cases with very low return the bank will have to repay its lenders with the capital it has from the start and the expected repayment should equal the risk-free return on an investment of size $z$ from time $0$ to time $t$. Hence, statutory gross repayment, $\rho$, which the bank pays on its borrowing should satisfy the following equation:

\[
\frac{1}{\sqrt{2\pi v}} \int_{-k}^{k} (k + x)e^{-\frac{(x-\rho t)^2}{2v}} dx + \rho \frac{1}{\sqrt{2\pi v}} \int_{\rho-k}^{\infty} e^{-\frac{(x-\rho t)^2}{2v}} dx = ze^{rt} \tag{12}
\]

\textsuperscript{16} An alternative way to calculate bank value is to use $V(k) = \int_{0}^{k} V'(s)ds$. 

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The first term on the left hand side is the repayment under bankruptcy weighted with the probability of default. The second term is statutory repayment multiplied with the probability that the bank can service its obligations.

It is clear that, for fixed values of \( c \) and \( r \), and a given function \( v(z) \), \( \rho \) is a function of \( z, k \) and \( t \), i.e., \( \rho = \rho(z,k,t| r,c) \). One observation can be made, namely if \( v = \sigma^2 z t \), that is if the variance is proportional to the size of bank investment, then \( \rho \) is homogeneous of the first degree in \( z \) and \( k \). That would imply that the bank’s value maximization problem has no solution. Here, we assume that the volatility \( v(z) \) is a strictly convex function of \( z \).

However, as there is no explicit solution for (12) in terms of \( \rho(z,k,t) \) we solve for the borrowing-cost numerically. First we standardize the problem in order to use the error function for the numerical solution of equation (12). After standardizing the normal distribution one obtains:

\[
\frac{1}{\sqrt{2\pi}} \left[ \int_{-\infty}^{\frac{k+ze^\beta}{\sqrt{2v}}} u e^{-u^2} du + \int_{\frac{k+ze^\beta}{\sqrt{2v}}}^{\frac{k+ze^\beta}{\sqrt{2v}}} e^{-u^2} du + \rho \int_{\frac{k+ze^\beta}{\sqrt{2v}}}^{\infty} e^{-u^2} du \right] = z e^{\rho}, \quad (12')
\]

which is the form of the equation used in the numerical analysis using the error function.\(^{17}\)

In the numerical analysis we assume that the variance of the repayment is the following explicit function:

\[
v(z) = \beta^2 e^{2\alpha z} \quad \text{ (13)}
\]

Parameter values \( \beta \) and \( \alpha \), are chosen such that the variance does not increase too fast with lending and that there is some variance for very low values of lending. It would

\[\text{The explicit expression is:}\]

\[
\sqrt{\frac{v}{2\pi}} e^{\frac{(k+ze^\beta)^2}{2v^2}} - e^{\frac{(k+ze^\beta)^2}{2v^2}} + \frac{1}{2} (k+ze^\beta) \left[ \text{erf} \left( \frac{\rho-k-ze^\beta}{\sqrt{2v}} \right) - \text{erf} \left( - \frac{k+ze^\beta}{\sqrt{2v}} \right) \right] + \frac{\rho}{2} \left[ 1 - \text{erf} \left( \frac{\rho-k-ze^\beta}{\sqrt{2v}} \right) \right] = ze^\rho
\]

\(^{17}\)
have been natural that the variance is zero at zero bank investment. However, without some variance for arbitrary low levels of lending, problems with existence might occur since the starting value for bank investment is the value that maximizes the ratio of drift to variance. Hence, zero variance is not a tractable feature. We view the assumption of a positive variance at $z = 0$ as similar to some start-up cost. That is, even if you lend to a very few you would be required to have some over-head costs, leading to banks starting at larger scale then they would otherwise. Having a strict positive variance service the same purpose: banks’ size will not be arbitrarily small.

For the diffusion (5) we need the continuous time gross bank borrowing cost of funds, $\rho(z, k)$. We calculate the $\rho(z, k)$ function for a loan of duration $t = 1$ so as to capture the risk in lending to the bank and to get risk-premiums at the same magnitudes as $c$ and $r$.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{drift.png}
\caption{The drift derived from equations (1') and (2)}
\end{figure}
Figure 1 shows the resulting net drift with parameter values
\[ \alpha = 0.1, \beta = 0.08, r = 0.045 \text{ and } c = 0.03. \] That is, the bank’s margin on its lending is 1.5 percent, which is not an unrealistic margin. It is clear that with the parameters set, the drift is relatively insensitive to equity at low levels of lending. This is due to the fact that: at low levels of lending the variance is low and most losses can be covered by the margin on borrowers that fulfills their contracts. Along the ridge with maximum drift given lending the effect from equity is rather substantial.

Figure 2 shows simulated bank investment functions with the same parameter values as those above. The function which maximizes bank value is \( z(k) \) and the function that maximizes current period profit is \( z^* \). Maximizing current period profit implies more risk-taking than the value maximizing strategy. The difference between the two functions reflects the risk aversion implied by the dynamic strategy. At bank capital equal to \( b \) the two strategies coincide.

**Figure 2:** The evolution of banks’ size (lending) using \( z(0) \) that maximizes risk standardized drift as starting value.
Figure 3 illustrates the resulting bank values. The low volatility for low levels of
bank investment implies that bank value increases steeply with increasing bank capital. At
higher levels of bank capital risk-taking and increased borrowing cost effectively balance
each other, resulting in an almost— but not entirely— linear value function.

![Figure 3: Value of bank that maximizes the current value of future expected dividends.](image)

2.3 Capital adequacy regulation

A pure capital adequacy regulation stipulates a lower bound on the ratio of reserve capital,
measured as common equity tier 1 capital, and risk weighted assets. As our model is a bare-
bone model with no other distinction between bank liability than that between equity capital
and debt, and only one type of asset, the constraint takes the following form.

\[ \frac{k}{z} \geq \gamma \]  \hspace{1cm} (14)
where $\gamma$ is the capital adequacy requirement. Moreover, perfect maturity matching between assets and debt implies that banks will never violate the capital adequacy constraint since cutting down on lending and investment is a possibility.

Figure 4 shows three different bank investment strategies. Bank investment under maximization of current period profit is illustrated by $z^*(k)$. That is, it is the solution to $m_z(z^*(k), k) = 0$. The dynamic strategy which satisfies equation (9) is $z(k)$. The straight line with the slope $1/\gamma$ shows the border of 10 percent capital adequacy regulation. The bank’s capital ratio is required to be to the right of that line. Hence, under the capital adequacy regulation the dynamic value maximizing strategy follows the straight line from $k = 0$ up to the point $k = ad$ where the capital adequacy constraint crosses $z(k)$. For larger $k : s$ it coincides with $z(k)$. Hence, when the capital adequacy regulation does not bind it does not affect the size of bank investment. Therefore, it has no effect on bank risk-taking when not binding. When the constraint binds, it results in lower bank investment than under the value maximizing dynamic strategy. Hence the regulation results in a more forceful credit squeeze than were there no regulation.
Figure 4: Optimal controls when capital adequacy rule of 10 percent is imposed.

The value of the bank under the capital adequacy constraint, binding or not, is lower than without the constraint. In principle, since the bank’s optimization will satisfy the Bellman equation (6) in the interval \((ad, b)\), the value of the bank can be calculated in the following way.

\[
V'(ad) = e^{-ad}
\]

In the interval \((0, ad)\), the bank’s dynamic value, \(\hat{V}\), satisfies the following equation.

\[
c\hat{V}(k) = m(k)\hat{V}'(k) + \frac{1}{2} v(k)\hat{V}''(k)
\]

with the initial condition \(\hat{V}(0) = 0\) and the endpoint condition \(\hat{V}'(ad) = V'(ad)\). However, to our knowledge there exists no closed form solution to (16) with the combination of initial and endpoint conditions. It is obvious that \(\hat{V} \leq V\) in the interval between 0 and \(ad\), since
\( z(k) = \frac{k}{\gamma} \) is a feasible choice in the unregulated case (5). The implication is that bank value is lower in the constrained case than in the first-best case also when the constraint does not bind. We return to bank value with a capital adequacy constraint in the next section with numerical procedure.

### 2.4 Dividend and Bonuses Constraints

A regulation that has been considered and also will be implemented within the Basle III regulation is constraints on dividends and bonuses. Within the context of the model in this paper there is no distinction between dividends and bonuses since there is no other explicit production factor than bank capital. However, in a richer model with banking expertise as an additional production factor, an issue is whether bonuses should be a part of the capital share of total return or if it is best viewed as part of return to labor. We write the bank’s value maximization problem as that in the first-best case but with the following additional constraint.

\[
w_t \leq \bar{w} \tag{17}
\]

In the continuous time case (17) is always binding for \( k \geq b^c \), where \( b^c \) is the level of capital at which it is optimal to commence to pay dividends when there is a constraint on the size of dividends. Note that \( b^c \) is lower than \( b \) because of the constraint on the size of dividends which implies that the first-best case bank value, \( V(b) \), cannot be obtained with a binding constraint on the size of dividends. Bank capital will evolve according to (5), with \( w_t = 0 \) when \( k < b^c \) and \( w_t = \bar{w} \) when \( k \geq b^c \). In accordance with that bank investment, \( z_2(k) \), for \( k > b^c \) develops according to the following differential equation.
\[ z_z'(k) = \frac{2(h - \bar{w}) \frac{m}{v^'} - h_k + c}{h_z} \] (17)

with \( z_z(b^c) = z(b^c) \).

Note also that \( \tilde{V} < V \) for \( k \leq b^c \). Hence, like in the case with capital adequacy requirements, dividend regulation lowers bank value also in situations where the constraint is not binding. In addition, in the case with dividends and bonuses constraints, optimal bank capital and bank investment will be lower than in the unconstrained case.

In our numerical analysis we adopt a slightly different form of the dividend constraint, namely that in cases where bank capital is above the optimal level of capital in the discrete time case, dividends are capped only when the realization is such that bank capital is larger than \( b^c + \bar{w} \).

3 Numerical procedure

Some important results, such as bank value with restricted maximum dividends and bonuses, cannot be characterized in a meaningful way in the continuous time model. Bank value under capital adequacy regulation is also not possible to characterize in a transparent way in the continuous time model. Therefore, we perform numerical procedures in time domain to shed light on those results. In addition to those results, the time domain simulations also deliver results on expected lifetime of banks under the different regimes. Quite unexpectedly they also show that bank value is affected by the change of time periods. Going from continuous time to discrete time is similar to imposing a binding constraint on the timing on payments of dividends and such a binding constraint lowers bank value for two reasons. The first is discounting; owners would have to wait for dividends. The second is that during a time period there may be occasions at which bank capital exceeds the threshold level for dividend payment but over the full period they may be balanced by occasions with below
threshold levels of capital. Hence, the amount of dividends paid out decreases the longer the time is between dividend payments.

In the simulations in time domain we use the results from the continuous time model on revenue and cost but with a yearly compounding. This means that we use the calculated gross bank borrowing cost $\rho(z, k)$ and we match it with the yearly continuous revenue from bank investment.

Using the optimal controls for size of bank investment, $z(k)$, given common equity, $k$, and the optimal common equity level, $b$, we simulate the paths for many banks starting at various specific current common equity levels. The value function is traced out by repeating the simulation for current common equity levels between $0$ and $b$. Each bank’s path is a maximum of 250 periods long, this as the assumed discount rate of 3 percent yields a discount factor of 0.000618 in the 250th period and hence, dividends after this time have very little impact on bank value.

3.1 Capital adequacy constraint

We showed in section 2.3 that bank value will decrease in the presence of capital adequacy constraints. In order to explore that decrease we first run a simulation with no constraints, that is, the first best case but in discrete time. This shows that when moving from continuous payments of dividends to discrete yearly payments, the optimal level of bank capital decreases from the optimum in the continuous time model, $b$, to $b^d$, which is shown in Figure 5, where the optimal level of capital have decreased with more than 25 percent relative to the continuous time case, from $b=8.256$ to $b^d=6.021$. The intuition for this decrease is as follows. First, when dividend is paid less frequently dividend value will be lowered by discounting. Second, at some occasions where bank capital exceeds the threshold value dividends are not paid.

\[s\]

18 Since the simulation is highly asymmetric due to the absorbing barrier, we draw 15 million paths for each initial level of common equity and average the resulting values.
paid out and bank capital may decrease down to the point where dividend may not be paid before the time for payment occurs. Hence, marginal value of bank capital is lower than one at bank capital \( b \). Therefore bank capital has to be lowered in relation to \( b \) to increase the marginal value of bank capital to one which is the level of capital where dividends are paid out, in discrete time.

When a capital adequacy regulation is adopted we showed that value of banks will be lowered as the first best solution is no longer viable for all levels of bank capital. It was also shown that this will not affect the optimal bank capital level. In Figure 5 the discrete time value functions are shown with and without capital adequacy constraint. Here the adopted capital adequacy regulation is a minimum common equity to assets is set to 10%.

![Figure 5: Simulated value functions for banks with and without capital adequacy regulation.](image)

Figure 5: Simulated value functions for banks with and without capital adequacy regulation.
It is obvious in Figure 5 that the value functions are in fact parallel for bank capital levels $k \geq ad$. Note that for levels of bank capital above $b^d$, $V'(k) = 1$ by construction as all excess bank capital above this level is distributed as dividends. The reduction of banks’ value in the presence of the constraint is 2.25 percent lower than the first best value at the optimal level of bank capital $b^d$. This implies that return on equity decreases with almost 6 percent with the inclusion of a capital adequacy constraint. The reason for this reduction is, as stated above, that when capital adequacy binds, marginal value of bank capital is lower since the constraint reduces the drift of bank capital when leverage is lowered.

Saving the financial system often have the focus on saving banks. Indeed a capital adequacy regulation increases longevity for banks. Table 1, shows that longevity is increased by 3.1 percent at the level of capital where capital adequacy starts to bind. The difference between the unregulated case and the case with the capital adequacy constraint decreases as we approach the optimal capital level where the difference in expected lifetime has decreased to 2.8 percent. Hence, capital adequacy constraints lead to lower bank value and longer expected survival, regardless whether it binds or not. That should be considered together with the results from section 2.3 where we showed that a binding capital adequacy constraint transforms a potentially small credit squeeze to a severe credit contraction. There is currently dawning evidence of a significant credit contraction occurring in which sovereigns and expanding firms are declined credits or receive extremely harsh terms on their contracts.

<table>
<thead>
<tr>
<th>Table 1</th>
<th>Simulated values for First best and Capital adequacy case</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>$K = 2.575$</td>
</tr>
<tr>
<td>No. of Div.</td>
<td>41.159</td>
</tr>
<tr>
<td>Longevity</td>
<td>180.292</td>
</tr>
</tbody>
</table>
To summarize, Table 1 shows that capital adequacy constraint decreases bank value and expected number of dividends and increases the waiting time for the first dividend and bank expected survival time.

### 3.2 Restriction on size of dividends and bonuses

Theoretical results for the case with a cap on dividends and bonuses are very hard to obtain, therefore we simulate all results for this case. We start the analysis by removing the possibility to payout the largest positive shocks directly. Figure 6 shows the distribution of dividends, including the case of a dividend of zero.

![Dividend distribution with bank capital at \( b^a \)](image)

**Figure 6:** The distribution of dividends in discrete time for a drawn sample of 10 000 random shocks when current bank capital is at the optimal level. Note that the first bin is very large as it also contains all zero dividend cases.

The median dividend is 0.58, mean dividend is 1.24 and our imposed cap on dividends is set at 4 to have a very marginal effect on the optimal control for size and optimal bank capital.
level. As seen in Figure 6 a very small number of states will lead to a binding restriction on dividend payment. Occasionally large positive shocks when banks are at their desired risk exposure in terms of bank capital over lending will lead to banks having more bank capital than optimal.

When we trace out the bank’s value function under the constraint we start by first iterating out the new optimal capital level $b^*$, it turns out that with such a loose restriction on dividends and bonuses the optimal level of bank capital decreases with 6.2 percent. The difference between the unconstrained value function and the constrained dividends case value function is shown in Figure 7.

![Figure 7: Simulated value functions for banks with and without constraint on dividend and bonuses. The cap on dividends is set to the 93<sup>rd</sup> percentile.](image)

It is obvious that the effect from a restriction that binds so seldom is almost negligible on a bank’s value function. An adverse effect occurs though: banks become less prudent in the sense that capital adequacy is decreased. Regulators have probably not considered that optimal level of bank capital decreases rather than increase when restrictions
on dividend sizes are imposed. In fact even with the modest constraint imposed here where only 7 percent of the states are affected the decrease is more than 6 percent. Note that the restriction is indeed very small as only about 7 percent of the states are affected. In Table 2 the numbers for some different levels of current bank capital with a restriction that dividends can at most be of size 4.

<table>
<thead>
<tr>
<th>K = 2.575</th>
<th>k = 4.81</th>
<th>k = 6.021</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of Div</td>
<td>41.159</td>
<td>44.009</td>
</tr>
<tr>
<td>W.T. First</td>
<td>11.949</td>
<td>11.003</td>
</tr>
<tr>
<td>Longevity</td>
<td>180.292</td>
<td>180.936</td>
</tr>
</tbody>
</table>

Note that the optimal level of bank capital under our dividend restriction is $b^c = 5.648$ but to make Table 1 and Table 2 comparable maximum current bank capital is set to 6.021 which explains the missing value in waiting time to first dividend when bank capital is at 6.021 as an instantaneous dividend is paid out as bank capital is “too large” but the restriction does not bind. It should be mentioned here that our restriction on dividend is different than the one proposed by Bank for International Settlements. They suggest that there should be a level of common equity to risk weighted assets that prevents the bank to pay dividend and bonuses. We have showed that such a restriction, at least under perfect maturity matching, is ineffective unless it always binds. The reason is that banks themselves, regardless of regulation, have incentive to keep bank capital as reserves to cushioning potential shocks to their investment portfolio in order to keeping their own financers indemnified.

Due to the rather infrequent reporting and dividend distribution the proposed restriction on dividend might even lead to a type of looting as banks that suspect or even
know that bank capital might be below the level which forbids dividends next period might be tempting to pay dividends (or too large dividends) already today. That is, bank capital will be either at some distance above or below the threshold but never just below unless the threshold coincides with the banks own desired capital level.

4. Concluding Comments

In this paper we have developed a dynamic model of banking aiming at exploring the relationships among bank capital, bank lending and dividends to bank owners. Necessarily, we have had to make assumptions about various components in the analysis. We have, for example, assumed that bank capital is used as collateral for bank debt, that the bank matches maturity of its debt to its investment perfectly, and that the growth of bank capital can be described as a Brownian motion. We also assume that the interest rate the bank has to pay on its borrowing includes a premium which reflects the risk lenders take to lend to the bank. Those assumptions can, of course, be questioned and altered. However, we think that the right way to analyze bank behavior is to assume that bank owners maximize bank value, that is the expected present value future dividends and since the model represents a novel approach to banking we think the specific technical are legitimate although they may be conceived of as restrictive.

An advantage of our analysis compared to analyses used in stress-tests and other similar analyses of banks sensitivity to negative shocks, in which threshold values on shocks for banks to violate capital adequacy constraints are calculated, is that in our analysis bank assets are adjusted so as to maintain value maximization after negative shocks have occurred. Such endogenous credit contractions reduce the frequency of violations of capital adequacy requirement which are central in stress tests.

Two results of the basic model seem to be particularly interesting One is that the bank’s dynamic value maximization significantly reduces incentives for excessive risk-taking
induced by limited liability. The other is, given risk neutral bank owners, an optimally
capitalized bank invests so as to maximize current period expected net return on investment.

We put the model to work by analyzing the functioning of two different components
in the Basle III accord. Several results obtained questions the efficacy of those regulations.
One result is that both capital adequacy regulation and constraints on dividends and bonuses
increase expected survival of banks. However, it is highly questionable whether increased
expected bank survival is a primary policy goal. Another result is that a capital adequacy
requirement will not improve equilibrium bank capitalization when the regulation does not
bind but lead to a more forceful credit contraction when it binds. That runs counter to a
primary policy goal, namely to keep up credit in downturns. Constraints on dividend and
bonuses payments will induce banks to pay dividends and bonuses at a lower level of
capitalization than they would, were there no such constraints. Hence, while such a constraint
prevents owners and management from eroding bank capital in bad times it may lower the
level of capitalization at which banks enter bad times. That is, of course, an unwanted side-
effect of the regulation.

One of the limitations on the analysis is that we have assumed perfect maturity
matching. It is likely that imperfect maturity matching, which increases bank profit through
the margins between short and long term interest rates, would increase the demand for bank
reserves. The reason is that bank investment cannot be decreased immediately and a bank
with a significant mismatch in combination with too low a level of bank capital therefore risks
violate the capital adequacy regulation and be forced into liquidation or recapitalization. A
stricter capital adequacy requirement would then possibly increase the optimal level of bank
reserves but also decrease maturity mismatch. Including endogenous maturity mismatch
represents an interesting development of the analysis in this paper. It should be noted that
maturity mismatch represents an unavoidable consequence of liquidity transformation whereby banks transform short term savings into long term investments.

As our analysis shows that regulations of capital adequacy and dividends and bonuses payments do not represent perfect instruments to control risk-taking in banks it may be valuable to consider additional means to affect incentives for risk-taking and to reduce the exposure of public finances to bank failures. “The Vicker’s report” (ICB 2011) considers additional measures such as separation between domestic retail banking and investment banking and stricter regulation of what kind of asset portfolios retail banks may have. Those measures may be suitable for the UK with its large internationally oriented investment banking sector. However, it may also be useful to more directly address incentives for risk-taking in banks by making premiums to deposit insurance better reflect risks, to formulate a credible bail-out policy for wholesale bank debt and possibly to tilt the priority order of claims in the favor of retail depositors and the deposit insurance in the case of bank bankruptcy so that incentives for wholesale depositors to evaluate investment risk get stronger. A move in that direction is implied from our results, namely, with funding at actuarial fair costs, maximization of fundamental value reduces risk-taking as compared to short-term profit maximization.
References


**Appendix**

For technical reasons we exclude the possibility for banks to use equity for lending/investing. That allows us to solve the dynamic banking problem. However, banks are of course not restricted to keep equity capital as liquid assets. According to Edlin and Jaffee (2009) US banks tend to keep large quantities at FED at low interest rather than lending up to their limits which implies that banks might follow a strategy similar to the one imposed here. In Figure A1 we show the return on equity for a bank which uses equity capital as collateral and a bank that uses equity capital for lending/investment.
Figure A1: The picture shows both the drift function for a bank that is restricted to “store” its equity as liquid assets not to be used in the banking and the drift for an all equity bank. Due to leverage the drift is much higher for the leveraged bank for the equity levels of interest. With leverage the return-on-equity is boosted. However, due to the convex volatility function the drift is decaying as risk increases with size and the cost of capital for the bank is therefore increasing with size and eventually the all equity bank will yield higher return on equity but that will, with our parameters, occur at equity levels above the optimal