Does Hidden Information Make Trade Liberalization More Fragile?*

Mathias Herzing[†]

Abstract

This paper focuses on the impact of hidden information on strategic interaction in the context of trade agreements. In the presence of informational asymmetry it is possible that a tradeoff between liberalization and sustainability of cooperation emerges. It is shown that it may be optimal to agree on a degree of liberalization associated with a strictly positive ex ante probability of deviation occurring. In that case, cooperation will break down in finite time, and the optimal degree of liberalization cannot be applied indefinitely. (JEL: F13, C72, C73, D82)

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[†]Department of Economics, Stockholm University, S-10691 Stockholm, Sweden; phone: +46-8-163043; fax: +46-8-159482; e-mail: Mathias.Herzing@ne.su.se

1 Introduction

A commonly raised objection against too far-reaching trade liberalization is that it may increase the incentive for non-cooperative (protectionist) behavior. Hence, liberalization should be restrained, in order to meet the objective of making liberalization sustainable over time. The implicit assumption behind such an argument is that there exists a tradeoff between liberalization and the sustainability of cooperation and that, while a higher degree of liberalization yields a higher expected short-term return, the loss stemming from an increased risk of a breakdown in cooperation is sufficiently large to outweigh the expected short-term gain. Hence, liberalization should be limited by the requirement of sustaining cooperation and avoiding protectionist measures in all contingencies.

The main contribution of this paper is to provide a model that incorporates uncertainty and asymmetric information within a trade policy framework and to explore how this source of uncertainty affects the scope for trade liberalization and its sustainability. The starting point is the notion that *ex ante*, when a trade agreement is negotiated, it is typically impossible to know with certainty how large the incentive to deviate will be, once the agreement is in place. There may be various sources of uncertainty, but this paper will examine a situation where two countries are exposed to a random variable, the realization of which is only privately observable. Due to this informational asymmetry, a government must infer the likelihood of its trading partner choosing cooperation from the commonly known distribution of the random variable, and the degree of liberalization agreed upon.

For the sake of analytical tractability, a model is employed where the weight attributed to present vis-à-vis future payoffs is stochastically determined and non-observable to the trading partner. Thus a situation, in which there exists uncertainty about the shortsightedness of politicians is modelled. The weight attributed to present vis-à-vis future payoffs can be regarded as

capturing the general uncertainty about the future under which decisions are taken.

It is shown that in the presence of this hidden-information-type of uncertainty, the scope for liberalization will decrease. Moreover, it is demonstrated how countries may face a tradeoff, when higher degrees of liberalization are associated with decreasing probabilities of cooperation being maintained. It may nevertheless be optimal to agree on a degree of liberalization such that there is a strictly positive likelihood of cooperation breaking down in finite time, since the short-term gain from increasing liberalization may outweigh the long-term loss of cooperation eventually breaking down.

The following section reviews the literature on strategic interaction under various types of uncertainty. Section 3 introduces the model. The scope for liberalization under uncertainty is examined in section 4, and optimality under uncertainty is addressed in section 5. Section 6 concludes.

2 Background

The literature on the implementability and sustainability of trade agreements typically examines the strategic interrelationship between two trading countries that can influence world prices through their import tariffs. The countries are in a Prisoner's Dilemma situation, where both would benefit from mutually reducing tariffs but where, from a short-term point of view, each country prefers to apply its best-response tariff vis-à-vis its trading partner. With repeated interaction between the trading partners, it is possible to sustain lower tariffs, however. In standard fashion, by threatening to punish current-period deviations in future periods, the incentive to deviate can be balanced and cooperation be sustained.

The establishment of cooperation hinges on two factors. First, the discount factor must be sufficiently large for the future loss from being punished to outweigh the current gain from

deviating. The lower the cooperative tariffs are set, the larger need the discount factor be. Second, there must be a sufficiently high degree of trust between the two parties for cooperation to be established. A country will opt for cooperation only if it attributes a sufficiently high probability to cooperative behavior by its trading partner; believing that the trading partner will deviate makes deviation the preferred choice. The lower the cooperative tariffs are set, the larger must the degree of trust in the trading partner be.

Introducing some sort of uncertainty into the conditions, under which decisions of complying with or breaching commitments made under a trade agreement are taken not only makes the analysis more complicated, but may also lead to different implications for the prospects of sustaining a cooperative arrangement. Several attempts have been made to incorporate uncertainty into the Prisoner's Dilemma setting of trade agreements. Obviously it is easy to find close correspondences to the literature on collusion under uncertainty. There are, broadly, three categories of uncertainty that have been addressed in the industrial organization literature and, to a lesser extent, also in the literature on trade agreements.

First, strategically interacting parties may be subject to *ex ante* uncertainty regarding a commonly observed shock which has an impact on the incentive to deviate from a cooperative arrangement. In Bagwell and Staiger (1990), the case of negotiating an agreement in the presence of commonly observable trade volume fluctuations is analyzed. Periods of high trade volumes are associated with stronger incentives to deviate so as to make terms-of-trade gains and therefore, a cooperative agreement will have to allow for the cooperative tariff to adjust in order to dampen trade volume fluctuations and hence the incentive to deviate. This type of trade management can thus be seen as an attempt by countries to maintain the self-enforcing nature of existing international cooperation.¹

¹The setting and the results in Bagwell and Staiger (1990) are similar to Rotemberg and Saloner (1986), who

Second, uncertainty may also emerge because the strategic partner's actions are unobservable. Assuming that commonly observed outcomes are not only influenced by the actions taken, but also by some stochastic variable, it is even ex post not possible to verify or correctly infer the action taken by the opponent in the previous period. A worse-than-expected outcome can thus be the consequence of either deviation on behalf of the opponent, or a bad realization of the stochastic variable. Riezman (1991) models a trade agreement between two countries in the presence of a random component attached to home imports, reflecting shocks to preferences or endowments, and assumes that protection is not perfectly observable.² When import trigger strategies are applied, reversionary (high tariff) episodes are triggered by the random variable. These Nash reversions are not the result of deviation against the low-tariff agreement, but necessary to provide the incentives for sustaining cooperative low tariff episodes.³

Finally, uncertainty can also arise when each party is exposed to a random variable, the realization of which cannot be observed by the opponent. Fudenberg et al. (1994) demonstrate that Nash superior equilibria exist in repeated games under this adverse-selection-type of uncertainty if sufficient weight is attributed to the future. Athey and Bagwell (2001) and Athey et al. (2004) elaborate on the impact of hidden information in infinitely repeated Bertrand games between competing firms when prices are publicly observed, but cost shocks are only privately observed.

With regard to uncertainty regarding only privately observable shocks in the context of trade cooperation, the case of one-stage games with implicitly assumed cooperative behavior has been examined by Feenstra (1987) and Feenstra and Lewis (1991). Given the underlying assumption

model cooperative arrangements between firms that are subject to ex ante unknown, but commonly observable fluctuations in demand.

²The same type of uncertainty also appears in Hungerford (1991) and Kovenock and Thursby (1992), which both focus on the dispute settlement procedures of the GATT.

³This result corresponds to the one obtained in Green and Porter (1984) where low realizations of the price trigger reversionary episodes when two firms strategically interact in a Cournot duopoly that is affected by unobservable demand shocks influencing the price level. See also Abreu et al. (1986), who modify and generalize the Green-Porter model, and Fudenberg et al. (1994), who provide folk theorems for moral-hazard-type of games.

that countries prefer to cooperate and apply agreed-upon policies, the problem addressed is how hidden information creates incentives to misrepresent in order to make gains. While these two contributions focus on the effect of hidden information on outcomes in cooperative games, Jensen and Thursby (1990) investigate the effect of private information on noncooperative equilibria and how the incentives for governments to establish tariff reputations might be influenced. Bagwell and Staiger (2005) examine the inclusion of safeguards and Martin and Vergote (2008) explore the role of retaliation in trade agreements in the presence of private information.

Hidden-information-type of uncertainty in the context of trade cooperation can be described as a situation where a government is exposed to some random variable that only it, and no one else, can observe, such that the true incentives faced by the government with regard to choosing to comply with or deviate from liberalization commitments are unknown to the trading partner(s). There exist various ways of introducing this type of uncertainty in the government's objective function. Baldwin (1987) introduces a politically realistic objective function (PROF) and shows it to be equivalent to the payoff functions derived from a wide range of political economy models. The PROF attributes different weights to consumer surplus, tariff revenues and different industry profits. A commonly used way of introducing uncertainty is to let one of these weights, typically profits of an import-competing sector, be randomly determined.

A simpler and analytically more tractable way of investigating the impact of uncertainty on the conditions for strategic interaction and cooperation in an infinitely repeated setting is to let the government weigh current-period and future-period payoffs and let one of these weights (and hence, the relative weight) be randomly determined in every period. In fact, attributing more weight to profits in the import-competing sector is similar to giving more weight to present vis-à-vis future payoffs. In both cases the incentives for protection increase.

Infinitely repeated games with stochastically determined discount factors have previously

been examined by Baye and Jansen (1996).⁴ In their model both parties are exposed to the same discount factor in every period, unlike in the present model where the two governments' realizations of the weight attributed to present payoffs are uncorrelated and only privately observable. Dal Bo (2007) examines the case when the interest rate is stochastically determined in every period in a repeated Bertrand oligopoly setting. In contrast to the present model it is however assumed that the random variable applies to all players and therefore is common knowledge. Baye and Kovenock (2004) model a Bertrand duopoly supergame where each firm's per-period discount factor is stochastically determined and only privately observable at the time prices are set.

The present paper models cooperation under adverse-selection-type of uncertainty in an infinitely repeated setting. While previous work on this type of uncertainty in the industrial organization literature has primarily analyzed Bertrand oligopolies, the present model also has implications for repeated Cournot competition games under informational asymmetry, because tariff setting is strategically similar to Cournot-type of interaction. With regard to the literature on trade agreements under uncertainty, which hitherto has mostly focused on commonly observable shocks and moral hazard, the present paper contributes by analyzing the implications of hidden information for trade cooperation and providing a framework for examining how trade agreements could be designed to make liberalization more sustainable.

3 The Model

Let there be two symmetric countries, home and foreign (distinguished by an asterix), each with one sector. In standard fashion the two countries interact in an infinitely repeated tariff setting game, and each government is assumed to be subject to a random variable that is only privately

⁴Bernheim and Dasgupta (1995) analyze the impact on cooperation in infinitely repeated games with asymptotically falling discount factors.

observable and i.i.d. across countries and periods. Due to symmetry, it suffices to consider the home country. The random variable γ is assumed to enter, such that the home government's payoff function W^γ in the current period is given by:

$$W^\gamma = \gamma w + (1 - \gamma)v,$$

where w is the payoff of the current period, v the expected discounted future flow of payoffs, and $\gamma \in [\gamma_{\min}, \gamma_{\max}] \subseteq [0, 1]$ the weight attributed by the government to present payoffs, the residual $1 - \gamma$ being the weight given to future payoffs. Let $\bar{\gamma} \equiv E(\gamma) \in [\gamma_{\min}, \gamma_{\max}]$ be the expected value of γ . Ex ante, there is uncertainty concerning what weight a government will attribute to present payoffs in every period. The density function $\varphi(\gamma)$, which is assumed to be continuous for $\gamma \in [\gamma_{\min}, \gamma_{\max}]$, and the associated cumulative distribution function $\Phi(\gamma)$ are common knowledge, however. Hence, $\bar{\gamma}$ can be seen as a measure for the shortsightedness of a government (a high value of $\bar{\gamma}$ implies a low discount factor), while $\gamma_{\max}$ represents the most myopic realization of γ possible.

For analytical tractability, a partial equilibrium setting where the current-period payoff w is additively separable in the home tariff t and the foreign tariff t^* , is assumed

$$w(t, t^*) = u(t) + \hat{u}(t^*).$$

There exists a best-response function $t_D(t^*) \equiv \arg \max_t w(t, t^*)$. From the additive separability of w , it immediately follows that the within-period best-reply tariff t_D is independent of t^* and hence constant, i.e. $t_D(t^*) = \arg \max_t u(t)$.

In the absence of cooperation, both countries apply the optimal tariff vis-à-vis each other, i.e. $t = t^* = t_D$, and both receive the Nash equilibrium current-period payoff $w_N = w(t_D, t_D)$. Since

the current-period payoff in every future period will be equal to w_N , the government's payoff in the absence of cooperation will be given by $W_N^\gamma = w_N$, i.e. it will be independent of γ .⁵

However, both countries can be made better off by agreeing to mutually lower their tariffs. What prevents the implementation of a cooperative tariff $t_C < t_D$, associated with a current-period payoff $w_C = w(t_C, t_C) > w_N$, is that the two countries are stuck in a Prisoner's Dilemma. If one country decides to break its commitment by applying the optimal tariff vis-à-vis its trading partner, it gets the current-period payoff $w_D = w(t_D, t_C) > w_C$, while its trading partner receives the payoff $w_S = w(t_C, t_D) < w_N$. Thus, both countries will apply t_D , and their current-period payoff will be given by w_N in the one-shot game.

The following assumptions regarding the properties of $w(t, t^*)$, reflecting features of typical trade models, will be made. The payoff under mutual cooperation w_C is concave in t_C and has a unique maximum for $t_C = t_C^{opt} < t_D$. It immediately follows that there exists a $t' < t_C^{opt}$ such that $w_C = w_N$ for $t_C = t'$.⁶ Thus, $w_C > w_N$ if and only if $t' < t_C < t_D$. The deviator's payoff w_D decreases unambiguously in t_C , i.e. the incentive to deviate increases as t_C decreases.⁷ The decrease in w_D is equal to the decrease in w_C (i.e. w_D and w_C are tangent) at $t_C = t_D$ and unambiguously larger for $t_C < t_D$. Thus, $w_D - w_C$ increases, and it does so at an increasing rate as t_C decreases (see also figures in the Appendix).

For analytical purposes define

$$\tau \equiv \frac{w_D - w_C}{w_D - w_N}.$$

It is easily shown that $\lim_{t_C \rightarrow t_D} \tau = 0$ and that τ increases monotonously as t_C decreases below t_D . Hence, τ can be seen as a measure of trade liberalization. A low value of t_C corresponds

⁵For the derivation of $W_N^\gamma = w_N$, see the Appendix.

⁶Depending on the how w is specified, t' may be negative (i.e. an import subsidy).

⁷Here, a deviation is defined as breaching the agreement on liberalization. However, from a game theoretic point of view choosing to apply the best-response tariff need not constitute a deviation from the equilibrium path.

to a high value of τ and thus, a high degree of trade liberalization. Since $w_C^{opt} > w_N$, the optimal degree of liberalization τ_{opt} , corresponding to $t_C = t_C^{opt}$, is strictly smaller than unity and, because $w_C = w_N$ for $t_C = t'$, the degree of liberalization τ' , corresponding to $t_C = t'$, equals unity. The relevant range of cooperative tariffs to consider is given by (t', t_D) , corresponding to degrees of liberalization in the range $(0, 1)$. In this range, it is the case that $w_D > w_C > w_N > w_S$, and strategic interaction is thus of Prisoner's Dilemma type.

As mentioned above, mutual deviation is the only equilibrium outcome in the one-shot Prisoner's Dilemma game. Under an infinite horizon, it is possible to create cooperation, however.⁸ By threatening to punish deviations and thus associate the one-period gain from deviating with a future loss, it is possible to induce cooperative behavior. There exist many different ways of conceiving punishment phases. For simplicity grim-trigger strategies will be considered throughout the paper. Hence it will be assumed that deviations from the cooperative tariff will trigger infinite reversion to the Nash equilibrium.

In the absence of uncertainty, $\gamma = \bar{\gamma}$ and, assuming a propensity for cooperative behavior across countries⁹, choosing cooperation yields $W_C = \bar{\gamma}w_C + (1 - \bar{\gamma})w_C = w_C$, while opting for deviation yields $W_D = \bar{\gamma}w_D + (1 - \bar{\gamma})w_N$. Cooperation is thus sustainable, if and only if

$$w_D - w_C \leq \frac{1 - \bar{\gamma}}{\bar{\gamma}}(w_C - w_N). \quad (1)$$

The left-hand side represents the short-term (current-period) gain from deviation, while the right-hand side represents the long-term loss from deviation. Since $w_C > w_N$ under an agreement, rearranging terms yields the following relationship between the degree of liberalization and the

⁸In fact, cooperative behavior can be established in all periods but the last in a finitely repeated game, if there exist multiple Nash equilibria. As pointed out by Dixit (1987), this is the case when a tariff setting game includes the possibility of reversion to autarky.

⁹It is easily established that both countries choosing to deviate constitutes an equilibrium outcome, independent of the degree of liberalization and the weight attributed to current-period payoffs.

discount factor, which is given by $1 - \bar{\gamma}$, i.e. the weight attributed to the future flow of payoffs (see the Appendix).

$$(1) \Leftrightarrow \frac{w_D - w_C}{w_C - w_N} \leq \frac{1 - \bar{\gamma}}{\bar{\gamma}} \Leftrightarrow \frac{\tau}{1 - \tau} \leq \frac{1 - \bar{\gamma}}{\bar{\gamma}} \Leftrightarrow \tau \leq 1 - \bar{\gamma} \equiv \tau_{\max} \quad (1')$$

Equation (1') tells us that in order to sustain cooperation, t_C can only be reduced to the degree where τ does not exceed the weight attributed to the future flow of payoffs $1 - \bar{\gamma}$. A lower $\bar{\gamma}$, i.e. a smaller weight given to present payoffs, implies that the upper bound for liberalization $\tau_{\max}$ increases, and it is thus possible to sustain a lower t_C . The restriction given by (1') thus imposes an upper limit on the scope for liberalization.

Two well-known results immediately follow from condition (1'). First, it is always possible to find some $\tau > 0$ ($t_C < t_N$) that is sustainable for $\bar{\gamma} < 1$ (i.e. a strictly positive discount factor). Second, the optimal degree of liberalization τ_{opt} can be sustained, if governments are sufficiently patient, i.e. if the weight attributed to current payoffs is not too large ($\bar{\gamma} \leq 1 - \tau_{opt}$).

4 The Scope for Trade Liberalization under Uncertainty

In what follows, the weights attributed to present payoffs will be stochastically and independently determined in both countries in every period. If γ and γ^* were stochastically determined, but perfectly observable by both governments, then countries could agree to adjust the degree of trade liberalization in every period such that no country would have an incentive to deviate.¹⁰ Here, it will, however, be assumed that the realizations of γ and γ^* are only privately observable. The obtained equilibrium outcomes will crucially depend on this informational asymmetry.

Introducing uncertainty about the weight the trading partner attributes to current vis-à-

¹⁰This is similar to the case of managed trade, explored by Bagwell and Staiger (1990).

vis future payoffs significantly complicates the analysis. The incentive to deviate will not only depend on the ex ante unknown realization of γ and the degree of liberalization, but also on the likelihood p of the trading partner choosing cooperation. In equilibrium, a government's ex ante probability of opting for cooperation must equal its belief regarding the other government's likelihood of choosing cooperation. Hence, beliefs must be consistent such that Baysean Nash equilibria can be attained.

This also applies in the absence of uncertainty. However, consistent solutions are much easier to derive under certainty. Any prior regarding the likelihood of cooperative behavior of the trading partner either results in deviation or, possibly (i.e. for sufficiently low degrees of liberalization), cooperation being a consistent solution.¹¹ Hence, cooperation over the infinite horizon is only a matter of establishing trust, such that both countries coordinate on the cooperative solution.

In the face of uncertainty, it is also the case that a propensity for maximally cooperative behavior is required for the most cooperative outcome to be attained, but it may not be sufficient to sustain cooperation in all contingencies. Knowing that the trading partner may deviate under certain circumstances feeds back into the decision for when opting for cooperation is optimal. Anticipating that the foreign country's government must similarly infer its optimal strategy from a belief concerning the likelihood of cooperation by the home country's government, a process of updating of initial priors will yield consistent solutions and thus, Baysean Nash equilibria are obtained. The derivation of consistent solutions turns out to be analytically non-trivial.

As above in the case of certainty, it will be assumed that grim-trigger strategies are applied. Let v_C and v_D be the continuation values of the game if both countries cooperate and if at least

¹¹Actually, whenever cooperation is a consistent outcome, there also exists a mixed strategy (i.e. randomizing between cooperation and deviation) that is consistent.

one country deviates, respectively. For any strictly positive degree of liberalization, the gain from deviating, denoted by Γ , is thus given by

$$\Gamma = p[\gamma(w_D - w_C) + (1 - \gamma)(v_D - v_C)] + (1 - p)\gamma(w_N - w_S).$$

Since w is additively separable, the one-period gain of deviating is independent of what action the trading partner takes, and in particular $w_N - w_S = w_D - w_C$. Therefore, any decision will solely depend on the domestic shock and the likelihood attributed to cooperation being chosen by the trading partner. Hence, the gain from opting for deviation is expressed as follows

$$\Gamma = \gamma(w_D - w_C) + p(1 - \gamma)(v_D - v_C).$$

The condition for when cooperation is chosen is thus given by

$$\Gamma \leq 0 \Leftrightarrow \gamma \leq \frac{p(v_C - v_D)}{w_D - w_C + p(v_C - v_D)} \equiv \eta.$$

A country will opt for cooperation as long as realizations of γ are smaller than the threshold value η . This threshold value, in turn, implies an ex ante likelihood of this country choosing cooperation of $\Phi(\eta)$. In fact, η can be regarded as a reaction function of p , i.e. the probability attributed to the trading partner choosing cooperation. By introducing the concept of consistency in beliefs regarding the likelihood of cooperation being chosen, solutions for η can be derived by treating p as a prior regarding the likelihood of the trading partner opting for cooperation and updating it.

Symmetry across countries and consistency require that the probability of the trading partner choosing cooperation must equal the implied likelihood of the own country opting for cooperation,

i.e. $p = \text{prob}(\gamma \leq \eta) = \Phi(\eta)$. Consistent solutions are thus given by solutions to the following equation

$$\eta = \frac{(v_C - v_D)\Phi(\eta)}{w_D - w_C + (v_C - v_D)\Phi(\eta)}. \quad (2)$$

The continuation values v_C and v_D are given by (see the Appendix for derivation)

$$v_D = w_N \quad (3)$$

$$v_C(\tau, \eta) = w_N + \frac{\bar{\gamma}\Phi(\eta) - \tau \int_{\gamma_{\min}}^{\eta} \gamma d\Phi}{1 - \Phi(\eta)[\Phi(\eta) - \int_{\gamma_{\min}}^{\eta} \gamma d\Phi]} (w_D - w_N). \quad (4)$$

Hence, v_C is affected by both the degree of liberalization and the associated threshold value, above which deviation will be chosen. Plugging the expressions for v_C and v_D into equation (2) and rearrangement of terms then yield the following equation, henceforth referred to as the consistent solution equation (CSE)¹²

$$\eta = \Phi(\eta) \frac{\bar{\gamma}\Phi(\eta) - \tau \int_{\gamma_{\min}}^{\eta} \gamma d\Phi}{\bar{\gamma}\Phi^2(\eta) + \tau[1 - \Phi^2(\eta)]} \equiv f(\tau, \eta). \quad (5)$$

The right-hand side of the CSE is given by

$$f(\tau, \eta) = \begin{cases} 0 & \eta \leq \gamma_{\min} \\ \Phi(\eta) \frac{\bar{\gamma}\Phi(\eta) - \tau \int_{\gamma_{\min}}^{\eta} \gamma d\Phi}{\bar{\gamma}\Phi^2(\eta) + \tau[1 - \Phi^2(\eta)]} & \eta \in (\gamma_{\min}, \gamma_{\max}) \\ 1 - \tau & \eta \geq \gamma_{\max} \end{cases}.$$

Naturally the properties of $f(\tau, \eta)$ depend on the underlying distribution function and it is

¹²Having introduced τ it is avoided that both w_D and w_C appear in the CSE. Hence, the CSE has only one term reflecting the extent of trade liberalization, which simplifies the analysis.

therefore difficult to make general statements regarding its shape. However, it is easy to see that $f(\tau, \eta)$ decreases monotonously in τ at a decreasing rate, i.e. $\frac{\partial f(\tau, \eta)}{\partial \tau} < 0$ and $\frac{\partial^2 f(\tau, \eta)}{\partial \tau^2} > 0$, for $\eta \in (\gamma_{\min}, \gamma_{\max})$. The partial derivatives with respect to η yield no clear results. Depending on the distribution of γ , $f(\tau, \eta)$ can both increase and decrease in segments of the interval $(\gamma_{\min}, \gamma_{\max})$. As τ increases, the slope of $f(\tau, \eta)$ decreases for sufficiently low η ($\frac{\partial^2 f(\tau, \eta)}{\partial \eta \partial \tau} < 0$).

Define $\Omega(\tau) \equiv \{\eta | \eta = f(\tau, \eta)\}$ as the set of consistent solutions. It is easy to see that $\Omega(\tau) \neq \emptyset$ for any degree of cooperation. Since $f(\tau, 0) = 0$, $\eta_0 \equiv 0$, implying that cooperation is never chosen, is a consistent solution for any degree of liberalization irrespective of the realization of γ . Naturally, this is also true in the case of no uncertainty. As mentioned above, the degree of trust in the opponent is of crucial importance for sustaining a cooperative regime. Having low faith in the opponent results in deviation being the best response, under certainty as well as under uncertainty.¹³ The following lemma follows by introspection of the CSE.

Lemma 1 *For sufficiently low degrees of liberalization ($\tau \leq 1 - \gamma_{\max} \equiv \tilde{\tau}_{\max}$), there exists a consistent solution ($\eta = 1 - \tau \geq \gamma_{\max}$) associated with a likelihood of cooperation equal to unity. The range of degrees of liberalization, for which cooperation is always chosen, is, however, unambiguously smaller under uncertainty than under certainty ($\tilde{\tau}_{\max} < \tau_{\max}$).*

Hence, it is also possible to sustain cooperation over the infinite horizon for sufficiently low degrees of liberalization, just as in the case of uncertainty, if $\gamma_{\max} < 1$. However, the range of degrees of liberalization, for which this is true, is smaller in the case of uncertainty, because $\bar{\gamma} < \gamma_{\max}$.

If the degree of liberalization is pushed further than $\tilde{\tau}_{\max}$, always opting for cooperation can no longer be a consistent solution, because the incentive to deviate will be too strong for large

¹³The degree of trust initially required, i.e. the value of the prior before updating, may naturally differ under certainty and uncertainty.

realizations of γ . There are two possibilities. Either cooperation completely breaks down, i.e. deviation is the preferred option for any γ , as in the case of certainty for $\tau > \tau_{\max}$; or it will be the case that cooperation is only chosen for sufficiently low realizations of γ , i.e. the threshold value η will lie in the interval $(\gamma_{\min}, \gamma_{\max})$, implying a probability of cooperation being chosen strictly smaller than one, but also strictly larger than zero. In the latter case, deviation will thus occur in finite time. Let $\tilde{\tau}'_{\max}$ be the highest degree of liberalization for which the ex ante probability of choosing cooperation is strictly positive.

In what follows, agreements under which cooperation can be sustained forever will be referred to as safe, while agreements under which the ex ante likelihood of cooperation being chosen is strictly positive, but also strictly smaller than one, will be referred to as self-destructive. Whether there exist self-destructive agreements crucially depends on how the CSE is affected by pushing τ above $\tilde{\tau}_{\max}$. Continuity of the density function implies that $f(\tau, \eta)$ is continuous in η for $\tau > 0$. The figure below demonstrates the impact of τ on the solution(s) of the CSE. Using a triangular density function and setting $\bar{\gamma} = 0.5$, $\gamma_{\min} = 0.1$ and $\gamma_{\max} = 0.9$, $f(\tau, \eta)$ is plotted for different values of τ (thick lines). Consistent solutions, i.e. solutions to the CSE, are given by intersections with the upward-sloping thin line, which represents the left-hand side of the CSE. The dashed vertical line denotes $\gamma_{\max}$; any intersections to its right thus imply a solution with an associated probability of cooperation being chosen equal to one.

When the degree of liberalization is zero¹⁴, $f(\tau, \eta) = 0$ for $\eta \in [0, \gamma_{\min}]$ and $f(\tau, \eta) = 1$ for all $\eta \in (\gamma_{\min}, 1]$. Hence, $\eta_0 = 0$ and $\eta = 1$ are consistent solutions. As can be easily inferred from the CSE, letting τ increase above zero will lead to a decrease in $f(\tau, \eta)$ for all $\eta \in (\gamma_{\min}, 1]$. Hence, $f(\tau, \eta) < 1$, but there exist $\eta \in (0, 1)$, for which $f(\tau, \eta) > \eta$. Thus, there will be at least two intersections between η and $f(\tau, \eta)$ in this interval. Let $\eta_1 \equiv \max\{\eta \in [0, 1] | \eta = f(\tau, \eta)\}$ be

¹⁴Note that the plot for $\tau = 0$ is actually the one obtained for $\lim_{\tau \rightarrow 0} f(\tau, \eta)$.

the largest and $\eta_2 \equiv \min\{\eta \in (0, 1] | \eta = f(\tau, \eta)\}$ the smallest strictly positive solutions to the CSE, respectively. An increase in τ will reduce $f(\tau, \eta)$ for all $\eta \in (\gamma_{\min}, 1]$ and thus, lead to a decrease in η_1 and an increase in η_2 . As long as $\tau \leq \tilde{\tau}_{\max}$, $\eta_1 \geq \gamma_{\max}$ and always cooperating is a consistent solution (as previously stated in Lemma 1). As τ is increased beyond $\tilde{\tau}_{\max}$, η_1 falls below $\gamma_{\max}$, while η_2 continues to increase.¹⁵ Eventually, η_1 and η_2 will coincide, i.e. $f(\tau, \eta) < \eta$ for all $\eta > 0$ except $\eta = \eta_1 = \eta_2$ and $f(\tau, \eta)$ will be tangent to η for $\eta = \eta_1$. Naturally the degree of liberalization at which η_1 and η_2 coincide is $\tilde{\tau}'_{\max}$, the highest degree of liberalization, for which the ex ante probability of choosing cooperation is strictly positive. Pushing the degree of liberalization beyond $\tilde{\tau}'_{\max}$ will lead to $f(\tau, \eta) < \eta$ for all $\eta > 0$, i.e. $\eta_0 = 0$ will be the only consistent solution. Hence, η_2 no longer exists and η_1 coincides with η_0 for $\tau > \tilde{\tau}'_{\max}$.

Generally, it will thus be the case that for sufficiently low degrees of liberalization, there exist (at least) three different solutions, η_0 , η_1 and η_2 . Henceforth, it will be assumed that

¹⁵It may also be the case that $\eta_1 = 0$ and η_2 is non-existent for any $\tau > \tilde{\tau}_{\max}$, in which case there exists no self-destructive agreements.

governments wish to behave as cooperatively as possible and thus apply η_1 for any given degree of liberalization. Thus, each government has an interest in fostering a belief about itself acting as cooperatively as possible, such that η_1 can be derived.¹⁶

Naturally, there are many possibilities for the shape of the f -function. But to derive a sufficient condition for when self-destructive agreements are feasible, it suffices to examine the case when there exist at most three solutions to the CSE, as in the figure above. Whether it is possible to increase liberalization beyond $\tilde{\tau}_{\max}$ without cooperation breaking down, will in this case depend on the marginal likelihood of the most myopic realization of γ occurring, as demonstrated by the following proposition.

Proposition 1 *Self-destructive agreements exist (i.e. $\tilde{\tau}'_{\max} > \tilde{\tau}_{\max}$), if*

$$\varphi(\gamma_{\max}) < \frac{1}{(1 - \gamma_{\max})(1 + \frac{\gamma_{\max}}{\bar{\gamma}})} \equiv \overline{\varphi(\gamma_{\max})}. \quad (6)$$

Proof. See the Appendix. ■

The intuition behind this result is the following. A low density for $\gamma = \gamma_{\max}$ implies that a threshold value η slightly smaller than $\gamma_{\max}$ will be associated with a probability of cooperation being chosen negligibly smaller than unity. Hence, the effect on the likelihood of cooperation is only negligibly different from when there is no uncertainty, in which case cooperation can be sustained when τ is marginally increased beyond $\tilde{\tau}_{\max}$, because $\tau_{\max} > \tilde{\tau}_{\max}$. However, if $\varphi(\gamma_{\max})$ is too large, this will no longer be the case. The probability of cooperation being chosen will fall by sufficiently much to make opting for deviation the only consistent solution.¹⁷

¹⁶The issue of how wide a range of priors support any of the solutions of the CSE and hence, how sensitive each solution is to changes in the prior, is not addressed here.

¹⁷In the case of certainty, the marginal likelihood of $\bar{\gamma}$ occurring is infinitely large and hence, the probability of cooperation has to fall to zero as τ is increased beyond $\tau_{\max}$.

It is straightforward that a similar condition as stated in the previous proposition, can easily be derived for the case when there exist more than three solutions to the CSE when $\tau = \tilde{\tau}_{\max}$. For self-destructive agreements to be feasible as τ is increased beyond $\tilde{\tau}_{\max}$, it is necessary that there exists at least one consistent solution $\eta > \gamma_{\min}$, for which the density is sufficiently low.

From condition (6) we can infer that $\overline{\varphi(\gamma_{\max})}$ increases unambiguously in $\bar{\gamma}$ and increases in $\gamma_{\max}$ for $\gamma_{\max} \geq \frac{1-\bar{\gamma}}{2}$. Thus, the scope for self-destructive agreements increases as $\bar{\gamma}$ and/or $\gamma_{\max}$ attain large values. Letting $\gamma_{\max}$ go to one, $\overline{\varphi(\gamma_{\max})}$ becomes infinitely large, which implies that self-destructive agreements will always exist. Hence, when there is a positive marginal probability that governments can become entirely short-sighted, self-destructive agreements are always feasible. In fact, in this case all agreements are self-destructive for any strictly positive degree of liberalization.

Qualitatively, the results above can also be obtained when finite Nash reversionary punishment phases are applied. Compared to the case when the grim-trigger strategy is applied, $v_C - v_D$ is smaller for $\tau > 0$ and $\eta > \gamma_{\min}$ under a finite punishment phase. From (2) it then immediately follows that $f(\tau, \eta)$ decreases for $\tau > 0$ and $\eta > \gamma_{\min}$. Thus, a finite punishment phase will decrease the range of safe agreements, under which breaches against liberalization commitments never occur (unless in the case of $\gamma_{\max} = 1$, when safe agreements are not feasible for any $\tau > 0$), i.e. $\tilde{\tau}_{\max}$ will be smaller. Moreover, the range of agreements with a strictly positive ex ante probability of cooperation being chosen will also diminish, i.e. $\tilde{\tau}'_{\max}$ will be smaller. However, just as in the case when the grim-trigger strategy is applied, a tradeoff between liberalization and its sustainability may emerge when liberalization is pushed beyond a certain threshold value.

5 The Optimal Degree of Liberalization under Uncertainty

In the previous section, a sufficient condition for when liberalization can be pushed beyond $\tilde{\tau}_{\max}$ without cooperation instantly breaking down was derived. This section explores what degree of liberalization is optimal whenever this condition is satisfied and hence, self-destructive agreements are feasible.

First, consider the case when $\gamma_{\max} \leq 1 - \tau_{opt}$. Then $\tilde{\tau}_{\max} \geq \tau_{opt}$, and the optimal degree of liberalization under certainty can be attained with an associated likelihood of cooperation being chosen equal to unity. In this case it is obviously optimal to apply τ_{opt} , because $v_C = w_C$ for $\tau \leq \tilde{\tau}_{\max}$ and hence, an increase in τ beyond τ_{opt} will lead to a fall in v_C . Thus, the optimal agreement under certainty can be replicated for sufficiently small values of $\gamma_{\max}$, i.e. when governments always attribute sufficient weight to the future.

If, however, $\gamma_{\max} > 1 - \tau_{opt}$, then $\tilde{\tau}_{\max} < \tau_{opt}$, and the optimal degree of liberalization under certainty cannot be attained with an associated likelihood of cooperation being chosen equal to unity. In this case, the continuation value increases unambiguously in the degree of liberalization for $\tau \leq \tilde{\tau}_{\max}$, because no deviations will ever occur and hence, there is no tradeoff between liberalization and sustainability of cooperation within this range of degrees of liberalization. But will the continuation value increase when the degree of liberalization is pushed beyond $\tilde{\tau}_{\max}$ and the probability for breaching the agreement becomes strictly positive? To assess the impact of an increase of the degree of liberalization beyond $\tilde{\tau}_{\max}$, consider the derivative of v_C with respect to τ , which is given by

$$\frac{dv_C}{d\tau} = \frac{\partial v_C}{\partial \tau} + \frac{\partial v_C}{\partial \eta} \frac{d\eta}{d\tau}. \quad (7)$$

An increase in the degree of liberalization has two effects. First, there is the direct effect on the outcome under mutual cooperation and thus, on the continuation value, measured by the

first term. Second, there is the indirect impact, stemming from the decrease in η due to the increase in the degree of liberalization, discussed in the previous section. This effect depends on $\frac{d\eta}{d\tau}$, which is determined by the CSE.

It is easily established that (see the Appendix)

$$\frac{\partial v_C}{\partial \tau} \Big|_{\tau=\tilde{\tau}_{\max}} = \frac{\partial w_C}{\partial \tau} \Big|_{\tau=\tilde{\tau}_{\max}}. \quad (8)$$

Since $\tilde{\tau}_{\max} < \tau_{opt}$, the direct effect of increasing τ beyond $\tilde{\tau}_{\max}$ is unambiguously positive.

To assess the second effect, it is necessary to calculate the partial derivative of v_C with respect to η and to set $\tau = \tilde{\tau}_{\max} = 1 - \gamma_{\max}$ (see the Appendix)

$$\frac{\partial v_C}{\partial \eta} \Big|_{\tau=\tilde{\tau}_{\max}} = [w_C(\tilde{\tau}_{\max}) - w_N] \left[\frac{1}{\gamma_{\max}} + \frac{1}{\bar{\gamma}} - 1 \right] \varphi(\gamma_{\max}). \quad (9)$$

This expression is positive for any $\gamma_{\max}$. From the previous section, we know that an increase in τ leads to a decrease in the largest solution of the CSE. The formal expression for $\frac{d\eta}{d\tau}$ when $\tau = \tilde{\tau}_{\max}$ is obtained by calculating the derivative of the CSE with respect to τ and setting $\tau = \tilde{\tau}_{\max}$ and $\eta = \gamma_{\max}$ (see the Appendix)

$$\frac{d\eta}{d\tau} \Big|_{\tau=\tilde{\tau}_{\max}} = - \frac{1}{1 - \frac{\varphi(\gamma_{\max})}{\varphi(\gamma_{\max})}}. \quad (10)$$

Since it is assumed that $\varphi(\gamma_{\max}) < \overline{\varphi(\gamma_{\max})}$, such that self-destructive agreements exist, this term is negative. Thus, the second effect of increasing τ beyond $\tilde{\tau}_{\max}$ is unambiguously negative, as expected.

It is easy to see that an increase in $\varphi(\gamma_{\max})$ makes (9) larger and (10) more negative and hence, the combined effect is unambiguously negative. Thus, the larger is $\varphi(\gamma_{\max})$ and hence,

the marginal likelihood of breaching the agreement, the stronger is the negative impact of an increase in τ beyond $\tilde{\tau}_{\max}$. The marginal likelihood of $\gamma_{\max}$ occurring therefore determines whether the negative, indirect effect dominates the positive, direct effect of pushing τ beyond $\tilde{\tau}_{\max}$. The following proposition provides a sufficient condition for when increasing τ beyond $\tilde{\tau}_{\max}$ is optimal.

Proposition 2 *When $\gamma_{\max} > 1 - \tau_{opt}$, in which case $\tilde{\tau}_{\max} < \tau_{opt}$, there exists a threshold value $\overline{\varphi(\gamma_{\max})}' \in (0, \overline{\varphi(\gamma_{\max})})$ such that a self-destructive agreement is preferred to a safe agreement ($\tilde{\tau}_{opt} > \tilde{\tau}_{\max}$) if*

$$\varphi(\gamma_{\max}) < \frac{\frac{\frac{dw_C}{d\tau}|_{\tau=\tilde{\tau}_{\max}}}{w_C(\tilde{\tau}_{\max})-w_N}}{[\frac{1}{\gamma_{\max}} + \frac{1}{\bar{\gamma}} - 1]\overline{\varphi(\gamma_{\max})} + \frac{\frac{dw_C}{d\tau}|_{\tau=\tilde{\tau}_{\max}}}{w_C(\tilde{\tau}_{\max})-w_N}} \overline{\varphi(\gamma_{\max})} \equiv \overline{\varphi(\gamma_{\max})}'. \quad (11)$$

Proof. For a self-destructive agreement to be preferred to the most far-reaching safe agreement, a sufficient condition is that $\frac{dv_C}{d\tau}|_{\tau=\tilde{\tau}_{\max}} > 0$. By plugging (8), (9) and (10) in (7) and rearranging terms, condition (11) follows immediately (see the Appendix). ■

Hence, given that $\gamma_{\max} > 1 - \tau_{opt}$, pushing liberalization beyond $\tilde{\tau}_{\max}$ will be worthwhile, if the marginal likelihood of $\gamma_{\max}$ occurring is sufficiently small. In this case the unambiguously positive effect on the outcome under mutual cooperation outweighs the negative impact, stemming from the decrease in η below $\gamma_{\max}$ such that the probability of a breakdown of the agreement becomes strictly positive.

The threshold value $\overline{\varphi(\gamma_{\max})}'$ solely depends on $\bar{\gamma}$ and $\gamma_{\max}$; the term $\frac{\frac{dw_C}{d\tau}|_{\tau=\tilde{\tau}_{\max}}}{w_C(\tilde{\tau}_{\max})-w_N}$ is, in fact, a function of $\gamma_{\max}$. By plugging expression (6) into (11), an expression for $\overline{\varphi(\gamma_{\max})}'$ can be

obtained such that the impact of changes in $\bar{\gamma}$ and $\gamma_{\max}$ can be more easily assessed

$$\overline{\varphi(\gamma_{\max})}' = \frac{1}{\left(\frac{1}{\gamma_{\max}} + \frac{1}{\bar{\gamma}} - 1\right) \frac{w_C(\tilde{\tau}_{\max}) - w_N}{\frac{dw_C}{d\tau}|_{\tau=\tilde{\tau}_{\max}}} + (1 - \gamma_{\max})\left(1 + \frac{\gamma_{\max}}{\bar{\gamma}}\right)}. \quad (11')$$

Since $\tilde{\tau}_{\max} < \tau_{opt}$, it is the case that $\frac{w_C(\tilde{\tau}_{\max}) - w_N}{\frac{dw_C}{d\tau}|_{\tau=\tilde{\tau}_{\max}}} \geq 0$. An increase in $\gamma_{\max}$ implies a lower $\tilde{\tau}_{\max}$ and thus a lower value for $\frac{w_C(\tilde{\tau}_{\max}) - w_N}{\frac{dw_C}{d\tau}|_{\tau=\tilde{\tau}_{\max}}}$, because w_C is concave in τ . Therefore, the first term of the denominator will unambiguously decrease when $\gamma_{\max}$ is increased. The effect on the second term of the denominator is unambiguously negative if and only if $\gamma_{\max} > \frac{1-\bar{\gamma}}{2}$. Hence, if $\gamma_{\max} \geq \frac{1-\bar{\gamma}}{2}$, an increase in $\gamma_{\max}$ unambiguously increases $\overline{\varphi(\gamma_{\max})}'$.¹⁸

A high value of $\gamma_{\max}$ implies a high $\overline{\varphi(\gamma_{\max})}'$ and hence, increasing τ beyond $\tilde{\tau}_{\max}$ will be optimal except when the marginal likelihood of $\gamma_{\max}$ occurring is very high; in fact it will always be beneficial to increase τ beyond $\tilde{\tau}_{\max}$ if $\gamma_{\max} = 1$, in which case $\tilde{\tau}_{\max} = 0$ and no safe agreement with a strictly positive degree of liberalization is feasible. The intuition is as follows. A higher $\gamma_{\max}$ implies a lower $\tilde{\tau}_{\max}$, i.e. the scope for safe agreements is smaller. The positive effect of increasing τ beyond $\tilde{\tau}_{\max}$ thus increases in $\gamma_{\max}$. The negative effect of an increase in τ beyond $\tilde{\tau}_{\max}$ on the likelihood of cooperation, which depends on $\varphi(\gamma_{\max})$, can thus be larger the larger is $\gamma_{\max}$, without the overall impact of an increase in τ beyond $\tilde{\tau}_{\max}$ becoming negative.

The effect of an increase in $\bar{\gamma}$ on $\overline{\varphi(\gamma_{\max})}'$ is unambiguously positive. The underlying reason is that a larger $\bar{\gamma}$ implies a lower expected weight attributed to the future. Since the risk of breakdown occurring increases in the number of future periods, a lower expected weighting of the future implies a lower weighting of the negative impact of increasing τ beyond $\tilde{\tau}_{\max}$. The following lemma summarizes the above findings.

¹⁸Note that $\gamma_{\max} \geq \frac{1-\bar{\gamma}}{2}$ will always hold for $\bar{\gamma} \geq \frac{1}{3}$. However, even if $\gamma_{\max} < \frac{1-\bar{\gamma}}{2}$, the effect of an increase in $\gamma_{\max}$ on $\overline{\varphi(\gamma_{\max})}'$ may be unambiguously positive (it will depend on the specification of w_C , however).

Lemma 2 *The scope for self-destructive agreements being preferred to safe agreements, increases (i) if $\bar{\gamma}$ is sufficiently large; (ii) if $\gamma_{\max}$ is sufficiently large; and (iii) if $\varphi(\gamma_{\max})$ is sufficiently small.*

6 Conclusion

The answer to the question posed in the title is affirmative: it may be optimal to agree on a degree of liberalization such that cooperation will eventually break down. A preference for self-destructive rather than safe agreements may arise in the present context of an infinitely repeated Prisoner's Dilemma tariff setting game between two symmetric countries, where the stochastically determined weight each government attributes to current vis-à-vis future payoffs is only privately observable. The model demonstrates how trade liberalization can suddenly break down and is thus consistent with the occurrence of trade wars.

The optimal agreement under certainty can be replicated under uncertainty if governments will always attribute sufficient weight to the future. Hence, applying the degree of liberalization that is optimal under certainty, while maintaining certainty of cooperation, will only be possible when the weight given to current payoffs is sufficiently low even under the most myopic realization of the random variable. If the latter is not the case, the degree of liberalization should at least be set at the level where the probability of deviation occurring just becomes strictly positive. Pushing the degree of liberalization further, the positive effect of more liberalization will have to be weighed against the negative impact of the ex ante likelihood of cooperation breaking down becoming strictly positive. If the latter outweighs the former, implementing the most far-reaching safe agreement is optimal. Else, it is optimal to implement a self-destructive agreement with a higher degree of liberalization than under the most far-reaching safe agreement. The latter outcome is more likely for a large ex ante expected weight given to current payoffs, for a

large maximum possible weight attributed to current payoffs, and a small marginal likelihood of the maximum possible weight given to current payoffs.

The results obtained in this paper apply to the case of infinite Nash reversion following breaches against liberalization commitments, but can easily be generalized to a situation where punishments for deviations from liberalization are finite. In this case the same tradeoff between more liberalization and a higher risk for deviations from liberalization occurring may emerge, and it may be optimal to implement an agreement, under which there is a strictly positive likelihood for protectionism occurring along the equilibrium path. Hence deviations from the cooperative degree of liberalization followed by retaliatory phases will occur in this framework of hidden information, just as in the presence of hidden-action-type of uncertainty, but, contrary to the case of hidden action, Nash reversions will be the consequences of such deviations having actually occurred.

This paper examines the simplest form of a trade agreement, one that merely specifies tariff levels and does not include instruments to deal with unforeseen events. While this is clearly not the case in the context of the General Agreement on Tariffs and Trade (GATT) and the World Trade Organization, the main aim is to shed light on the tradeoff between liberalization commitments and the possibility to sustain liberalization in the long run. Thus, the present model provides a theoretical framework for analyzing how bilateral and multilateral trade agreements could be designed to ensure that higher degrees of liberalization are sustainable in the presence of only privately observable incentives to breach liberalization commitments. In fact, there are ways of overcoming the problem that agreements will not always be sustainable over time. Safeguard provisions similar to those found in trade agreements such as the GATT, for example, allow signatory countries to withdraw liberalization commitments under the agreement in order to protect certain overriding interests under specified conditions, thereby possibly eliminating the

risk of breakdown. The model presented here thus provides a justification for the inclusion of safeguard provisions in trade agreements.

The findings generated by the trade-policy framework presented here can easily be generalized to any type of repeated Prisoner's-Dilemma-type of strategic interaction taking place under hidden information. For example, in a Cournot duopoly firms may find it optimal to apply a degree of cooperation that cannot be upheld indefinitely in the presence of asymmetric information concerning the incentives to deviate from the agreed-upon degree of cooperation. Just as in the case of hidden-action-type of uncertainty, the optimal degree of cooperation may be associated with deviations from the cooperative regime along the equilibrium path.

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Proof of $W_N^\gamma = w_N$

$$W_N^\gamma = \gamma w_N + (1 - \gamma)[\bar{\gamma} w_N + (1 - \bar{\gamma})[\bar{\gamma} w_N + \dots]] = \gamma w_N + (1 - \gamma)\bar{\gamma} w_N \sum_{n=0}^{\infty} (1 - \bar{\gamma})^n = w_N.$$

Condition (1')

$$\frac{w_D - w_C}{w_C - w_N} = \frac{w_D - w_C}{(w_D - w_N) - (w_D - w_C)} = \frac{\frac{w_D - w_C}{w_D - w_N}}{1 - \frac{w_D - w_C}{w_D - w_N}} = \frac{\tau}{1 - \tau}$$

Hence,

$$\frac{w_D - w_C}{w_C - w_N} \leq \frac{1 - \bar{\gamma}}{\bar{\gamma}} \Leftrightarrow \frac{\tau}{1 - \tau} \leq \frac{1 - \bar{\gamma}}{\bar{\gamma}} \Leftrightarrow \tau \leq 1 - \bar{\gamma} \equiv \tau_{\max}.$$

Derivation of v_D and v_C

$$\begin{aligned}
v_D &= \bar{\gamma}w_N + (1 - \bar{\gamma})v_D \Leftrightarrow v_D = w_N \\
v_C &= \Phi(\eta)\left\{\int_{\gamma_{\min}}^{\eta} [\gamma w_C + (1 - \gamma)v_C]d\Phi + \int_{\eta}^{\gamma_{\max}} [\gamma w_D + (1 - \gamma)v_D]d\Phi\right\} \\
&\quad + [1 - \Phi(\eta)]\left\{\int_{\gamma_{\min}}^{\eta} [\gamma w_S + (1 - \gamma)v_D]d\Phi + \int_{\eta}^{\gamma_{\max}} [\gamma w_N + (1 - \gamma)v_D]d\Phi\right\} \\
&= \Phi(\eta)\left\{(w_C - w_D)\int_{\gamma_{\min}}^{\eta} \gamma d\Phi + w_D\int_{\gamma_{\min}}^{\gamma_{\max}} \gamma d\Phi + (v_C - v_D)\int_{\gamma_{\min}}^{\eta} (1 - \gamma)d\Phi\right\} \\
&\quad + [1 - \Phi(\eta)]\left\{(w_S - w_N)\int_{\gamma_{\min}}^{\eta} \gamma d\Phi + w_N\int_{\gamma_{\min}}^{\gamma_{\max}} \gamma d\Phi + v_D\int_{\gamma_{\min}}^{\gamma_{\max}} (1 - \gamma)d\Phi\right\} \\
&= \Phi(\eta)\left\{(w_C - w_D)\int_{\gamma_{\min}}^{\eta} \gamma d\Phi + \bar{\gamma}w_D + (v_C - v_D)\left[\Phi(\eta) - \int_{\gamma_{\min}}^{\eta} \gamma d\Phi\right]\right\} \\
&\quad + [1 - \Phi(\eta)]\left\{(w_C - w_D)\int_{\gamma_{\min}}^{\eta} \gamma d\Phi + \bar{\gamma}w_N\right\} + (1 - \bar{\gamma})v_D \\
&\stackrel{(3)}{=} w_N + (w_D - w_N)\bar{\gamma}\Phi(\eta) - (w_D - w_C)\int_{\gamma_{\min}}^{\eta} \gamma d\Phi \\
&\quad + (v_C - w_N)\Phi(\eta)\left[\Phi(\eta) - \int_{\gamma_{\min}}^{\eta} \gamma d\Phi\right] \\
&\Leftrightarrow \{1 - \Phi(\eta)\left[\Phi(\eta) - \int_{\gamma_{\min}}^{\eta} \gamma d\Phi\right]\}(v_C - w_N) = [\bar{\gamma}\Phi(\eta) - \tau\int_{\gamma_{\min}}^{\eta} \gamma d\Phi](w_D - w_N) \\
&\Leftrightarrow v_C = w_N + \frac{\bar{\gamma}\Phi(\eta) - \tau\int_{\gamma_{\min}}^{\eta} \gamma d\Phi}{1 - \Phi(\eta)\left[\Phi(\eta) - \int_{\gamma_{\min}}^{\eta} \gamma d\Phi\right]}(w_D - w_N).
\end{aligned}$$

Proof of Proposition 1

Given that there exist no more than three solutions to the CSE, and since $f(\gamma_{\max}) = \gamma_{\max}$ for $\tau = \tilde{\tau}_{\max}$, it must be that $\lim_{\eta \rightarrow \gamma_{\max}} f'(\eta) < 1$ (note that $f'(\eta) = 0$ for $\eta > \gamma_{\max}$) for $\tau = \tilde{\tau}_{\max}$ for there to exist strictly positive solutions to the CSE when τ is increased beyond $\tilde{\tau}_{\max}$. The

first derivative of $f(\eta)$ for $\eta \in (\gamma_{\min}, \gamma_{\max})$ is calculated as follows

$$\begin{aligned}
f'(\eta) &= \frac{[(\bar{\gamma} - \tau)\Phi^2(\eta) + \tau][2\bar{\gamma}\Phi(\eta)\varphi(\eta) - \tau\Phi(\eta)\varphi(\eta)\eta - \tau\varphi(\eta)\int_{\gamma_{\min}}^{\eta} \gamma d\Phi] \\
&\quad - 2(\bar{\gamma} - \tau)\Phi(\eta)\varphi(\eta)[\bar{\gamma}\Phi^2(\eta) - \tau\Phi(\eta)\int_{\gamma_{\min}}^{\eta} \gamma d\Phi]}{\{\bar{\gamma}\Phi^2(\eta) + \tau[1 - \Phi^2(\eta)]\}^2} \\
&= \frac{(\bar{\gamma} - \tau)\Phi^2(\eta)[- \tau\Phi(\eta)\varphi(\eta)\eta + \tau\varphi(\eta)\int_{\gamma_{\min}}^{\eta} \gamma d\Phi] \\
&\quad + \tau[2\bar{\gamma}\Phi(\eta)\varphi(\eta) - \tau\Phi(\eta)\varphi(\eta)\eta - \tau\varphi(\eta)\int_{\gamma_{\min}}^{\eta} \gamma d\Phi]}{\{\bar{\gamma}\Phi^2(\eta) + \tau[1 - \Phi^2(\eta)]\}^2} \\
&= \frac{2\bar{\gamma}\Phi(\eta) - \tau[\Phi(\eta)\eta + \int_{\gamma_{\min}}^{\eta} \gamma d\Phi] - (\bar{\gamma} - \tau)\Phi^2(\eta)[\Phi(\eta)\eta - \int_{\gamma_{\min}}^{\eta} \gamma d\Phi]}{\{\bar{\gamma}\Phi^2(\eta) + \tau[1 - \Phi^2(\eta)]\}^2} \tau\varphi(\eta).
\end{aligned}$$

Hence,

$$\begin{aligned}
\lim_{\eta \rightarrow \gamma_{\max}} f'(\eta) &= \frac{2\bar{\gamma} - (1 - \gamma_{\max})(\gamma_{\max} + \bar{\gamma}) - (\bar{\gamma} + \gamma_{\max} - 1)(\gamma_{\max} - \bar{\gamma})}{\bar{\gamma}^2} (1 - \gamma_{\max})\varphi(\gamma_{\max}) \\
&= (1 - \gamma_{\max})(1 + \frac{\gamma_{\max}}{\bar{\gamma}})\varphi(\gamma_{\max}).
\end{aligned}$$

Thus, if $\varphi(\gamma_{\max}) < \frac{1}{(1 - \gamma_{\max})(1 + \frac{\gamma_{\max}}{\bar{\gamma}})} \equiv \overline{\varphi(\gamma_{\max})}$, then $f'(\gamma_{\max})|_{\tau=\tilde{\tau}_{\max}} < 1$ for $\tau = \tilde{\tau}_{\max}$ and

$$\tilde{\tau}'_{\max} > \tilde{\tau}_{\max}.$$

Equation (8)

Because $v_C - w_C$ for $\tau \leq \tilde{\tau}_{\max}$, we have $\lim_{\tau \rightarrow \tilde{\tau}_{\max}} \frac{\partial v_C}{\partial \tau} = \frac{\partial w_C}{\partial \tau} \big|_{\tau=\tilde{\tau}_{\max}}$.

For $\tau > \tilde{\tau}_{\max}$, the continuation value v_C is given by (4), and hence,

$$\lim_{\tau \rightarrow \tilde{\tau}_{\max}^+} \frac{\partial v_C}{\partial \tau} = \lim_{\tau \rightarrow \tilde{\tau}_{\max}^+} \left[\frac{-\int_{\gamma_{\min}}^{\eta} \gamma d\Phi}{1 - \Phi(\eta)[\Phi(\eta) - \int_{\gamma_{\min}}^{\eta} \gamma d\Phi]} (w_D - w_N) + \frac{\bar{\gamma}\Phi(\eta) - \tau \int_{\gamma_{\min}}^{\eta} \gamma d\Phi}{1 - \Phi(\eta)[\Phi(\eta) - \int_{\gamma_{\min}}^{\eta} \gamma d\Phi]} \frac{\partial w_D}{\partial \tau} \right].$$

Since $\eta = \gamma_{\max}$ for $\tau = \tilde{\tau}_{\max}$, we get

$$\lim_{\tau \rightarrow \tilde{\tau}_{\max}^+} \frac{\partial v_C}{\partial \tau} = -(w_D - w_N) \big|_{\tau=\tilde{\tau}_{\max}} + (1 - \tau) \frac{\partial w_D}{\partial \tau} \big|_{\tau=\tilde{\tau}_{\max}}. \quad (\text{A1})$$

Rearranging $\tau = \frac{w_D - w_C}{w_D - w_N}$ yields

$$w_C - \tau w_N = (1 - \tau)w_D \Leftrightarrow w_D = \frac{w_C - \tau w_N}{1 - \tau}, \quad (\text{A2})$$

and hence,

$$\frac{\partial w_D}{\partial \tau} = \frac{(1 - \tau) \left(\frac{\partial w_C}{\partial \tau} - w_N \right) + (w_C - \tau w_N)}{(1 - \tau)^2} = \frac{w_C - w_N + (1 - \tau) \frac{\partial w_C}{\partial \tau}}{(1 - \tau)^2}.$$

Plugging this expression into (A1) gives us

$$\begin{aligned} \lim_{\tau \rightarrow \tilde{\tau}_{\max}^+} \frac{\partial v_C}{\partial \tau} &= -(w_D - w_N) \big|_{\tau=\tilde{\tau}_{\max}} + \frac{w_C - w_N + (1 - \tau) \frac{\partial w_C}{\partial \tau}}{1 - \tau} \big|_{\tau=\tilde{\tau}_{\max}} \\ &= -(w_D - w_N) \big|_{\tau=\tilde{\tau}_{\max}} + \frac{w_C - w_N}{1 - \tau} \big|_{\tau=\tilde{\tau}_{\max}} + \frac{\partial w_C}{\partial \tau} \big|_{\tau=\tilde{\tau}_{\max}} \end{aligned}$$

Because $w_D - w_N \stackrel{(M2)}{=} \frac{w_C - \tau w_N}{1 - \tau} - w_N = \frac{w_C - w_N}{1 - \tau}$, it immediately follows that $\lim_{\tau \rightarrow \tilde{\tau}_{\max}^+} \frac{\partial v_C}{\partial \tau} =$

$\frac{\partial w_C}{\partial \tau} \big|_{\tau=\tilde{\tau}_{\max}}$ and hence $\frac{\partial v_C}{\partial \tau} \big|_{\tau=\tilde{\tau}_{\max}} = \frac{\partial w_C}{\partial \tau} \big|_{\tau=\tilde{\tau}_{\max}}$ 2

Equation (9)

Deriving the expression for v_C with respect to η and setting $\tau = \tilde{\tau}_{\max} = 1 - \gamma_{\max}$ (implying $\eta = \gamma_{\max}$) yields

$$\begin{aligned}
\frac{\partial v_C}{\partial \eta} \Big|_{\tau=\tilde{\tau}_{\max}} &= \left\{ \left[\frac{\bar{\gamma}\varphi(\eta) - \tau\eta\varphi(\eta)}{\bar{\gamma}\Phi(\eta) - \tau \int_{\gamma_{\min}}^{\eta} \gamma d\Phi} - \frac{-2\Phi(\eta)\varphi(\eta) + \Phi(\eta)\varphi(\eta)\eta + \varphi(\eta) \int_{\gamma_{\min}}^{\eta} \gamma d\Phi}{1 - \Phi(\eta)[\Phi(\eta) - \int_{\gamma_{\min}}^{\eta} \gamma d\Phi]} \right] [v_C - w_N] \right\}_{\tau=\tilde{\tau}_{\max}} \\
&= \left[\frac{\bar{\gamma}\varphi(\gamma_{\max}) - (1 - \gamma_{\max})\gamma_{\max}\varphi(\gamma_{\max})}{\bar{\gamma} - (1 - \gamma_{\max})\bar{\gamma}} - \frac{-2\varphi(\gamma_{\max}) + \varphi(\gamma_{\max})\gamma_{\max} + \varphi(\gamma_{\max})\bar{\gamma}}{\bar{\gamma}} \right] [w_C(\tilde{\tau}_{\max}) - w_N] \\
&= \frac{\varphi(\gamma_{\max})}{\bar{\gamma}} \left[\frac{\bar{\gamma} - \gamma_{\max} + \gamma_{\max}^2}{\gamma_{\max}} + 2 - \gamma_{\max} - \bar{\gamma} \right] [w_C(\tilde{\tau}_{\max}) - w_N] \\
&= \frac{\varphi(\gamma_{\max})}{\bar{\gamma}} \frac{\bar{\gamma} + \gamma_{\max} - \bar{\gamma}\gamma_{\max}}{\gamma_{\max}} [w_C(\tilde{\tau}_{\max}) - w_N] \\
&= [w_C(\tilde{\tau}_{\max}) - w_N] \left[\frac{1}{\gamma_{\max}} + \frac{1}{\bar{\gamma}} - 1 \right] \varphi(\gamma_{\max}).
\end{aligned}$$

Derivation of $\frac{d\eta}{d\tau} \Big|_{\tau=\tilde{\tau}_{\max}}$

The CSE (5) can be rewritten as follows:

$$0 = \Phi(\eta) \frac{\bar{\gamma}\Phi(\eta) - \tau \int_{\gamma_{\min}}^{\eta} \gamma d\Phi}{\bar{\gamma}\Phi^2(\eta) + \tau[1 - \Phi^2(\eta)]} - \eta. \quad (5')$$

Let $\eta(\tau)$ solve the CSE and $\eta' = \eta'(\tau)$. Deriving (5') with respect to τ yields

$$\begin{aligned}
0 = & \frac{\{\bar{\gamma}\Phi^2(\eta) + \tau[1 - \Phi^2(\eta)]\} \{2\bar{\gamma}\Phi(\eta)\varphi(\eta)\eta' - \Phi(\eta) \int_{\gamma_{\min}}^{\eta} \gamma d\Phi - \tau\varphi(\eta) \int_{\gamma_{\min}}^{\eta} \gamma d\Phi\eta' - \tau\Phi(\eta)\varphi(\eta)\eta'\eta'\} \\
& - \{\bar{\gamma}\Phi^2(\eta) - \tau\Phi(\eta) \int_{\gamma_{\min}}^{\eta} \gamma d\Phi\} \{2\bar{\gamma}\Phi(\eta)\varphi(\eta)\eta' + 1 - \Phi^2(\eta) - 2\tau\Phi(\eta)\varphi(\eta)\eta'\}}{\{\bar{\gamma}\Phi^2(\eta) + \tau[1 - \Phi^2(\eta)]\}^2} - \eta'(\tau)
\end{aligned}$$

$$\begin{aligned}
& \Leftrightarrow \{\bar{\gamma}\Phi^2(\eta) + \tau[1 - \Phi^2(\eta)]\}\{2\bar{\gamma}\Phi(\eta)\varphi(\eta)\eta' - \Phi(\eta)\int_{\gamma_{\min}}^{\eta} \gamma d\Phi - \tau\varphi(\eta)\int_{\gamma_{\min}}^{\eta} \gamma d\Phi\eta' - \tau\Phi(\eta)\varphi(\eta)\eta\eta'\} \\
& \quad - \{\bar{\gamma}\Phi^2(\eta) - \tau\Phi(\eta)\int_{\gamma_{\min}}^{\eta} \gamma d\Phi\}\{2\bar{\gamma}\Phi(\eta)\varphi(\eta)\eta' + 1 - \Phi^2(\eta) - 2\tau\Phi(\eta)\varphi(\eta)\eta'\} \\
& = \{\bar{\gamma}\Phi^2(\eta) + \tau[1 - \Phi^2(\eta)]\}^2\eta' \\
& \Leftrightarrow \left[\begin{aligned} & \{\bar{\gamma}\Phi^2(\eta) + \tau[1 - \Phi^2(\eta)]\}\{2\bar{\gamma}\Phi(\eta)\varphi(\eta) - \tau\varphi(\eta)\int_{\gamma_{\min}}^{\eta} \gamma d\Phi - \tau\Phi(\eta)\varphi(\eta)\eta\} \\ & - \{\bar{\gamma}\Phi^2(\eta) - \tau\Phi(\eta)\int_{\gamma_{\min}}^{\eta} \gamma d\Phi\}\{2\bar{\gamma}\Phi(\eta)\varphi(\eta) - 2\tau\Phi(\eta)\varphi(\eta)\} - \{\bar{\gamma}\Phi^2(\eta) + \tau[1 - \Phi^2(\eta)]\}^2 \end{aligned} \right] \eta' \\
& = \{\bar{\gamma}\Phi^2(\eta) + \tau[1 - \Phi^2(\eta)]\}\Phi(\eta)\int_{\gamma_{\min}}^{\eta} \gamma d\Phi + \{\bar{\gamma}\Phi^2(\eta) - \tau\Phi(\eta)\int_{\gamma_{\min}}^{\eta} \gamma d\Phi\}\{1 - \Phi^2(\eta)\} \\
& \quad \{\bar{\gamma}\Phi^2(\eta) + \tau[1 - \Phi^2(\eta)]\}\Phi(\eta)\int_{\gamma_{\min}}^{\eta} \gamma d\Phi + \{\bar{\gamma}\Phi^2(\eta) - \tau\Phi(\eta)\int_{\gamma_{\min}}^{\eta} \gamma d\Phi\}\{1 - \Phi^2(\eta)\} \\
& \Leftrightarrow \eta' = \frac{\{\bar{\gamma}\Phi^2(\eta) + \tau[1 - \Phi^2(\eta)]\}\{2\bar{\gamma}\Phi(\eta)\varphi(\eta) - \tau\varphi(\eta)\int_{\gamma_{\min}}^{\eta} \gamma d\Phi - \tau\Phi(\eta)\varphi(\eta)\eta\} - \{\bar{\gamma}\Phi^2(\eta) - \tau\Phi(\eta)\int_{\gamma_{\min}}^{\eta} \gamma d\Phi\}\{2\bar{\gamma}\Phi(\eta)\varphi(\eta) - 2\tau\Phi(\eta)\varphi(\eta)\} - \{\bar{\gamma}\Phi^2(\eta) + \tau[1 - \Phi^2(\eta)]\}^2}{\{\bar{\gamma}\Phi^2(\eta) + \tau[1 - \Phi^2(\eta)]\}\{2\bar{\gamma}\Phi(\eta)\varphi(\eta) - \tau\varphi(\eta)\int_{\gamma_{\min}}^{\eta} \gamma d\Phi - \tau\Phi(\eta)\varphi(\eta)\eta\} - \{\bar{\gamma}\Phi^2(\eta) - \tau\Phi(\eta)\int_{\gamma_{\min}}^{\eta} \gamma d\Phi\}\{2\bar{\gamma}\Phi(\eta)\varphi(\eta) - 2\tau\Phi(\eta)\varphi(\eta)\} - \{\bar{\gamma}\Phi^2(\eta) + \tau[1 - \Phi^2(\eta)]\}^2}
\end{aligned}$$

Hence, for $\tau = \tilde{\tau}_{\max} = 1 - \gamma_{\max}$ and the optimal solution $\eta = \eta_1(\tilde{\tau}_{\max}) = \gamma_{\max}$,

$$\begin{aligned}
\frac{d\eta}{d\tau}|_{\tau=\tilde{\tau}_{\max}} &= \frac{\bar{\gamma}^2}{\bar{\gamma}\{2\bar{\gamma}\varphi(\gamma_{\max}) - (1 - \gamma_{\max})\bar{\gamma}\varphi(\gamma_{\max}) - (1 - \gamma_{\max})\gamma_{\max}\varphi(\gamma_{\max})\} - \{\bar{\gamma} - (1 - \gamma_{\max})\bar{\gamma}\}\{2\bar{\gamma}\varphi(\gamma_{\max}) - 2(1 - \gamma_{\max})\varphi(\gamma_{\max})\} - \bar{\gamma}^2} \\
&= \frac{\bar{\gamma}^2}{2\bar{\gamma}^2\varphi(\gamma_{\max}) - \bar{\gamma}^2\varphi(\gamma_{\max}) + \bar{\gamma}^2\gamma_{\max}\varphi(\gamma_{\max}) - \bar{\gamma}\gamma_{\max}\varphi(\gamma_{\max}) + \bar{\gamma}\gamma_{\max}^2\varphi(\gamma_{\max}) - 2\bar{\gamma}^2\gamma_{\max}\varphi(\gamma_{\max}) + 2\bar{\gamma}\gamma_{\max}\varphi(\gamma_{\max}) - 2\bar{\gamma}\gamma_{\max}^2\varphi(\gamma_{\max}) - \bar{\gamma}^2} \\
&= \frac{\bar{\gamma}^2}{\bar{\gamma}^2\varphi(\gamma_{\max}) - \bar{\gamma}^2\gamma_{\max}\varphi(\gamma_{\max}) + \bar{\gamma}\gamma_{\max}\varphi(\gamma_{\max}) - \bar{\gamma}\gamma_{\max}^2\varphi(\gamma_{\max}) - \bar{\gamma}^2} \\
&= \frac{1}{\varphi(\gamma_{\max}) - \gamma_{\max}\varphi(\gamma_{\max}) + \frac{\gamma_{\max}}{\bar{\gamma}}\varphi(\gamma_{\max}) - \frac{\gamma_{\max}^2}{\bar{\gamma}}\varphi(\gamma_{\max}) - 1} \\
&= \frac{1}{(1 - \gamma_{\max})(1 + \frac{\gamma_{\max}}{\bar{\gamma}})\varphi(\gamma_{\max}) - 1} \\
&= -\frac{1}{1 - \frac{\varphi(\gamma_{\max})}{\varphi(\gamma_{\max})}}.
\end{aligned}$$

Proof of Proposition 2

$$\begin{aligned}
\frac{dv_C}{d\tau}|_{\tau=\tilde{\tau}_{\max}} &> 0 \Leftrightarrow \frac{\partial v_C}{\partial \tau}|_{\tau=\tilde{\tau}_{\max}} + \frac{\partial v_C}{\partial \eta}|_{\tau=\tilde{\tau}_{\max}} \frac{d\eta}{d\tau}|_{\tau=\tilde{\tau}_{\max}} > 0 \\
&\stackrel{(8,9,10)}{\Leftrightarrow} \frac{\partial w_C}{\partial \tau}|_{\tau=\tilde{\tau}_{\max}} - [w_C(\tilde{\tau}_{\max}) - w_N] \left[\frac{1}{\gamma_{\max}} + \frac{1}{\bar{\gamma}} - 1 \right] \varphi(\gamma_{\max}) \frac{1}{1 - \frac{\varphi(\gamma_{\max})}{\varphi(\gamma_{\max})}} > 0 \\
&\Leftrightarrow \frac{\frac{\partial w_C}{\partial \tau}|_{\tau=\tilde{\tau}_{\max}}}{w_C(\tilde{\tau}_{\max}) - w_N} > \left[\frac{1}{\gamma_{\max}} + \frac{1}{\bar{\gamma}} - 1 \right] \frac{\varphi(\gamma_{\max}) \overline{\varphi(\gamma_{\max})}}{\varphi(\gamma_{\max}) - \varphi(\gamma_{\max})} \\
&\Leftrightarrow \frac{\frac{\partial w_C}{\partial \tau}|_{\tau=\tilde{\tau}_{\max}}}{w_C(\tilde{\tau}_{\max}) - w_N} [\overline{\varphi(\gamma_{\max})} - \varphi(\gamma_{\max})] > \left[\frac{1}{\gamma_{\max}} + \frac{1}{\bar{\gamma}} - 1 \right] \varphi(\gamma_{\max}) \overline{\varphi(\gamma_{\max})} \\
&\Leftrightarrow \varphi(\gamma_{\max}) \left\{ \left[\frac{1}{\gamma_{\max}} + \frac{1}{\bar{\gamma}} - 1 \right] \overline{\varphi(\gamma_{\max})} + \frac{\frac{\partial w_C}{\partial \tau}|_{\tau=\tilde{\tau}_{\max}}}{w_C(\tilde{\tau}_{\max}) - w_N} \right\} < \frac{\frac{\partial w_C}{\partial \tau}|_{\tau=\tilde{\tau}_{\max}}}{w_C(\tilde{\tau}_{\max}) - w_N} \overline{\varphi(\gamma_{\max})} \\
&\Leftrightarrow \varphi(\gamma_{\max}) < \frac{\frac{\frac{\partial w_C}{\partial \tau}|_{\tau=\tilde{\tau}_{\max}}}{w_C(\tilde{\tau}_{\max}) - w_N}}{\left[\frac{1}{\gamma_{\max}} + \frac{1}{\bar{\gamma}} - 1 \right] \overline{\varphi(\gamma_{\max})} + \frac{\frac{\partial w_C}{\partial \tau}|_{\tau=\tilde{\tau}_{\max}}}{w_C(\tilde{\tau}_{\max}) - w_N}} \overline{\varphi(\gamma_{\max})}.
\end{aligned}$$