Endogenous Product Differentiation, Market Size and Prices*†

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First Version: October 2008
This Version: November 2010

Abstract

Recent empirical evidence suggests that prices for many goods and services are higher in larger markets. This paper provides an explanation for this phenomenon when firms can choose how much to differentiate their products in a monopolistically competitive environment. The model proposes that consumers’ love of variety makes them more sensitive to product differentiation efforts by firms, which leads to higher prices in larger markets. Larger markets lead to greater variety and products that are more differentiated, which provides consumers with greater welfare despite the adverse effect of product differentiation on prices. The social planner does not charge a markup, which allows it to differentiate products more than is possible in the competitive equilibrium. The model also provides an explanation for why prices do not always fall when trade is liberalized.

JEL Classification Codes: D43, F12, L13.

Keywords: Endogenous Technology, Market Size Effect, International Trade.

* I thank Gregory Corcos, Karolina Ekholm, Rikard Forslid, Henrik Horn, Toshihiro Okubo Philippe Martin and seminar participants at Stockholm University, the Institute of Industrial Economics and the European Trade Study Group for valuable comments and suggestions. Financial support from the Social Sciences and Humanities Research Council of Canada (SSHRC) and the Widar Bagges foundation is greatly appreciated.
† This paper was earlier circulated under the name "A Model of Ideal Differentiation and Trade".
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1 Introduction

Conventional wisdom would suggest that prices should always be lower in larger markets as long as firms enter and competition becomes more intense. However, recent empirical evidence using within-country and cross-country data suggests that prices for many goods and services are in fact higher in larger markets. Studies of consumer price data collected across geographic regions in the same country find that prices are higher in regions with larger populations. Studies of firm-level exports to different countries find that firms sell at higher prices in countries with larger populations. This evidence is at odds with all mainstream models of differentiated goods in the literature, which generally predict that prices are lower in larger markets or that prices are unaffected by market size. This clear disconnect between theory and evidence suggests that there are mechanisms at play that are not captured by current models of monopolistic competition.

This paper provides an explanation for why prices can be higher in larger markets by assuming that firms can choose how much to differentiate their products. The basic model provides a simple tractable general equilibrium result, with the prediction that product differentiation increases with market size. Larger markets encourage product differentiation in the model because the "love of variety" property of the utility function. Love of variety leads individual consumers to consume more varieties and less of each variety in larger markets. I show that this behavior makes them more sensitive to firms' spending on product differentiation, with the prediction that products are more differentiated and sell at higher prices in larger markets.

The theoretical model in this paper is based on monopolistic competition with endogenous technology choice. I assume that consumer utility follows a generalized CES utility function originating in Spence (1976). The model allows firms to spend
on fixed costs in order to differentiate their product. Firms choose their product differentiation spending from a continuum, with a more differentiated product requiring higher fixed cost spending. The fixed cost spending can be considered to be persuasive advertising or product development that differentiates one’s own product from that of other firms. Firms then set prices via monopolistic competition.

Product differentiation in this paper is modeled in a new way compared to previous literature. In the model, product differentiation affects products’ elasticity of substitution. This in turn affects the concavity of utility for each variety. A greater concavity of utility increases consumers marginal utility of consumption at low levels of consumption. Larger markets encourage product differentiation because individuals consume more varieties and less per variety per capita. Larger markets lead consumers to move down their utility curves for each variety and become more sensitive to firms’ spending on product differentiation.

I show that endogenous product differentiation modeled in this way can have either a positive or negative effect on entry as markets expand, depending on the cost to differentiate goods. If the cost to differentiate goods is sufficiently high then the model predicts a positive and concave relationship between market size, entry and prices. Previous studies typically interpret a concave relationship between market size and entry as a pro-competitive effect of market size. My framework suggests, however, that market size and firm numbers alone do not provide sufficient information to make inferences about competition when product differentiation is endogenous.

In the welfare analysis I show that product differentiation has two countervailing effects. While individuals like to consume varieties that are more differentiated, there is also a negative effect due to higher price-cost markups. I am able to show,

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1 This contrasts with most of the previous literature on trade and endogenous technology, such as Markusen and Venables (1997), Ekholm and Midelfart (2005) and Bustos (2010) assume a trade-off between lower marginal costs and higher fixed costs.
however, that the beneficial effect of product differentiation and variety outweighs the adverse effect of higher markups in larger markets. The model thus predicts that market size has an unambiguously positive effect on consumer welfare. I also compare the competitive equilibrium with the social optimum and show that the competitive equilibrium leads to an underinvestment in product differentiation compared to the social planner equilibrium. The social planner does not charge a markup, which allows it to differentiate products more than the competitive equilibrium.

I extend the basic model to include firm heterogeneity. In this case firms are assumed to vary with respect to their efficiency in differentiating their product, which leads to a distribution of prices in the economy and a lower bound on prices. In larger markets the least efficient firms with the lowest prices exit, which increases the lower bound price and hence the average price. I also extend the basic model to include two countries and trade costs. A new prediction regarding trade liberalization in this model is that price-cost markups and fixed cost spending is highest at an intermediate level of per-unit trade costs. The intuition behind this result is that firms’ total trade costs are greatest when per-unit trade costs are at an intermediate level. Since a lower elasticity of substitution reduces the negative impact of total trade costs on export demand, firms have the strongest incentive to differentiate their product and reduce their substitution elasticity when total trade costs make up the largest proportion of firms’ output. The model thus predicts that prices can either rise or fall with trade liberalization. This result can help to make sense of the mixed evidence concerning whether prices fall when trade liberalizes, as is found by Revenga (1997), Trefler (2004) and Feenstra (2006). The prediction that the elasticity of substitution is decreasing with market size and trade costs can also reconcile the Broda and Weinstein (2006) finding that elasticities of substitution have decreased over time.

While there are several recent papers that deal with various aspects of product
quality in differentiated goods markets\textsuperscript{2}, only a few predict that prices increase with market size\textsuperscript{3}. Helble and Okubo (2008) propose a supply-side explanation for higher prices in larger markets, whereby the home market effect bids up the price of skilled labor in the larger country, which increases quality and prices. Similarly, Tabuchi and Yoshida (2000) suggest that the productivity benefits of agglomeration can lead to higher nominal wages, which increases rentals and hence leads to higher prices and hence lower real wages. In contrast, my model proposes a demand-side explanation for price differences between large and small markets. The other model that predicts higher prices in larger markets is by Lorz and Wrede (2009), who model the decision of multiproduct firms to differentiate their products. Their model predicts that firms differentiate a larger proportion of their products in larger markets.

The rest of the paper is organized as follows: The basic model in a closed economy and the effect of market size on product differentiation, entry and welfare are presented in Section 2. A comparison of the competitive solution to the social optimum is given in Section 3. Firm heterogeneity is added to the model in Section 4. The model is expanded to include two countries and trade costs in Section 5. Conclusions follow in Section 6.

\textsuperscript{2}Prominent examples include Baldwin and Harrigan (2007), Simonovska (2010) and Khandelwal (2010).

\textsuperscript{3}The model in this paper describes endogenous horizontal product differentiation, in contrast to the models of endogenous vertical differentiation in the Industrial Organization literature. Prominent examples include Shaked and Sutton (1987), Mazzeo (2002) and Berry and Waldfogel (2010).
2 Basic Model

2.1 Setting

I begin by describing the model in a closed economy. There are two industries: a differentiated goods industry $M$ characterized by increasing returns to scale and a constant returns industry $A$. The differentiated goods industry is composed of a continuum of firms and there is no strategic interaction between firms. Any individual is endowed with one unit of labor.

2.2 Consumer Preferences:

Consumer utility is defined as:

$$U = U[A, M], \quad M = \left( \int_{i=1}^{N} c_i^{\theta_i} \, di \right)^{\frac{1}{\theta_m}}$$

where $c_i$ is the quantity of good $i$ consumed by the representative consumer. The utility for differentiated goods is based on the generalized CES utility function by Spence (1976), where $\theta_i \in (0, 1)$ is a firm-specific parameter that determines the price elasticity of demand for good $i$. $\theta_m \in (0, 1)$ is a preference parameter that is not firm-specific.

The representative consumer’s utility maximization problem for differentiated goods is defined as:

$$\max_{c_i} \left( \int_{i=1}^{N} c_i^{\theta_i} \, di \right)^{\frac{1}{\theta_m}} \quad \text{s.t.} \quad \int_{i=1}^{N} p_i c_i \, di = \mu w$$

(1)

$^{4}$Since $\theta_i$ is a parameter in the consumer’s utility function it may be argued that it is not observable. I assume that firms are able to invest in product differentiation, which is assumed to have a mapping into $\theta_i$. 

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where \( p_i \) is the price of good \( i \), \( \mu \) is the share of income expended on the manufacturing good and \( w \) is the representative consumer’s wage. The demand for a good by a representative consumer is thus:

\[
c_i (p_i, \theta_i, N, \mu, \lambda) = \frac{\mu w \left( \frac{M^{1-\theta_m} \theta_i}{\theta_m \lambda p_i} \right)^{\frac{1}{1-\theta_i}}}{\int_{i=1}^{N} p_i \left( \frac{M^{1-\theta_m} \theta_i}{\theta_m \lambda p_i} \right)^{\frac{1}{1-\theta_i}} di}
\]  

(2)

where \( \lambda \) is the representative consumer’s marginal utility of income. One can also derive an expression for the marginal utility of income:

\[
\lambda = \frac{M^{1-\theta_m} \theta_i c_i^{\theta_i-1}}{\theta_m p_i}.
\]  

(3)

As in Krugman (1979), firms will take \( \lambda \) as given.

2.3 Technology in IRS Industry:

2.3.1 Production Technology

Labor is the only input in this economy, and each firm’s total labor requirement \( l_i \) includes an endogenously determined fixed labor cost \( F_i \), and an exogenous variable amount of labor cost \( \beta \) in the production process:

\[
l_i = F_i + \beta x_i
\]

where \( x_i \) is the total quantity demanded of good \( i \).

2.3.2 Endogenous Product Differentiation

The model in this paper assumes that the fixed cost, \( F_i \), is a function of the preference parameter \( \theta_i \) and imposes the following assumption:
ASSUMPTION 1:

For all $\theta_i \in (0, 1)$,

$$F_i = F_i(\theta_i)$$

is twice continuously differentiable in $\theta_i$, with the following properties:

$$F'_i(\theta_i) < 0, \quad F''_i(\theta_i) > 0, \quad \lim_{\theta_i \to 0^+} F_i(\theta_i) = \infty, \quad \lim_{\theta_i \to 1^-} F_i(\theta_i) = 0.$$ 

The assumption is simply that costs are increasing in product differentiation and convex.

The concept that fixed costs affect consumer preferences follows the work of Sutton (1991). Differentiating one's own product from others (i.e. lowering the preference parameter, $\theta_i$) requires higher fixed costs. These fixed costs could be persuasive advertising or product development that differentiates a firm's own product from that of other firms. I do not assume a functional form at the moment, only that it is upward sloping and convex as $\theta_i$ decreases, so that fixed cost spending exhibits decreasing returns. I later assume a particular function form for $F_i(\theta_i)$ which satisfies all of these properties and allows for an analytical solution, but many properties can be shown with this more general assumption. I refer to (4) as the "advertising function" throughout the rest of the paper.

2.3.3 Markup Pricing

Firms enter, then they choose their optimal level of product differentiation, then they set prices via monopolistic competition. The equilibrium is found by backward induction. Each firm sets price in order to maximize profit, and takes endogenous
fixed costs, $F_i$, as given:

$$\pi_i = p_i x_i - w \beta x_i - w F_i (\theta_i).$$  \hfill (5)

Firms maximize (5) with respect to $p_i$, yielding following first order condition:

$$p_i = \frac{w \beta}{\theta_i}. \hfill (6)$$

One thus obtains markup pricing, with a constant markup for a given $\theta_i$. The markup is endogenous, however, since $\theta_i$ is an endogenous variable chosen by the firm. Firms will take (6) into consideration when choosing $\theta_i$.

### 2.3.4 Optimal Product Differentiation

Each firm chooses their preference parameter, $\theta_i$, to maximize operating profits less the fixed cost to differentiate. Firm $i$’s demand is the sum of consumer demands, $x_i = L c_i$, where $L$ is the number of consumers in the economy. Operating profits are concave in $\theta_i$, so a maximum exists as long as $F_i (\theta_i)$ is chosen such that profits are non-negative. Using (2), the demand for a good in the manufacturing industry is:

$$x_i (p_i; \theta_i, N, \mu, L, \lambda) = \frac{\mu L w \left( \frac{M^{1-\theta_m} \theta_i}{\theta_m \lambda p_i} \right)^{\frac{1}{1-\sigma_i}}}{\int_{i=1}^{N} p_i \left( \frac{M^{1-\theta_m} \theta_i}{\theta_m \lambda p_i} \right)^{\frac{1}{1-\sigma_i}} di} \quad i = 1, ..., N. \hfill (7)$$

Firms take $M, \lambda$ and the denominator in (7) as given when setting their preference parameter. The first order condition for product differentiation is derived by
substituting (6) and (7) into (5), then maximizing firm profits with respect to \(\theta_i\):

\[
(p_i(\theta_i) - w\beta) x_i(p_i, \theta_i) \left[ \frac{1}{(1-\theta_i)^{2}} \ln \left( \frac{M^{1-\theta}}{\theta_{m,\lambda}^{1}} \frac{1}{w\beta} \right) \right] = wF'(\theta_i) \tag{8}
\]

Note that (8) equates the marginal revenue and marginal cost of increasing \(\theta\), which explains why both sides of (8) are negative. Given Assumption 1, the marginal cost of product differentiation increases and approaches infinity as \(\theta\) decreases towards zero.

### 2.3.5 Equilibrium with Identical Firms

The rest of this section assumes that firms are identical, meaning that they will all choose the same level of product differentiation. Firms enter until profits equal zero for each firm. Combining the markup pricing condition, (6), and the zero profit condition, \(p(\theta) = w\beta + wF(\theta) / x(\theta)\), one obtains an expression for output per firm:

\[
x(\theta) = \frac{F(\theta) \theta}{\beta (1 - \theta)}. \tag{9}
\]

The functional form of \(F(\theta)\) can be chosen such that \(x(\theta)\) is an increasing, decreasing, or constant function of \(\theta\).

The full employment of labor condition determines the number of firms in the manufacturing industry:

\[
N = \frac{\mu L}{F(\theta) + \beta x}. \tag{10}
\]

Overall, equations (3), (4), (6), (8), (9) and (10) make up the basic model. This includes the same equations as Krugman (1980) for profit maximization in price, zero profits, and full employment of labor, plus (3), (4) and (8). The unknowns are \(p, x,\)
The next lemma establishes that firms choose a unique level of product differentiation that maximizes their profit, which holds under very general assumptions about the nature of the advertising function.

**Lemma 1** Suppose that Assumption 1 holds and firms set prices via monopolistic competition. Then for a large enough $L$ there exists a unique symmetric equilibrium value of $\theta$ with positive fixed cost spending given by the following expression:

$$
\frac{F'(\theta)}{1 - \theta} \left[ \ln \left( \frac{F(\theta) \theta}{\beta L} \right) + \frac{1}{\theta} \right] = F'(\theta).
$$

**Proof.** See appendix.

Equation (11) provides an implicit solution for the equilibrium level of product differentiation, $\theta$, which is a function of market size $L$, marginal cost $\beta$ and the form of the advertising function.

### 2.4 Market Size Effects

The model predicts that firms’ spending on product differentiation is affected by market size, which has important implications for prices and entry. The next proposition describes how market size affects equilibrium product differentiation.

**Proposition 1** Suppose that Assumption 1 holds. Then product differentiation is increasing in market size, i.e.:

$$
\frac{d\theta}{dL} < 0.
$$

**Proof.** See appendix.

The intuition for this result falls directly from the love-of-variety property of the generalized CES utility function. The elasticity of substitution affects the concavity
of utility for each variety and a greater concavity of utility increases consumers marginal utility of consumption at low levels of consumption. Larger markets encourage product differentiation because consumers consume more varieties and less per variety per capita. Consumers thus move down their utility curves for each variety and become more sensitive to firms’ spending on product differentiation.

The market size effect on $\theta$ affects all of the other endogenous variables in the model. As market size increases, fixed cost spending will accordingly increase via the relationship specified in equation (4). Prices rise via the markup pricing rule (6).

The prediction that prices are higher in larger markets has been confirmed by several studies. Roos (2006) finds that consumer prices are higher in regions of Germany with larger populations, while Tabuchi and Yoshida (2000) find that doubling city size in Japan reduces real wages by approximately 7-12%. Recent evidence on export prices at the firm-level suggests a similar pattern in differentiated goods markets across countries differing in population size. Manova and Zhang (2009) find that Chinese firms set higher prices in larger markets. Ottaviano and Mayer (2008) examine the pattern of trade across several European countries using firm-level data and find that larger countries export and import higher-priced goods on average. They conclude that prices "defy gravity", but cannot offer an explanation for this phenomenon.

This new evidence on the pattern of prices does not square with any workhorse models of monopolistic competition in the International Trade and New Economic Geography literature. The intra-industry trade models of Krugman (1980) and Helpman and Krugman (1987) employ Dixit-Stiglitz (1977) preferences and exhibit no market size or distance effect on prices. The heterogeneous firm model by Melitz (2003) exhibits no market size effects and average export prices that are decreasing in distance. Models based on Melitz and Ottaviano (2008) assume quadratic utility
and exhibit "pro-competitive effects", whereby markups and hence prices are lower in larger markets due to greater competition. Models based on Baldwin and Harrigan (2007) exhibit the desired characteristic that higher-priced goods are sold to more distant countries, but lack any market size effect. To the best of my knowledge there are no models that predict both an "anti-competitive" market size effect and that average export prices increase with distance.

The prediction that prices increase in market size is the opposite conclusion of the "ideal variety" approach to modelling monopolistic competition, whether product differentiation is exogenous as in Lancaster (1979, 1980) or endogenously determined as in Weitzman (1994). The reason for this discrepancy is that product differentiation imposes a disutility on consumers in the "ideal variety" approach, while it enhances consumers' utility in the "love of variety" approach. Adding endogenous product differentiation in the manner described above can thus provide a very different result compared to previous models.

The predictions here also conflict the evidence from the market for ready-mixed concrete by Syverson (2004). Ready-mixed concrete is a homogeneous product, which makes switching between suppliers easier when markets are more dense. In contrast, my framework deals with markets for differentiated products, and going without a particular good is more difficult in larger markets because goods are more differentiated.

The relationship between the number of firms, market size and product differentiation can be seen by substituting (9) into (10):

$$N = \frac{\mu L (1 - \theta)}{F(\theta)}. \tag{12}$$

Equation (12) illustrates that the effect of market size on variety depends not
only on the direct effect of $L$ but also on $F$ and $\theta$. In contrast to Krugman (1980),
the number of firms need not increase linearly with market size. It is unclear whether
product differentiation has a positive or negative effect on entry without making
a further assumption about the functional form of $F(\theta)$. However, one can make
a more precise statement about how variety changes with market size by totally
differentiating (12) with respect to $L$:

$$
\frac{dN}{dL} = \mu \frac{1 - \theta}{F(\theta)} \left( 1 - \frac{\theta}{1 - \theta} \frac{d\theta}{dL} \frac{L}{\theta} + \frac{\theta F'(\theta)}{F(\theta)} \frac{d\theta}{dL} \frac{L}{\theta} \right)
$$

(13)

One can see in (13) that variety is affected by market size via three distinct channels.
The first channel is the linear relationship between market size and variety that occurs
when product differentiation is exogenous. The second channel is the effect of market
size on variety via product differentiation, which is positive since Proposition 1 showed
that $d\theta/dL$ is negative. The third channel is the effect of market size on variety due
to fixed cost spending, which depends on the elasticity of the advertising function and
the market size effect on product differentiation. The third channel is negative since
$F'(\theta)$ is negative by Assumption 1. While the first channel is the linear relationship
found in Krugman (1980), other two channels capture the two countervailing effects
of product differentiation on variety. On the one hand, product differentiation on
its own has positive effects on variety, but the fixed costs associated with increasing
product differentiation reduces variety by leading to fewer larger firms. These effects
can be summarized in the following proposition:

**Proposition 2** Entry increases or decreases with market size under the following
condition:
\[
\frac{dN}{dL} \leq 0 \iff 1 - \frac{d}{dL} \frac{L}{\theta} \left( \frac{\theta}{1 - \theta + \frac{\theta F'(\theta)}{F(\theta)}} \right) \leq 0
\]

Proof. See appendix.

The important message in Proposition 2 is that endogenous product differentiation can lead to a concave relationship between market size and entry. This contrasts with the oligopoly framework by Bresnahan and Reiss (1991) that infers increased competition and lower markups when entry is increasing and concave in market size. For example, Campbell and Hopenhayn (2005) found that retailing firms are larger in larger cities, and concluded that firms were larger due to falling markups in larger markets, although the authors had no direct evidence on markups. My framework suggests, however, that market size and firm numbers or firm size alone do not provide sufficient information to make inferences about competition. The endogenous sunk cost literature argues a similar point. In contrast to Sutton (1991), however, my framework predicts an anti-competitive market size effect without requiring a high concentration of firms.

2.5 An Analytical Solution

One can fully solve the basic model using a specific functional form for \( F(\theta_i) \). I thus impose:

ASSUMPTION 2:

\[
F(\theta_i) = \alpha \frac{1 - \theta_i}{\theta_i}, \quad \theta_i \in (0, 1), \quad \alpha < \beta L
\]

This functional form has all of the properties in Assumption 1. The constraint on \( \alpha \) ensures the existence of a solution. Using Assumption 2 we can find an analytical
solution to the system of equations (3) - (10):

\[ p = \frac{w \beta}{2} \ln \left( \frac{\beta L}{\alpha} \right) \]

\[ x = \frac{\alpha}{\beta} \]

\[ N = \mu L \frac{2}{\ln \left( \frac{\beta L}{\alpha} \right)} \]

\[ \theta = \frac{2}{\ln \left( \frac{\beta L}{\alpha} \right)} \]

\[ F = \alpha \frac{\ln \left( \frac{\beta L}{\alpha} \right) - 2}{2} \]

This particular formulation of the advertising function is tractable because it eliminates the problem of having a logged \( \theta \) term in (8). One can see in (9) that this particular function has the unique property that \( x = \alpha \beta^{-1} \), a constant.

The analytically solvable model is also characterized by product differentiation and prices increasing in \( \beta \), the marginal cost parameter. Thus more expensive materials lead to higher prices and more product differentiation.

2.6 Welfare Effect of Market Size

I use the direct utility function for manufactures, \( M \), to examine the welfare effects of market size. In the symmetric equilibrium with \( \theta_m = \theta \), and substituting (12) and \( c = x/L \) into \( M \) provides an expression of utility in terms of market size and product differentiation:

\[ M (L, \theta) = L^{\frac{1}{\beta} - 1} \left( \frac{1 - \theta}{\mu F(\theta)} \right)^{\frac{1}{\beta}} x(\theta) \]  

(14)

The direct effect of \( L \) on utility from manufactures is positive. The effect of product differentiation on utility, however, is unclear because reductions in \( \theta \) have two counter-
vailing effects: utility becomes more concave in consumption, but prices increase as
markups widen. This contrasts with Krugman (1980) that assumes exogenous tech-
nology, where prices are constant and welfare effects occur exclusively via increased
variety. One can show, however, that utility is increasing as products become more
derifferentiated and this leads to utility increasing in market size. These results are
summarized in the following proposition:

Proposition 3 Suppose that Assumption 2 holds. Then utility per-capita is increasing in market size, i.e.:

\[
\frac{dM}{dL} = \frac{\partial M}{\partial L} + \frac{\partial M}{\partial \theta} \frac{d\theta}{dL} > 0.
\]

Proof. See appendix.

Utility per-capita is thus increasing as products become more differentiated, so
endogenous product differentiation modeled in this way has a net positive effect on
welfare when the market expands, despite the adverse effect of higher markups.

3 Competitive Equilibrium vs. Social Optimum

An important task is to compare the level of product differentiation and variety pro-
vided in the competitive equilibrium with the social optimum. The social optimum is
constrained only by the full employment condition and the technologies for producing
the differentiated good and for differentiating these goods as defined by Assumption
1. The social planner maximizes the representative consumer’s utility by choosing
the optimal variety, output per firm and level of product differentiation in the differ-
entiated goods industry, subject to the full employment of labor constraint and the
advertising function:

\[ U \left[ w - \frac{N}{L} (F(\theta) - \beta x), N u(N, \theta) \frac{x}{L} \right] \]

where \( u(N, \theta) \) represents the "taste for variety", which captures consumers' love of variety:

\[ u(N, \theta) = \frac{M_N(x, \ldots, x)}{M_1(Nx)} = \frac{M_N(1, \ldots, 1)}{N} \]

The concept of separating the love of variety effect from the markup condition originates in Benassy (1996). I extend Benassy's formulation so that the love of variety depends not only on the number of varieties but also on how differentiated they are from each other. Differentiating with respect to \( x, N, \) and \( \theta \) yields, respectively:

\[ U_1[\beta] = U_2[u(N, \theta)] \quad (15) \]

\[ U_1[F(\theta) + \beta x] = U_2 \left[ x u(N, \theta) + x N \frac{\partial u}{\partial N} \right] \quad (16) \]

\[ U_1[F'(\theta)] = U_2 \left[ \frac{\partial u}{\partial \theta} x \right] \quad (17) \]

where \( U_1 \) and \( U_2 \) are the partial derivatives of \( U \) with respect to its first and second arguments. The solutions to equations (15), (16) and (17) give \( x^{opt}, N^{opt}, \) and \( \theta^{opt} \). Dividing (15) by (16) we obtain:

\[ \frac{\beta x}{F(\theta) + \beta x} = 1 + \frac{1}{\frac{\partial u}{\partial N} \frac{N}{v(N, \theta)}} \quad (18) \]

This result, originating in Benassy (1996), illustrates how the socially optimal production decision depends crucially on consumer's love of variety. Dividing (15) and (16) by (17) respectively and combining these with (18) provides a new expression
that describes the social planner’s optimal trade-off between product differentiation and variety:

$$\frac{\partial \nu}{\partial N} = NF'(\theta)$$  \hspace{1cm} (19)

The social planner sets the optimum level of product differentiation where the marginal rate of substitution between product differentiation and variety equals the percentage change in the cost to differentiate all available goods.

Comparing the social planner and competitive equilibrium is eased by assuming a particular utility function. In the case of CES utility the love of variety expression is $v(N, \theta) = N^{1-\theta}$. Plugging this into (19) pins down the socially optimal level of product differentiation:

$$\frac{F'(\theta)}{1-\theta} \left[ \frac{1}{\theta} \ln \frac{F(\theta)}{\mu L (1 - \theta)} \right] = F'(\theta).$$  \hspace{1cm} (20)

Comparing equations (11) and (20), it is difficult to say without making further assumptions whether the competitive equilibrium differentiates products more or less than the social optimal. However, one can show that the social planner differentiates more than the competitive equilibrium if we assume a particular form of the advertising function. This result is described in the following proposition:

**Proposition 4** The competitive equilibrium results in less than the socially optimal product differentiation when consumers have CES utility and the advertising function is given by Assumption 2 if $\beta L > \alpha$.

**Proof.** See appendix.

This result, although dependent upon a particular advertising function, illustrates a key property of the model. The social planner does not charge a markup, which allows it to differentiate products more than is possible in the competitive equilibrium.
4 Firm Heterogeneity

The basic model can easily be extended to capture product differentiation heterogeneity across firms. The profit of firm $i$ is:

$$\pi_i = (p_i (\theta_i) - w \beta) x_i (p_i, \theta_i) - w F_i (\theta_i, \alpha_i)$$

where $\alpha_i$ is a parameter that determines the firm’s efficiency in product differentiation. A lower $\alpha_i$ makes the firm more efficient in differentiating its product. Following Melitz (2003), firms draw their efficiency parameter from a random distribution. Since operating profits are concave in $\theta_i$ this means that low-efficiency firms will make negative profits and leave the market. The "cutoff" firm $D$ with an efficiency draw $\alpha_D$ such that is indifferent between leaving the market and remaining earns zero profits:

$$(p_D (\theta_D) - w \beta) x_D (p_D, \theta_D) = w F_D (\theta_D, \alpha_D)$$

The first order condition for the cutoff firm is given by:

$$\frac{F_D (\theta_D)}{1 - \theta_D} \left[ \ln \left( \frac{F(\theta_D)}{\beta L} \right) + \frac{1}{\theta_D} \right] = F_D' (\theta_D)$$

As given by Proposition 1 and abstracting from general equilibrium effects, a larger market will be characterised by a cutoff firm with a greater degree of differentiation. This model thus predicts that higher-priced goods are sold in larger markets.
5 Two Country Model

5.1 Setting, Preferences and Technology

Extending the basic model to a two country model with trade costs yields new results regarding firms’ technology choice and the pattern of trade. Preferences and the firms’ problem are identical to the basic model. I assume that iceberg trade costs between the two countries, whereby $\tau$ units must be shipped in order for one unit to arrive at its destination. I assume that the markets are of equal size and that firms are identical. Moreover, I normalize the price of the agricultural good to unity, which equalizes the wage in both country. These simplifications allow us to more easily see the effect of trade costs on equilibrium product differentiation.

5.2 The Trade Friction Effect

The effect of trade costs on the equilibrium level of product differentiation is a unique property of the model. The first order condition for product differentiation under the special case where country sizes are identical (i.e. $L = L^*$) is:

$$
\frac{F(\theta)}{1-\theta} \left[ \ln \left( \frac{F(\theta) \theta}{\beta \left( 1 - \theta \right)} \right) + \frac{1}{\theta} \right] - \frac{F(\theta)}{(1-\theta)^2} \ln \left( \frac{\tau^{1/\sigma}}{1 + \tau^{1/\sigma}} \right) = F'(\theta). \quad (21)
$$

This equation effectively divides the first order condition into two parts, a "market size effect" and a "trade friction effect". The "market size effect" is almost identical to the left hand side of the first order condition in the basic model, (8), except for the additional term $1 + \tau^{1/\sigma}$ multiplying $L$ in the denominator. This term equals 1 under infinite trade costs and 2 under free trade, since free trade between two countries of equal size effectively doubles the market.
The "trade friction effect" is an additional term that is not present in the first order condition under autarky. If trade costs per unit are zero or infinite then no trade occurs and the trade friction effect will disappear. This term is positive for intermediate trade costs, reaching a single maximum. The marginal revenue of decreasing $\theta$ is thus maximized at some intermediate level of trade costs. Trade frictions thus affect more than just market potential in the model; the friction itself enhances the marginal revenue of product differentiation. The intuition is that lowering $\theta$ abates the loss of demand due to "melting", and the marginal benefit from this activity is greatest when "melting" is greatest (i.e. intermediate trade costs).

It can be helpful to analyze this result within a trade liberalization context. If trade costs are gradually lowered from autarky to free trade, $\theta$ first decreases, then increases as trade costs approach zero. Similarly, $F$ and $p$ first increase to a maximum at some intermediate level of trade costs, then decrease as trade costs approach zero. This contrasts with the monotonic market size effects that one observes in the basic closed economy model. The result that product differentiation effects are strongest at intermediate trade costs is akin to new economic geography literature, where agglomeration forces are strongest at intermediate trade costs.

When trade costs are low the model predicts that prices increase with distance, which agrees with the export price literature, such as Baldwin and Harrigan (2007) and Ottaviano and Mayer (2008). The result that the elasticity of substitution is decreasing can also reconcile the Broda and Weinstein (2006) result that the elasticity of substitution has been trending downwards at the same time as the process of "globalization" has expanded markets and reduced trade barriers.
6 Conclusion

The model presented in the paper takes a new look at product differentiation in a model of monopolistic competition. Moreover, the model has several attractive features that agree with recent empirical findings on the pattern of prices within and across countries. Prices are higher in larger countries and regions and prices increase with distance.

The model allows for firms to endogenously choose from a continuous set of technologies by creating a trade-off between fixed costs and product differentiation. This assumption is consistent with fixed costs that represent persuasive advertising or product development that differentiate one’s own product from others in the eyes of consumers. To the best of my knowledge, the idea of endogenizing the elasticity of substitution in order to endogenize product differentiation is novel. Fixed costs, markups, and output per firm are increasing functions of market size, a characteristic that agrees with the literature. The model thus generates "endogenous markups" that are a direct result of firms’ optimizing behavior.

The mechanism of endogenous product differentiation described in this paper may be part of the reason why we do not always see pro-competitive effects in differentiated goods markets. This model can be applied to many issues, including growth, trade, and economic geography. The prediction that markups increase with market size may be considered somewhat controversial, since the "conventional wisdom" is that markups will decrease as market size increases, firms enter, and the competition becomes tougher. However, the recent empirical evidence on prices suggests that pro-competitive effects of market size need not hold, especially in differentiated products. As Tirole (1988, p.289) puts it, "Though it will be argued that advertising may foster competition by increasing the elasticity of demand (reducing "differentiation"), it is
easy to find cases in which the reverse is true." It is hoped that this paper has given some theoretical foundation to this argument.

Appendix

A.1 Proof of Lemma 1:

Substitute (3) and (10) into (8) and rearrange to obtain (11). The right hand side of (11) is negative and monotonically increasing in $\theta$ by Assumption 1. If $L$ is sufficiently large then the left hand side of (11) is negative. ■

A.2 Proof of Proposition 1:

Totally differentiating and rearranging (11) we obtain an expression for the effect of market size on product differentiation:

$$
\frac{d\theta}{dL} = \frac{1}{L} \ln \left( \frac{F(\theta)}{F(\theta)} \right) + \frac{1}{\theta} - F''(\theta)
$$

The denominator is negative by Assumption 1 and since operating profits are concave in $\theta$. Thus $d\theta/dL < 0$. ■

A.3 Proof of Proposition 2:

Rearranging (13), endogenous product differentiation has a positive effect on entry if $\theta F'(\theta)/F(\theta) < -\theta/(1 - \theta)$. ■

A.4 Proof of Proposition 3:

It is clear from (14) that $\partial M/\partial L > 0$, $\partial \theta/\partial L < 0$ due to Proposition 1. The partial derivative of indirect utility with respect to $\theta$ is:

$$
\frac{\partial M}{\partial \theta} = \left( \frac{\mu L (1 - \theta)}{F(\theta)} \right)^{\frac{1}{2}} x(\theta) \left[ \frac{x'(\theta)}{x(\theta)} - \frac{1}{\theta^2} \ln \left( \frac{\mu L (1 - \theta)}{F(\theta)} \right) \right].
$$

In order for indirect utility to be decreasing in $\theta$ the following condition must hold:

$$
\frac{\partial M}{\partial \theta} < 0 \iff \frac{x'(\theta)}{x(\theta)} < \frac{1}{\theta^2} \ln \left( \frac{\mu L (1 - \theta)}{F(\theta)} \right).
$$
Using Assumption 2 one can show that this inequality holds:

\[ \frac{\partial M}{\partial \theta} < 0 \iff 0 < \frac{1}{\theta^2} \ln \left( \frac{\mu L \theta}{\alpha} \right) . \]

Utility per-capita is thus increasing in market size when using the advertising function given by Assumption 2.

**A.5 Proof of Proposition 4:**

Using (11) and (20), the social planner differentiates products more than the competitive equilibrium under the following condition:

\[ \frac{\ln \frac{F(\theta)}{\mu L (1-\theta)}}{\theta \ln \left( \frac{F(\theta)}{\frac{\theta}{L}} \right) + 1} < 1 \]

Substituting for \( F(\theta) \) using the advertising function in Assumption 2, using the analytical solution for \( \theta \), and using L'Hôpital’s rule reduces this condition to one that can be easily interpreted:

\[ \frac{-2}{\ln \frac{\beta L}{\alpha}} < 1 \]

The social planner thus differentiates products more than the competitive equilibrium as long as \( \beta L > \alpha \).

**References**


