On the development strategy of countries of intermediate size - An analysis of heterogenous firms in a multiregion framework *

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Abstract

This paper compares two policies: trade cost reduction and firm relocation cost reduction using a three-country version of a heterogeneous-firms economic geography model, where the three countries have different market (population) size. We show how the effects of the two policies differ, in particular, for the country of intermediate size. Unless the intermediate country is very small, it will gain industry when relocation costs are reduced, but lose industry when trade costs are reduced. The smallest country loses industry in both cases, but only experiences lower welfare in the case of lower relocation costs. Thus, the ranking of the policies from the point of view of the two small and intermediate countries tends to be the opposite.

JEL Classification: F12, F15, F21, R12

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1 Introduction

World-wide economic integration, often called globalisation, makes it easier to trade goods and, in many cases, makes it easier to set up plants and establishments in foreign countries. Models of economic geography and trade have focused on the effects of lower trade costs. They show how industries agglomerate to large core countries as trade costs are reduced (see e.g. Baldwin et al. (2003) and Fujita and Thisse (2002) for a survey). The analysis is generally performed in a two-country setting, but similar conclusions apply in a multi-country setting.1

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1For a multiregion analysis, see e.g. Baldwin et al. (2003, ch. 14), Forslid (2010), Krugman and Livas (1996) and Puga and Venables (1996).

Ago et al. (2006) and Okubo and Thisse (2008) show how industry may relocate towards the smallest region because of severe price competition in the larger markets when using the linear-demand monopolistic competition model of Ottaviano et al. (2002).
An important example of far reaching economic integration is the European Union. The focus here has been as much on lower barriers to the free mobility of production factors, such as labour and capital, as on lower trade costs for goods. The economic integration in Europe has influenced the geographical patterns of industries, but unlike what models of trade and economic geography tells us, there is no strong empirical evidence of an emerging core-periphery pattern in Europe, as shown by Midelfart-Knarvik et al. (2000) and Midelfart-Knarvik and Overman (2002). The location pattern in Europe is therefore better described by a multi-country than a two country framework, and by an framework where the reduction of relocation costs of factors of production are analysed together with reduction of trade costs.

The current research on firm location patterns focuses not only on geographical concentration, as mentioned above, but also on firm heterogeneity in productivity. The emergence of this literature is closely related to micro-econometric results based on firm level data sets. More precisely, the current theoretical advancement concerns how spatial location patterns are related to firm heterogeneity in labour productivity, and how firms are selected or sorted to markets of different size according their productivity. Baldwin and Okubo (2006) show how the most productive firms have the strongest incentives to move to the larger countries leading to spatial sorting, with the less productive firms left in the periphery when there is a fall in trade costs. On the empirical side Combes et. al. (2009) show how firm heterogeneity influences the productivity of French cities.

To analyse these issues, the present paper studies the effects of lower relocation costs of firms (capital) as well as lower trade costs and presents a three-country economic geography model with heterogeneous firms. Relocation costs in our model are any costs associated with the geographical movement of a production facility, such as e.g. regulatory barriers. Our analysis shows how the collapse of all industry to the core may be specific to two-country models analysing economic integration in the form of lower trade costs only. Here, we use a model with large, intermediate and small countries. In addition to lower trade costs, we analyse economic integration in the form of lower relocation costs. We show that, contrary to trade liberalisation, lower relocation costs can lead to firm relocation into both the large and the intermediate country.

The framework we use is a multi-country version of the heterogeneous firms trade and location model by Baldwin and Okubo (2006). We find several new results. Lower trade costs

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2 The first ‘pillar’ of the Maastricht Treaty includes the the Internal Market with its four freedoms: free movement of goods, services, workers and capital, as well as the Single Market Programme including harmonisation of standards.

3 Okubo and Rebeyrol (2005) analyse a fall in relocation costs but use a two country framework.

4 There is also a body of literature showing that workers and firms on average are more productive in larger markets (Head and Mayer, 2004; Redding and Venables, 2004; Syverson, 2004, 2006; and Amiti and Cameron, 2007).

5 Relocations costs could also encompass such phenomena as a malfunctioning housing market that makes it difficult to establish a factory in a new location.
tend to produce the usual concentration of economic activity to the core (large) country in our model. That is, industry from all countries moves towards the core. Despite this, welfare increases for all countries as a result of trade liberalisation.\textsuperscript{6} Lower relocation costs also lead to an increased concentration to the core but, unless the intermediate country is very small, it is only firms from the smallest country that move there. The intermediate country actually gains industry as a consequence of lower relocation costs. Welfare increases for the large and intermediate countries, whereas the small country that loses industry experiences declining welfare in this case.

A policy implication of our analysis is that European countries of intermediate size, in particular, may benefit from free mobility of production factors within EU. Turning to development strategies of poor countries, our analysis indicates that intermediate size developing countries may be better served by focusing on FDI than on trade. Lower barriers to FDI would lead to an inflow of industry, whereas lower trade costs could lead to the opposite. Our model could also be applied to a regional context within a country, where trade costs are interpreted as transportation costs only. An interpretation of our results, from a regional perspective, is that the long-run prospects of regional centers outside the largest core regions could be upgraded as a result of lower relocation costs.

2 The Model


2.1 Basics

There are \(n\) countries with an asymmetric population (market size). Countries are ordered so that \(Country\ 1\) is the largest and \(Country\ n\) the smallest. There are two types of factors of production, capital and labour. Capital, which is sector specific, can move between countries but capital owners do not. Workers can move freely between sectors but are immobile between countries. A homogeneous good is produced with a constant-returns technology only using labour. Differentiated manufactures are produced with increasing-returns technologies using both capital and labour. The mass of differentiated firms is normalised to one, \(N \equiv 1\).

All individuals have the utility function

\[
U = C_M^\mu C_A^{1-\mu}, \quad \text{where} \quad C_M = \left[ \int_{c \in \Psi} c_i^{(\sigma-1)/\sigma} \, dk \right]^{\sigma/(\sigma-1)}
\]  

\textsuperscript{6}Our results on trade liberalisation are related to those of Gopinath and Saito (2011) who analyse the effects of preferential trade liberalisation in a setting with two domestic regions trading with the outside world. They use a model with heterogeneous firms in a linear demand setting, and do not analyse the effects of lower relocation costs.
where $\mu \in (0,1)$, $\sigma > 1$ are constants and $\Psi$ is the set of consumed variety. $C_M$ is a consumption index of manufacturing goods and $C_A$ is consumption of the homogenous good. $c_l$ is the amount consumed of variety $l$. Country subscripts are suppressed when possible for ease of notation.

Each consumer spends a share $\mu$ of his income on manufactures. Total demand for a domestically produced variety $i$ is

$$x_i = \frac{p_i^{-\sigma}}{\int_{l \in \Psi} p_l^{1-\sigma} \, dk} \, l^\sigma,$$  \hspace{0.5cm} (2)

where $p_l$ is the price of variety $l$ and $Y$ income in the country.

Ownership of capital is assumed to be fully internationally diversified; that is, if one country owns $X$-percent of the world capital stock, it will own $X$-percent of the capital in each country. The income of each country is therefore constant and independent of the location of capital. World expenditure equals world factor income $E^W = wL^W + \mu E^W / \sigma$. Without loss of generality, we choose units so that $L^W \equiv 1$, which gives $E^W = \frac{1}{1-\mu/\sigma}$. Income of country $j$ is equal to its share of world expenditures given by

$$Y_j = s_j E^W = s_j \frac{\sigma}{\sigma - \mu}.$$  \hspace{0.5cm} (3)

$Y_j$ is thus constant irrespective of the location of capital; i.e. also out of long-run equilibrium.

Turning to the supply side, the homogeneous good sector is a constant returns and perfect competition sector. The unit factor requirement of the homogeneous good is one unit of labour. The good is freely traded and since it is also chosen as the numeraire, we have

$$p_A = w = 1,$$  \hspace{0.5cm} (4)

$w$ being the wage of workers in all countries.

In the production of differentiated goods, firms have a firm specific unit labour input coefficient ($a$) and uses one unit of capital, as in the standard footloose capital model. Fixed amount of capital endowments in the world leads to no entry and exit of firms, whilst international capital mobility allows firms to move between countries. Total costs for firm $i$ are specified as

$$TC_i = \pi_i + a_i x_i,$$  \hspace{0.5cm} (5)

where the fixed cost consists of capital, whereas the variable cost consists of labour. Importantly firms are heterogeneous and their firm-specific marginal production costs $a_i$ are distributed according to the cumulative distribution function $F(a)$.

Geographical distance is represented by trading costs. Shipping the manufactured good involves a frictional trade cost of the “iceberg” form: for one unit of good from country $j$ to arrive in country $k$, $\tau_{jk} > 1$ units must be shipped. Trade costs are symmetric between all countries $\tau_{jk} = \tau \forall j,k$.

Profit maximisation by manufacturing firms leads to a constant mark-up over marginal cost

$$p_i = \frac{\sigma}{\sigma - 1}a_i,$$  \hspace{0.5cm} (6)
and the export price is $p_i \tau_{jk}$, taking the iceberg trade costs into account.

### 2.2 Short-run equilibrium

In the short run equilibrium, the allocation of capital in each country is taken to be fixed. Capital owners hold capital in their country of origin. $s_j$ denotes the share of capital and the number (mass) of firms in Country $j$ since one unit of capital corresponds to one firm, and since $N^W = 1$.

Firm heterogeneity in labour requirements, $a_i$, is probabilistically allocated among firms. In order to analytically solve the model, we follow Helpman, Melitz and Yeaple (2004) and assume a Pareto cumulative density function of $a$:

$$F(a) = \frac{a^\rho}{a_0^\rho - a^\rho},$$  \hspace{1cm} (7)

where $\rho > 1$ is a shape parameter and $a_0^\rho - a^\rho$ is a scaling factor. We assume the distribution to be truncated at $0 < a < a_0$ so that the productivity of firms is bounded, and we normalise so that $a_0 = 1$. Figure 1 illustrates the distribution of firms in the three economies before capital can move.
The return to capital of a firm in a country \( j \) is the firm’s operating profit,

\[
\pi_j(a_i) = \frac{a_i^{1-\sigma}}{(\sigma - \mu)\mu} \left( s_j \frac{\Delta_j}{\Delta_j} + \sum_k \phi_{jk} s_k \frac{\Delta_k}{\Delta_k} \right),
\]

where the right-hand side follows from the demand functions in (2) and

\[
\Delta_j = s_j \int_a^1 a_i^{1-\sigma} dF(a) + \sum_k \phi_{jk} s_k \int_a^1 a_i^{1-\sigma} dF(a).
\]

The object \( \phi_{jk} \equiv \tau_{jk}^{1-\sigma} \), ranging between 0 and 1, stands for "free-ness" of trade between countries \( j \) and \( k \) (0 is autarky and 1 is zero trade costs). It is assumed that the labour stock is sufficiently large so that the agricultural sector, which pins down the wage, is active in all countries.

Consider now what would happen if firms were allowed to move between countries. From (8) the firms’ return to capital is convex and falling in \( a_i \). Firms with the highest labour productivity (the lowest \( a_i \)) have the largest profits and will be the most sensitive to market size and thus have the strongest incentives to move to the large market. Under reasonable assumptions of moving costs, this would lead to sorting with the most productive firms in the larger market, as shown by Baldwin and Okubo (2006).

More formally, a firm will move from \( k \) to \( j \) when

\[
\pi_j(a_i) - \pi_k(a_i) - \chi = \frac{a_i^{1-\sigma}}{(\sigma - \mu)} \left( 1 - \phi \right) \mu \left( \frac{s_j}{\Delta_j} - \frac{s_k}{\Delta_k} \right) - \chi \geq 0,
\]

where \( \chi \) is a per-unit of capital fixed relocation cost.\(^7\)

In the following we proceed with a three-country analysis, which is the simplest structure that enables us to focus on countries of intermediate size. Country 1 has the largest population, Country 2 is of intermediate size and Country 3 has the smallest population \( (s_1 > s_2 > s_3) \).

### 2.3 Relocation tendencies

Before moving to the full long-run solution of the model, we consider the relocation incentives faced by firms starting out from the initial equilibrium. Figure 2 shows \( \pi_j(a_i) - \pi_k(a_i) \) for all country pairs.

Note that we rule out that firms have infinite productivity by assuming \( a \) to be bounded from below at \( a_0 \).\(^8\) The incentive to relocate increases in firm size as well as in the market size difference between two countries. Higher productivity firms are more sensitive to market size difference and have stronger incentives to move to large markets. The largest size difference

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\(^7\)This specification differs from that of Baldwin and Okubo (2006), where the relocation cost is a function of the migration pressure.

\(^8\)No bounded fixed moving cost per unit of capital would be sufficiently high to prevent the infinitely productive firms to move.
corresponding to the highest curve in Figure 2 is always between the largest and the smallest country. Then, the curves will be ordered depending on the relative size of countries.

The effects of relocation costs can be seen from Figure 2. For a high moving cost, as illustrated by line $\chi_0$ in the figure, only the most productive firms from the smallest country will migrate to Country 1. As relocation costs are reduced, relocation will take place between more countries. The extent of relocation between different countries will depend on their relative size.

When turning to the long run equilibrium, firms start to move and we need to explicitly model the dynamics. With many countries, there will in general be a simultaneous relocation between several country pairs.

2.4 Long-run equilibrium

In the long run equilibrium, capital is fully mobile between countries and responsive to the incentives provided by the relative returns that can be obtained in the two countries. 9 Thus, firms are mobile internationally. However, note that capital owners are bound to their country of origin, and capital rewards are therefore repatriated to the country of origin. The value of relocation to a larger market is highest for the most productive firms since they have higher sales and are better equipped to cope with the higher competition in the large market. Relocation

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9Profit maximisation ensures that capital is located where its return is maximised.
therefore starts from the high end of the productivity distribution. Generally, the value of migrating for a firm depends on its own marginal cost and the mass of firms that have already migrated, \( a_R \). The value of migrating from smaller market (Country \( k \)) to the larger market (Country \( j \)) at a point in time is therefore

\[
v_{jk}(a_R) = \pi_j(a_R) - \pi_k(a_R) - \chi = \frac{a_R^{1-\sigma}}{(\sigma - \mu)}(1 - \phi)(B_j - B_k) - \chi,
\]

where

\[
B_j \equiv \frac{s_j}{\Delta_j(a_R)}, \quad \Delta_j(a_R) \equiv s_j \int_a^{a_R} a^{1-\sigma} dF(a) + \phi(1 - s_j) \int_0^1 a^{1-\sigma} dF(a) + s_k(1 - \phi) \int_a^{a_R} a^{1-\sigma} dF(a),
\]

and where \( B \) is a measure of the average per-firm market size that is independent of the firm's productivity, \( a_i \). The long-run equilibrium is determined by solving \( v_{jk}(a_R) = 0 \) for \( a_R \).

The relative size of countries will be of key importance in any multiple country setting. As mentioned above, we assume that \( s_1 > s_2 > s_3 \), which implies that relocation will start from Country 3 to Country 1. The long-run implications for the intermediate country, Country 2, will depend on its relative size. To highlight market size differences, we assume that \( \phi \) is the same between all country pairs.

3 The effect of reduced trade costs

Trade liberalisation (an increase in \( \phi \)) affects the value of relocation. A difficulty, when analysing trade liberalisation with many countries, is that it may be that firms from one country move to two other countries simultaneously or that firms from two countries simultaneously move to a third country. When firms are heterogeneous, it becomes difficult to keep track of the sorting of firms when this happens. To simplify the analysis, we assume in this section that, instead of a fixed relocation cost, there is a firm relocation cost à la Baldwin and Okubo (2006) that is related to the migration pressure. The relocation cost is high when many firms move out of a country at the same time, or when many firms move in to a country at the same time, but gradually declines as the migration pressure falls when we approach equilibrium. This assumption implies that the most productive firm is the first to relocate and that it moves to the location with the highest return. It will likewise be the firms with the highest gains that are the first to move into a country. This implies that as long as the gains from moving between country pairs are different, out-migration of firms from a country will go to one destination country at a time, while firms migrating into a country will come from one source country at a time.
3.1 The equilibrium path

Using the above logic, starting from autarchy the productive firms in the smallest country move to the other countries. Section 6.1 in the Appendix shows that

\[
\left. \frac{dv_{31}(a)}{d\phi} \right|_{\phi=0} > \left. \frac{dv_{21}(a)}{d\phi} \right|_{\phi=0} > 0.
\]  

This implies that relocation starts by firms in Country 3 relocating to Country 1, as trade liberalisation starts from autarky. Further trade liberalisation implies that there is a fall in \( \pi_1 \), as firms move into Country 1. Would it then be the case that after a while, firms instead move to Country 2? The answer is no as long as \( \phi < \phi^B \), where \( \phi^B \) stands for bifurcation point trade cost. Section 6.1 in the Appendix shows that

\[
\left. \frac{dv_{31}(a_R)}{d\phi} \right| > \left. \frac{dv_{32}(a_R)}{d\phi} \right| > 0 \text{ for } \phi < \phi^B,
\]  

where \( \phi^B = \frac{(1-s_2)^2}{3s_2-1} \). This implies that starting from any equilibrium \( a_R \) when \( \phi < \phi^B \), it is more profitable for a Country 3 firm to relocate to Country 1 than to Country 2. (Note that \( v_{31}(a_R) = v_{32}(a_R) \) in equilibrium, since the relocation costs related to congestion go to zero as the relocation of firms stops.)

This relocation pattern is illustrated in the left-hand part of Figure 3. However, successive trade liberalisations reduce the profit gap between Country 1 and 3 more than between Country 2 and 3: \( \frac{d^2v_{31}(a_R)}{d\phi^2} < \frac{d^2v_{32}(a_R)}{d\phi^2} < 0 \) as shown in section 6.1 in the Appendix. When we reach \( \phi = \phi^B \), we have \( \frac{dv_{31}(a_R)}{d\phi} = \frac{dv_{32}(a_R)}{d\phi} \), and here relocation also starts from Country 3 to Country 2. This is illustrated in Figure 3 by the hump-shape for Country 2.

Further liberalisation affects the profit differentials according to: \( \frac{d^2v_{31}(a)}{d\phi^2} < \frac{d^2v_{32}(a)}{d\phi^2} < 0 \), and at the level of \( \phi' \) where \( \frac{dv_{31}(a_R)}{d\phi} = \frac{dv_{32}(a)}{d\phi} > 0 \) (for \( a_R > a \)) relocation also starts from Country 2 to 1 (See section 6.1 in the appendix).\(^{10}\)

At the sustainpoint for Country 3, \( \phi' = \frac{s_2}{1-2s_3} \), all firms have left Country 3, and migration continues from Country 2 to Country 1 only. Country 2 thereafter gradually loses its industry as illustrated in Figure 3.

The end result is always that all firms concentrate in Country 1 for sufficiently low trade costs, as illustrated in the figure. This relocation pattern is, in a qualitative sense, very similar to the standard footloose capital model with three countries.\(^{11}\)

\(^{10}\)It may be the case that \( \phi' < \phi^B \) if Country 3 is very small, in which case there is no hump for Country 2. The exact condition for \( \phi' > \phi^B \) is \( s_3 > \frac{(1-s_2)^2}{1-s_2+2s_2^2} \).

\(^{11}\)See e.g. Forslid (2010).
3.2 Welfare effects of reduced trade costs

In the case of homogenous firms, the small and intermediate countries will always gain from trade liberalisation despite losing their entire manufacturing industry. Here, we consider heterogeneous firms and the welfare consequences are therefore potentially different because of the sorting of the least productive firms to the periphery.

The welfare of country $j$ could be measured by real income, $w_j / p_j$. The trade cost reduction has two effects on welfare. It reduces the price of imports, which is always positive for welfare, but it also leads to an outflow of firms for the smaller countries, which is negative for welfare since these varieties must now be imported.

Since $w = p_A = 1$ and capital is fully internationally diversified, it suffices to study the price index, $P_j = \Delta_j^{1/\varphi}$, to compare welfare between countries. As shown in section 6.2 in the Appendix, $\frac{\partial \Delta_j}{\partial \varphi} > 0$, implying that the small country will always gain from trade liberalisation in spite of losing its industry. That is, the effect of cheaper imports always outweighs the effect of the outflow of industry. The same logic applies for the country of intermediate size, which will gain from trade liberalisation even during the final phase of trade liberalisation when it loses its industry to the core country.

We next turn to the effects of reduced relocation costs. As will be seen, these effects can be very different than for reduced trade costs.

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\[12\] See e.g. Forslid (2010).
4 The effect of reduced relocation costs

Here, we discuss the general effects of reduced relocation costs keeping trade costs fixed. In the following subsections, we derive the critical levels of relocation costs where relocation changes nature (the sustain and bifurcation points).

4.1 The equilibrium path

Starting from a hypothetical situation with a given \( \phi \) and with high relocation costs \( \chi \), where the \( \chi \)-line does not intersect with any of the profit differential curves in Figure 2, there will be no relocation. Gradually reducing \( \chi \) we reach a point where the \( \chi \)-line reaches the first profit-differential curve and relocation starts. The first firms to move are the most productive firms in the smallest country, which move to the largest country; thus, from Country 3 to Country 1. Further reductions in \( \chi \) imply that successively less productive firms move. The relocation of firms into Country 1 reduces \( B_1 \) and thereby the incentives to move to Country 1. Despite this, firms from Country 3 will never prefer to move to Country 2 instead of Country 1, as shown in section 6.3 in the Appendix.

The marginal firm at equilibrium, \( a_R \), is defined by the condition that

\[
\frac{a_R^{1-\sigma}}{(\sigma - \mu)} (1 - \phi) (B_1 - B_3) = \chi. \tag{15}
\]

The relocation of firms from Country 3 to Country 1, as relocation costs are reduced, will reduce \( B_1 \) as competition increases in Country 1 and for the same reasons, it will increase \( B_3 \). However, \( B_2 \) and the profit of firms in Country 2 remain constant since the price index is unaffected when no firms relocate to or from Country 2 and when trade costs are unchanged. That is, prices of import goods in Country 2 are unchanged, even if firms relocate from Country 3 to Country 1, since the cost of import, determined by \( \phi \), is the same from both countries. This is illustrated in Figure 4, where the \( B_1 \) line falls and \( B_3 \) rises as successively as less productive firms relocate.

In the case illustrated in Figure 4, \( B_1 \) converges to \( B_2 \) while \( B_3 < B_1 = B_2 \). A sufficient condition for this to happen is that \( s_1 - s_2 < s_2 - s_3 \) as shown in section 6.3 in the Appendix. At the point where \( B_1 = B_2 \), we have that

\[
\pi_1 - \pi_3 = \pi_2 - \pi_3 \tag{16}
\]

since

\[
\frac{a_R^{1-\sigma}}{(\sigma - \mu)} (1 - \phi) (B_1 - B_3) = \frac{a_R^{1-\sigma}}{(\sigma - \mu)} (1 - \phi) (B_2 - B_3) \quad \text{for} \quad B_1 = B_2. \tag{17}
\]

Countries 1 and 2 are then equally attractive for a potential relocater from Country 3, and relocation from Country 3 therefore goes to both Country 1 and Country 2 from this point on. Successively lower relocation costs will lead to a gradual relocation from Country 3 until no
Figure 4: The effect of a lower relocation cost when the intermediate region is relatively large.
firms are left in that country. Further reductions in $\chi$ do not affect the location of firms as long as the relocation costs are positive since $B_1 = B_2$. The location pattern of firms as relocation costs, $\chi$, are reduced is illustrated in Figure 5, where $\chi$ decreases along the x-axis.\(^\text{(13)}\)

The effects of lower relocation costs are thus very different from the effects of reduced trade costs in the case where the intermediate country is not too small ($s_1 - s_2 < s_2 - s_3$). The main difference lies in the outcome for the country of intermediate size. Lower trade costs lead to a concentration of industry from all countries to the largest country, whereas reduced relocation costs lead to deindustrialisation of the smallest country only. Here, both the large and the intermediate country gain industry. The fundamental reason for this difference is that trade liberalisation always affects the price index of all countries, and therefore the relative attractiveness of the countries. In contrast, reduced relocation costs only affect the price index of countries where firms move in and out.

Our interest lies in the case where $s_1 - s_2 < s_2 - s_3$, as illustrated in Figure 5. There are two

\(^{13}\)When the intermediate country is smaller so that $s_1 - s_2 > s_2 - s_3$, it will instead be the case that the system reaches a point where $B_3 = B_2 < B_1$ as relocation costs are reduced. At this point, relocation starts from both Country 2 and 3 towards Country 1. Thus, this case resembles the core-periphery outcome that is the result of trade liberalisation, and this case will not be further analysed here.
phases in the relocation of firms to Country 1. In the first phase, firms from the small country relocate to the largest country only. Thereafter, in phase 2, when the point where $B_2 = B_1$ has been reached, relocation from Country 3 goes to both Country 1 and Country 2. We now turn to the calculation of the "bifurcation point" at which relocation switches from the first to the second phase and the "sustain point" at which all industry has left Country 3. From this point onwards, there is no more reallocation since there are positive reallocation costs, and since $B_2 = B_1$ with constant trade costs (constant $\phi$).

4.2 Phase 1 (relocation from Country 3 to Country 1)

We here analyse relocation under phase 1 when firms in Country 3 only relocate to Country 1. This phase continues until $B_1 = B_2$.

As noted above, the cut-off, $a_{R}$, is determined by the following equation:

$$
\frac{a_{R}^{1-\sigma}}{\sigma - \mu} (1 - \phi) (B_1^1 - B_3^1) = \chi
$$

(18)

where $B_1^1 = \frac{s_1}{\Delta_1}$, $B_2^1 = \frac{s_2}{\Delta_2}$ and $B_3^1 = \frac{s_3}{\Delta_3}$. Superscript "1" indicates phase 1. $\Delta_1^1$, $\Delta_2^1$ and $\Delta_3^1$ are given by
\[
\Delta_1^I = \lambda \{s_1 + \phi(1 - s_1) + (1 - \phi)s_3 \frac{(a_R^\alpha - a^\alpha)}{\gamma}\}
\]
\[
\Delta_2^I = \lambda \{s_2 + \phi(1 - s_2)\}
\]
\[
\Delta_3^I = \lambda \{s_3 + \phi(1 - s_3) - (1 - \phi)s_3 \frac{(a_R^\alpha - a^\alpha)}{\gamma}\}
\]

(19)

where \(\lambda \equiv \frac{\rho}{1 - \sigma + \rho} \frac{1 - a^\alpha}{1 - a^\alpha}, \alpha \equiv 1 - \sigma + \rho\) and \(\gamma \equiv 1 - a^\alpha\). Note that firm relocation never affects \(\Delta_2\) and \(B_2\) in phase 1. Using these definitions, we get \(\frac{d\Delta_2^I}{da_R} < 0\) and \(\frac{d\Delta_3^I}{da_R} > 0\). Thus, \(\frac{dB_2}{da_R} > 0\), \(\frac{dB_3}{da_R} < 0\). As a result of total differentiation, we get \(da_R < 0\) and \(\frac{dB_1}{d\chi} < 0\). Due to \(\frac{dB_1}{d\chi} > 0\) and \(\frac{dB_3}{d\chi} < 0\), a decline in relocation costs promotes relocation, which decreases \(B_1\) but increases \(B_3\). This relocation phase finishes when \(B_1 = B_2\). The cut-off at the bifurcation point is

\[
a_R^\alpha = \frac{\gamma \phi(s_1 - s_2)}{(1 - \phi)s_2 s_3} + a^\alpha,
\]

(20)

and the level of relocation costs is

\[
\chi = \frac{a_{B_2}^{1 - \sigma}}{(\sigma - \mu)}(1 - \phi) \left( \frac{s_1}{\Delta_1^B} - \frac{s_3}{\Delta_3^B} \right),
\]

(21)

where

\[
\Delta_1^B = \lambda \{s_1 + \phi(1 - s_1) + \frac{\phi(s_1 - s_2)}{s_2}\} = \lambda s_1 \{1 + \frac{\phi(1 - s_2)}{s_2}\},
\]
\[
\Delta_2^B = \Delta_2^I = \lambda \{s_2 + \phi(1 - s_2)\}
\]
\[
\Delta_3^B = \lambda \{s_3 + \phi(1 - s_3) - \frac{\phi(s_1 - s_2)}{s_2}\}.
\]

(22)

4.3 Phase 2 (Relocation from Country 3 to Country 1 and Country 2)

When relocation costs are lower than \(\chi\), \(B_1 = B_2\) and relocation from Country 3 goes to both Country 2 and Country 1. Relocation will now keep \(B_1 = B_2\) for all levels of \(\chi\). This will imply that a fraction \(\delta = \frac{s_1}{s_1 + s_2}\) of the moving firms moves to Country 1 and a fraction \((1 - \delta)\) moves to Country 2.

The cut-off, \(a_R\), is determined by the condition that

\[
\frac{a_{B_2}^{1 - \sigma}}{(\sigma - \mu)}(1 - \phi) (B_1^2 - B_3^2) = \frac{a_{B_1}^{1 - \sigma}}{(\sigma - \mu)}(1 - \phi) (B_2^2 - B_3^2) = \chi,
\]

where \(B_1^2 = \frac{s_1}{\Delta_1^B}, B_2^2 = \frac{s_2}{\Delta_2^B}\) and \(B_3^2 = \frac{s_3}{\Delta_3^B}\). Superscript "2" indicates phase 2. \(\Delta_1^2, \Delta_2^2\) and \(\Delta_3^2\) in phase 2 are given by
Figure 7: Phase 2 relocation
\[ \Delta_1^2 = \lambda \{ s_1 + \phi(1 - s_1) + (1 - \phi)s_3 \frac{(a_B^o - a_B^o)}{\gamma} + \delta(1 - \phi)s_3 \frac{(a_R^o - a_B^o)}{\gamma} \} \\
= \lambda \{ s_1 + \frac{s_1 \phi(1 - s_2)}{s_2} + \delta(1 - \phi)s_3 \frac{(a_R^o - a_B^o)}{\gamma} \} \\
\Delta_2^2 = \lambda \{ s_2 + \phi(1 - s_2) + (1 - \delta)(1 - \phi)s_3 \frac{(a_R^o - a_B^o)}{\gamma} \} \\
\Delta_3^2 = \lambda \{ s_3 + \phi(1 - s_3) - (1 - \phi)s_3 \frac{(a_R^o - a_B^o)}{\gamma} \} . \\
\]

Note that \( B_1^2 = B_2^2 \). Using these expressions, we get \( d\Delta_2^2 = d\Delta_3^2 = 0 \). Thus, \( dB^3 = dB^2 > 0 \). As a result of total differentiation, we can derive \( dB^3 = dB^2 > 0 \).

The relocation cost at the sustain point when all firms have left Country 3 is given by

\[ \chi_S = \frac{1}{(\sigma - \mu)}(1 - \phi)(B_2 - B_3) \]
\[ = \frac{1}{(\sigma - \mu)}(1 - \phi) \left( \frac{s_2}{s_2 + \phi(1 - s_2) + (1 - \delta)(1 - \phi)s_3 \left( \frac{1 - \phi(s_1 - s_2)}{(1 - \phi)s_2} \right)} - \frac{s_3}{\phi} \right) . \] (23)

All firms are concentrated in Country 1 and Country 2 when relocation costs are lower than \( \chi_S \). Importantly, when \( \chi = \chi_S \), the relocation process finishes. Even if relocation costs are reduced from \( \chi_S \), no firms relocate from Country 2 to Country 1 since the \( B_i \)'s are not directly affected by \( \chi \). From the sustain point and onwards, we have \( B_1^S = B_2^S \), where

\[ \Delta_1^S = \lambda \{ s_1 + \frac{s_1 \phi(1 - s_2)}{s_2} + \delta(1 - \phi)s_3 \frac{(1 - a_B^o)}{\gamma} \} , \]
\[ \Delta_2^S = \lambda \{ s_2 + \phi(1 - s_2) + (1 - \delta)(1 - \phi)s_3 \frac{(1 - a_B^o)}{\gamma} \} , \] and
\[ \Delta_3^S = \lambda \phi , \] (24)

where superscript "S" indicates the sustain point. This differs starkly from the usual core-periphery outcome. Full agglomeration never occurs as a result of reduced relocation costs in our model. Both Country 1 and Country 2 experience an inflow of industry.

### 4.4 Welfare effects of reduced relocation costs

Once more, the price index, \( P_j^3 = \Delta_j^3 \), can be used for welfare comparisons. The welfare analysis is simple in this case, since firm location is the sole factor affecting welfare with constant trade costs. The welfare effects of lower \( \chi \) are different in the two phases. Starting with phase 1 when \( B_1 > B_2 > B_3 \), firms relocate from Country 3 to Country 1, resulting in \( \frac{d\Delta_3^2}{d\chi} > 0 \) and \( \frac{d\Delta_1^3}{d\chi} < 0 \). This implies that welfare increases in Country 1 and decreases in Country 3. During phase 2, when \( B_1 = B_2 \), firms move from Country 3 to both Country 1 and Country 2, resulting
in $\frac{dA_3}{d\eta} > 0$, $\frac{dA_1}{d\eta} < 0$ and $\frac{dA_2}{d\eta} < 0$. This implies that welfare increases in both Country 1 and 2 and decreases in Country 3.

The welfare implication of lower trade costs and lower relocation costs are thus very different for the smaller countries. The smallest country loses its industry in both cases, but experiences increased welfare as trade costs are reduced, while a lower relocation cost leads to lower welfare as industry relocates. The intermediate country has increasing welfare in both cases, but gains industry as relocation costs are reduced, while it loses industry as trade costs are reduced.

5 Concluding Discussion

This paper analyses a three-country trade and location model with heterogeneous firms, where the effects of trade liberalisation and a reduction of firm relocation costs are compared. Trade liberalisation eventually leads to the usual core periphery outcome with all firms in the core, also in our case of multiple (three) countries and heterogeneous firms. However, this is no longer the case when considering the effect of lower relocation costs and multiple (three) countries. Unless the intermediate country is too small, it will grow (as will the largest country) as a result of reduced relocation costs.

The welfare implications of trade liberalisation and reduced relocation costs also differ. Trade liberalisation leads to welfare gains for all countries even if both smaller countries lose their industrial base to the core. Reduced relocation costs instead imply loss of welfare and industry for the smallest country, but gains in welfare for the intermediate country, as long as it is sufficiently large to gain industry. This means that the interests of intermediate and small countries may be very different when it comes to these two types of economic integration policies.

Our model may be applied in a national context, where the policy experiments are regional policies, or it may be applied in an international context where the policy experiments pertain to different aspects of globalisation. First, from a regional policy perspective, the above experiments imply that it is of great importance how the integration of different regions in a country is achieved. Regional policy may involve policies that make it easier for individuals and firms to move between regions, such as subsidies for movers or a better functioning real estate market, as well as policies that decrease transportation costs, such as better roads and trains. The first of these policies corresponds to a lower relocation cost in the model and the second to lower trade costs. From the perspective of the largest core region, these policies are both attractive as they lead to an increased concentration to the core and higher welfare. However, the interests of the two smaller regions differ. Lower relocation costs lead to higher welfare and more industry in the intermediate region (unless it is very small), whereas it leads to a loss of industry and welfare for the smallest region. Lower transportation costs lead to a deindustrialisation of the intermediate region along with the smallest one, while both regions gain in welfare.

Second, from an international perspective, our policy experiments imply that the development strategies of countries differ. In particular, the strategy may be different for very small
countries and for countries of intermediate size. Intermediate size countries, as e.g. some of
the fast growing Asian countries, may be best served by focusing on policies that facilitate
the relocation of capital to the country, e.g. policies that promote inward FDI. The smallest
developing countries should instead focus on trade liberalisation according to our model.
6 Appendix

6.1 The trade liberalisation experiment

6.1.1 Proof that \( \frac{dv_{31}(a)}{d\phi} > \frac{dv_{21}(a)}{d\phi} > 0 \)

First, we prove that, starting from autarky, firms in Country 3 always relocate to Country 1 rather than to Country 2:

\[
\frac{dv_{31}(a)}{d\phi} > \frac{dv_{21}(a)}{d\phi} > 0.
\]

\[
\frac{dv_{31}(a)}{d\phi} = -(B_1 - B_3)a^{1-\sigma} + (1 - \phi) \left( \frac{dB_1 d\Delta_1}{d\phi} - \frac{dB_3 d\Delta_3}{d\phi} \right) a^{1-\sigma}
\]

\[
= -(B_1 - B_3)a^{1-\sigma} + (1 - \phi) \psi \left( \frac{s_1}{\Delta_1^2} (1 - s_1) \gamma - s_3 \beta + \frac{s_3}{\Delta_3^2} (1 - s_3) \gamma + s_3 \beta \right) a^{1-\sigma}
\]

\[
= -(B_1 - B_3)a^{1-\sigma} + (1 - \phi) \psi \left( \frac{B_1}{\Delta_1} (1 - s_1) \gamma - s_3 \beta + \frac{B_3}{\Delta_3} (1 - s_3) \gamma + s_3 \beta \right) a^{1-\sigma}
\]

\[
= a^{1-\sigma} \left\{ - \left( 1 + (1 - \phi) \left( \frac{(1 - s_1) \gamma - s_2 \beta}{s_1 \gamma + s_2 \gamma + s_2 \beta(1 - \phi)} \right) B_1 + \left( 1 + (1 - \phi) \left( \frac{(1 - s_3) \gamma + s_3 \beta}{s_3 \gamma + s_3 \gamma + s_3 \beta(1 - \phi)} \right) B_3 \right) \right\}
\]

\[
= a^{1-\sigma} B_1 \left\{ - \frac{1}{\Delta_1} + \frac{1}{\Delta_3} \right\} > 0
\]

where \( \psi \equiv \frac{\nu}{1 - \sigma + \nu} \frac{1}{1 - \sigma}, \beta \equiv a_R^\alpha - a^\alpha \) and \( \gamma \equiv 1 - a^\alpha \).

Note that, in the trade cost reduction case, \( B_1 = B_3 \) in equilibrium since there are no congestion related relocation costs in equilibrium as migration stops (see Baldwin and Okubo (2006)).

Likewise

\[
\frac{dv_{21}(a)}{d\phi} = a^{1-\sigma} \left\{ - \frac{1}{\Delta_2} B_2 \right\} = a_R^{1-\sigma} B_1 \left\{ - \frac{1}{\Delta_1} + \frac{1}{\Delta_2} \right\} > 0.
\]

Since \( \Delta_1 > \Delta_2 \) and \( B_1 = B_2 \), we have that \( \frac{dv_{31}(a)}{d\phi} > \frac{dv_{21}(a)}{d\phi} > 0 \).

6.1.2 Proof that \( \frac{dv_{33}(a_R)}{d\phi} > \frac{dv_{32}(a_R)}{d\phi} > 0 \)

Second, we prove that firms in Country 3 always relocate to Country 1 rather than to Country 2 for a range of trade costs:

\[
\frac{dv_{31}(a_R)}{d\phi} > \frac{dv_{32}(a_R)}{d\phi} > 0 \quad \text{for} \quad \phi < \phi^B
\]

\[
\frac{dv_{32}(a_R)}{d\phi} = a_R^{1-\sigma} \left\{ - \frac{1}{\Delta_2} B_2 + \frac{1}{\Delta_3} B_3 \right\} = a_R^{1-\sigma} B_1 \left\{ - \frac{1}{\Delta_2} + \frac{1}{\Delta_3} \right\} > 0.
\]

Since \( \Delta_1 > \Delta_2 \), we have that \( \frac{dv_{31}(a_R)}{d\phi} > \frac{dv_{32}(a_R)}{d\phi} > 0 \).
6.1.3 Proof that \( \frac{d^2v_{31}(a)}{d\phi^2} < \frac{d^2v_{21}(a)}{d\phi^2} < 0 \), and that \( \frac{d^2v_{31}(a)}{d\phi^2} - \frac{d^2v_{22}(a)}{d\phi^2} < 0 \)

We here derive the relative size of the second derivatives \( \frac{dv(a)}{d\phi} \) and \( \frac{dv(a)}{d\phi} \).

\[
\frac{d^2v_{31}(a)}{d\phi^2} = a^{1-\sigma} B_1 \left\{ -1 + \frac{1}{\Delta_1} \right\} + a^{1-\sigma} B_1 \left\{ \frac{1}{(\Delta_1)^2} \frac{d\Delta_1}{d\phi} - \frac{1}{(\Delta_3)^2} \frac{d\Delta_3}{d\phi} \right\} < 0
\]

\[
\frac{d^2v_{21}(a)}{d\phi^2} = a^{1-\sigma} B_1 \left\{ -1 + \frac{1}{\Delta_2} \right\} + a^{1-\sigma} B_1 \left\{ \frac{1}{(\Delta_2)^2} \frac{d\Delta_1}{d\phi} - \frac{1}{(\Delta_3)^2} \frac{d\Delta_3}{d\phi} \right\} < 0
\]

\[
\frac{d^2v_{32}(a)}{d\phi^2} = a^{1-\sigma} B_1 \left\{ -1 + \frac{1}{\Delta_3} \right\} + a^{1-\sigma} B_1 \left\{ \frac{1}{(\Delta_3)^2} \frac{d\Delta_1}{d\phi} - \frac{1}{(\Delta_2)^2} \frac{d\Delta_2}{d\phi} \right\} < 0
\]

because of \( \Delta_1 > \Delta_2 > \Delta_3 \), \( \frac{dB_1}{d\phi} < 0 \), and \( \frac{d\Delta_1}{d\phi} = \psi\{(1-s_3)\gamma + s_3\beta\} > \frac{d\Delta_1}{d\phi} = \psi\{(1-s_2)\gamma\} > \frac{d\Delta_1}{d\phi} = \psi\{(1-s_1)\gamma - s_1\beta\} > 0 \).

Furthermore, using these two derivatives, we can get

\[
\frac{d^2v_{31}(a)}{d\phi^2} - \frac{d^2v_{21}(a)}{d\phi^2} = a^{1-\sigma} B_1 \left( \frac{1}{\Delta_3} - \frac{1}{\Delta_2} \right) + a^{1-\sigma} B_1 \left( \frac{1}{(\Delta_2)^2} \frac{d\Delta_2}{d\phi} - \frac{1}{(\Delta_3)^2} \frac{d\Delta_3}{d\phi} \right) < 0.
\]

Thus, we can derive \( \frac{d^2v_{31}(a)}{d\phi^2} < \frac{d^2v_{21}(a)}{d\phi^2} < 0 \). Likewise we get \( \frac{d^2v_{31}(a)}{d\phi^2} - \frac{d^2v_{22}(a)}{d\phi^2} < 0 \).

6.1.4 Proof that \( \frac{dv_{31}(a_R)}{d\phi} > \frac{dv_{21}(a)}{d\phi} > 0 \)

We here derive the point of the top of the hump-shaped firms’ location in Country 2. Because of the congestion cost when entering Country 1, there is no movement from Country 2 to Country 1 as long as

\[
\frac{dv_{31}(a_R)}{d\phi} > \frac{dv_{21}(a)}{d\phi} > 0,
\]

where \( \frac{dv_{31}(a_R)}{d\phi} = a_R^{1-\sigma} B_1 \left( -\frac{1}{\Delta_1} + \frac{1}{\Delta_3} \right) \) and \( \frac{dv_{21}(a)}{d\phi} = a^{1-\sigma} B_1 \left( -\frac{1}{\Delta_1} + \frac{1}{\Delta_2} \right) \).

Straightforward calculation shows that this condition holds for

\[
a_R < \tilde{a}_R = \left( \frac{s_1 - s_3}{s_1 - s_2} \right) \left( \frac{s_2}{s_3} \right)^{\frac{1}{\sigma-1}} a.
\]

When \( \tilde{a}_R = a_R \), the most productive firms in Country 2 will start to relocate to Country 1 since \( \frac{dv_{31}(a_R)}{d\phi} = \frac{dv_{21}(a)}{d\phi} > 0 \).
6.2 The welfare effect of trade liberalisation for the small country

Here, we prove that trade liberalisation always improves the per-capita welfare of the smallest country (Country 3). Once Country 3 has been completely deindustrialised, it will obviously gain from further reductions in trade costs, since lower trade costs reduce the price index due to cheaper imports. However, we need to be shown the welfare consequences during the phase when Country 3 loses industry due to trade liberalisation.

In the long-run equilibrium, \( B_1 = B_3 \) always holds as long as there is a manufacturing industry in both countries. The following equation must therefore be satisfied in the equilibrium:

\[
s_1 \Delta_3 - s_3 \Delta_1 = 0 \iff F \equiv (s_1 - s_3)\phi \gamma - s_1 s_3 (1 - \phi) \beta - s_3^2 (1 - \phi) \beta - s_3 s_2 (1 - \phi) \eta = 0,
\]

where \( \Delta_1 = \psi \{s_1 + (1 - s_1)\phi\} \gamma + s_3 (1 - \phi) \beta + s_2 (1 - \phi) \eta \), and \( \Delta_3 = \psi \{s_3 + (1 - s_3)\phi\} \gamma - s_3 (1 - \phi) \beta \). \( \gamma \equiv 1 - a^\alpha, \beta \equiv a^{\alpha}_{R31} - a^\alpha \) and \( \eta \equiv a^{\alpha}_{R21} - a^\alpha \). Note that \( a_{R21} \) denotes the cut-off level of firms in Country 1 which were relocated from Country 2 to Country 1. When there is no relocation from Country 2 to Country 1 (with sufficiently high trade costs), then \( \eta = 0 \). But here we more generally consider the case of the firm relocation from Country 3 to Country 1 as well as from Country 2 to Country 1. We note that \( \eta \) only influences \( \Delta_1 \).

There is no relocation from Country 3 to Country 2. The reason for this is that \( B_1 > B_2 > B_3 \), which implies that \( \pi^1(a_R) - \pi^3(a_R) > \pi^2(a_R) - \pi^3(a_R) \) for all \( a_R \). This means that firms in Country 3 would always relocate to Country 1 rather than Country 2.

Next, differentiating \( F \) w.r.t. \( \phi \) and \( a_{R31} \) gives

\[
\frac{dF}{d\phi} = (s_1 - s_3) \gamma + s_3 (s_1 + s_3) \beta + s_2 s_3 \eta > 0
\]

and

\[
\frac{dF}{da_{R31}} = -s_3 (s_1 + s_3) (1 - \phi) a^{\alpha-1}_{R31} < 0.
\]

Using these two differentials, we get

\[
\frac{da_{R31}}{d\phi} = \frac{(s_1 - s_3) \gamma + s_3 (s_1 + s_3) \beta + s_2 s_3 \eta}{s_3 (s_1 + s_3) (1 - \phi) a^{\alpha-1}_{R31}} > 0.
\]

Since \( \frac{d\Delta_3}{da_{R31}} = -s_3 (1 - \phi) a^{\alpha-1}_{R31} \psi \) and \( \frac{d\Delta_1}{d\phi} = \{(1 - s_3) \gamma + s_3 \beta\} \psi \), we have

\[
\frac{d\Delta_3}{d\phi} = \frac{d\Delta_3}{d\phi} + \frac{\partial \Delta_3}{\partial a_R} \frac{da_R}{d\phi} = \{(1 - s_3) \gamma + s_3 \beta\} \psi - \{(1 - s_3) \gamma + s_3 (1 - s_3) \beta + s_3 s_3 \eta\} \psi
\]

which yields

\[
> \{(1 - s_1 - s_2 s_3) \gamma + s_3 s_2 \beta\} \psi > 0
\]

22
where $\gamma > \eta$ and $1 - s_1 - s_2 s_3 > 1 - s_1 - s_2 = s_3 > 0$.

Thus, a rise of $\phi$ always increases $\Delta_3 \left( \frac{d\Delta_3}{d\phi} > 0 \right)$. Hence, trade liberalisation always improves per-capita welfare in the smallest country.

### 6.3 Reduced relocation costs

This Appendix proves the following inequality:

$$v_{31}(a_R) > v_{21}(a),$$

which may be rewritten as

$$(B_1 - B_3)a_R^{1-\sigma} > (B_1 - B_2)a_R^{1-\sigma}.$$  

In the following, $(B_1 - B_3)a_R^{1-\sigma}$ is called LHS and $(B_1 - B_2)a_R^{1-\sigma}$ is called RHS. Note that because of the fixed per capital relocation cost, it will here typically not be the case that $B$'s are the same in equilibrium.

1) First we check whether it always holds at the initial short-run equilibrium, i.e. with no relocation. When $a_R = a$, it will always be the case that LHS > RHS because $B_2 > B_3$.

2) Second we show that LHS and RHS are both decreasing (continuous) functions of $a_R$.

**LHS:**

$$\frac{dLHS}{da_R} = \left( \frac{dB_1}{da_R} - \frac{dB_3}{da_R} \right) a_R^{1-\sigma} + (1 - \sigma)(B_1 - B_3)a_R^{-\sigma} < 0$$

since

$$B_1 > B_3,$$

$$\frac{dB_1}{da_R} = \frac{dB_1}{d\Lambda} \frac{d\Lambda}{da_R} = -\frac{B_1}{\Delta_1} \left( \frac{d\Lambda}{da_R} \right) < 0,$$

$$\frac{dB_3}{da_R} = \frac{dB_3}{d\Lambda} \frac{d\Lambda}{da_R} = B_3 \left( \frac{d\Lambda}{da_R} \right) > 0,$$

where

$$\Lambda = \frac{(1 - \phi)}{(1 - a^\phi)}(a_R^a - a^a),$$

and

$$\frac{d\Lambda}{da_R} = \frac{a(1 - \phi)}{(1 - a^\phi)}a_R^{a-1} > 0.$$

Turning to RHS, we get

$$\frac{dRHS}{da_R} = \left( \frac{dB_1}{da_R} \right) a_R^{1-\sigma} = -\frac{B_1}{\Delta_1} \frac{d\Lambda}{da_R} a_R^{1-\sigma} < 0.$$
3) No crossing point property: We here show that RHS and LHS never cross in terms of $a_R$. This means that there does not exist any $a_R (\geq \underline{a})$ for $1 > \phi > 0$ such that RHS and LHS are equal.

First we assume that there is an $a_R$ such that RHS and LHS are equal. To equalise RHS and LHS, we need $B_1 = B_2 = B_3$. This happens only when $\phi = 0$, since $s_1 > s_2 > s_3$. However, this is a contradiction to our setting.

4) We show that for $a_R = a_B$, which implies $B_1 = B_2$, we have LHS larger than RHS. RHS is zero since

$$(B_1 - B_2)a^{1-\sigma} = 0.$$ 

Since $B_1 = B_2$, we have $\Lambda = \phi \frac{s_1 - s_2}{s_2}$. LHS can be written as

$$(B_1 - B_3)a^{1-\sigma} = (B_2 - B_3)a^{1-\sigma}.$$ 

The difference in $B'$s is

$$B_1 - B_3 = B_2 - B_3 = \frac{s_2}{s_2 + \phi(1 - s_2)} \psi - \frac{s_3}{s_3 + \phi(1 - s_3) - \Lambda} \psi$$

$$= \frac{s_2/\psi}{s_2 + \phi(1 - s_2)} - \frac{s_2s_3/\psi}{s_2s_3 + \phi s_2(1 - s_3) - \phi(s_1 - s_2)}$$

$$= \frac{s_2/\psi}{s_2 + \phi(1 - s_2)} - \frac{s_2s_3/\psi}{s_2s_3 + \phi s_2(1 - s_3) - \phi(s_1 - s_2)}$$

$$= \frac{s_2}{s_2 + \phi(1 - s_2)} \{s_2s_3 - \phi s_2(1 - s_3) - \phi(s_1 - s_2)\}$$

$$= \frac{s_2}{s_2 + \phi(1 - s_2)} \{s_2s_3 - \phi(s_2(1 - s_3) - \phi(s_1 - s_2))\}$$

$$= \frac{s_2}{s_2 + \phi(1 - s_2)} \{s_2s_3(1 - \phi) + \phi(2s_2 - s_1)\}$$

$$= \frac{s_2}{s_2 + \phi(1 - s_2)} \{s_2s_3(1 - \phi) + \phi(2s_2 - s_1)\}$$

Therefore, for $s_1 - s_2 < s_2 - s_3$, $B_1 - B_3 > 0$ always holds. Note that if $s_1 - s_2 < s_2 - s_3$, then $s_3 < 2s_2 - s_1$. Hence $2s_2 - s_1 > 0$ always holds, which indicates that the denominator is always positive.

From 1) to 4),

$$\frac{(1 - \phi)}{(\sigma - \mu)}(B_1 - B_3)a^{1-\sigma} > \frac{(1 - \phi)}{(\sigma - \mu)}(B_1 - B_2)a^{1-\sigma}$$

holds for any $a_R \in (\underline{a}, a_B)$.
References


