Corporate Tax Systems and the Location of Industry

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Abstract

This paper analyzes the effects of different corporate tax systems on the location of industry within an economic geography model with regional size asymmetries. Both the North and the South gain industry by adopting a tax regime that produces the lowest tax level. As the share of expenditures in the North increases, the Nash equilibrium has this region setting regressive taxes, while the South introduces progressive taxation. The unilateral welfare-maximizing tax structure in the North (South) is the regressive (progressive) system when expenditures in the North increase. Welfare in the North (South) is however maximized if both regions set regressive (progressive) taxes, while regressive (progressive) taxation in both regions represents a joint welfare maximizing outcome if the economic size of the North is higher (lower) than a certain threshold value. As trade is liberalized, the equilibrium tax regime adopted depends on how profits respond to lower trade costs. Proportional taxation is never an equilibrium, neither as regional spending changes, nor as trade is liberalized.

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1 Introduction and Related Literature

There is an extensive line of theoretical and empirical research which examines the effects of corporate taxation levels on industry location in an international setting. At the same time, there are many differing views about the social and economic impacts of the corporate income tax structure, but few or no studies of how different tax systems interact with the location decisions of firms in the international economy. Given that suggestions to reform the tax structure are regularly made, it appears like an important research direction to consider how corporate tax systems affect the location of economic activity. The current paper investigates the effects of progressive, proportional, and regressive corporate taxation on industry agglomeration.

Most countries have a mixture of progressive and regressive taxes, both because different principles of fairness are applied to different taxes and because of different levels of government (Graddy et al., 2008). Porcano (1986) argues that although the U.S. corporate income tax rate essentially is defined to be progressive at low income levels (up to the first USD 335,000 of taxable income), and proportional on taxable income for large firms (producing a flat tax rate of 35 percent on a corporation’s entire taxable income once it exceeds USD 18.33 million), effectively it is regressive and very low.1

In fact, local governments generally do not make much effort to apply progressive taxation principles. The reason is that they are subject to tax competition—a local government that tries to impose high taxes on capital might find industry moving to other locations where taxes are lower. Clearly, this is also a concern at the national level in a globalized economy, since the international capital can easily migrate to where it is treated more favorably.2

Accordingly, standard principles of international taxation suggest that the tax burden should fall most heavily on those factors of production which are least mobile, in order to maximize government income and minimize the disincentives to economic growth. Hence, there has been a corresponding shift in the incidence of taxation from capital to labor as governments have tried to maintain levels of both fiscal revenue and private investment. As a result, it is not surprising that the main conclusion of the basic tax competition models states that corporate tax rates will be inefficiently low, leading

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1The Baltic countries apply an explicit regressive corporate tax rule.
2The corporate tax in the U.S. is imposed both by the federal government and by most state governments. Germany’s corporate tax is a joint federal-state tax; the surtax is a federal tax, and the trade tax is a municipal tax.
to an underprovision of public goods.\(^3\) This happens because capital or firms respond immediately to tax differentials and governments engage in race-to-the-bottom tax competition—corporate taxation is competed downwards. According to the theoretical predictions of the tax competition literature, a high and increasing degree of economic integration should lead to severe pressures on local and national tax policies. Moreover, corporate taxation should become increasingly more important as a determinant of industry location as the extent of economic integration increases, since capital becomes more sensitive-elastic to tax differentials as trade is liberalized.

However, recent applications of new economic geography models to tax competition have modified the neoclassical race-to-the-bottom result. In case of full agglomeration, capital might earn a premium that can be taxed by local authorities without any distortions. This agglomeration rent makes the mobile factor quasi-inelastic over a certain range of trade costs, so that tax rates on capital income might rise in proportion to deepened market integration.\(^4\)

The implicit assumption in this literature is that the average tax rate is independent of the tax base, that is, taxes are taken to be proportional. Hence, these studies do not take into account aspects of tax structure on the economic geography; for example, how agglomeration rents might vary with the specific tax regime. It is therefore crucial to extend the current literature on corporate taxation and industrial location by considering widely adopted alternative tax schedules other than the proportional scheme.

This paper analyzes the effects of different corporate tax systems on the location of industry. Specifically, the paper introduces progressive, proportional, and regressive taxation into a modified version of the economic geography model of Martin and Rogers (1995). In that respect, the paper closest to the present is that of Commedatore and Kubin (2008), who study the choice between taxing capital according to the residence and source principle within the Martin and Rogers (1995) setting.

The analysis shows that the relatively larger region (the North) prefers a regressive tax system for a given shift in regional expenditures, while the smaller region (the South) benefits from adopting progressive taxation. Both regions gain industry by adopting a tax regime that produces the lowest tax

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\(^3\)See for example Wilson (1999) for a review of the theoretical tax competition literature.

\(^4\)Such argument has for example been made by Ludema and Wooton (2000), Kind et al. (2000), and Baldwin and Krugman (2004).
level. As expenditures in the North increase, the North (South) attracts more industry compared to the location equilibrium without tax structure by setting regressive (progressive) taxes. However, welfare in the North (South) is maximized if both regions set regressive (progressive) taxes. The reason is that symmetric regional regressive (progressive) taxation, through lower taxes in the North (South) and higher taxes in the South (North), gives rise to relatively higher (lower) agglomeration economies in the North. This in turn decreases prices in the North (South) and saves the transportation cost facing the North (South) consumers when trade costs have to be paid on a lower number of goods.

Furthermore, as trade is liberalized, and provided that the North’s share of regional expenditures is lower (higher) than a certain threshold value, the North optimally sets regressive (progressive) taxes, while the South prefers to apply a progressive (regressive) tax scheme. In this case, welfare of the North is maximized if the South also sets regressive (progressive) taxes, since this increases the agglomeration economies in the North over and above what is implied by the Nash equilibrium that prescribes different tax systems in the North and the South. A combination of tax schedules that maximizes the location of production in the North (South) is a joint efficient outcome if the relative endowment of labor of the North is higher (lower) than a certain critical value. The reason is that an endowment above (below) this threshold level implies that the North (South) gains (loses) more in terms of total welfare from agglomeration of production than the South (North) loses (gains).

It is also shown that a proportional tax system is never an equilibrium; as to be seen, this tax regime replicates the location outcome without regional tax structures. This is an interesting result considering its widespread international support and usage.

The paper is organized as follows. The next section presents the economic geography framework with regional corporate taxation levels, and solves for the equilibrium location of industry. Section 3 defines the specific structures of the tax policy instrument; determines industry agglomeration under progressive, proportional, and regressive tax systems; derives the tax schedules that prevail in equilibrium, and those that maximize regional and joint welfare. Section 4 concludes.
2 The Economic Geography Model


2.1 Assumptions

There are two regions, two sectors and two factors. Specifically, the two regions, North (N) and South (S), belong to the same country—or federation of countries—and are endowed with two factors, labor (L) and capital (K). The regions are symmetric in terms of tastes, technology, openness to trade, but are separate tax jurisdictions and differ in their factor endowments; the North is a scaled-up version of the South. Thus the regions may be of different size, but they have identical capital-labor ratios. In particular, the North’s endowment of both capital and labor is $\lambda > 1$ times the South’s endowment.

The two sectors are referred to as agriculture (A) and manufacturing or industry (M). The agricultural sector is assumed to produce a homogenous good under constant returns to scale and perfect competition using $a_A$ units of labor per unit of output. Labor is the only input and this good is chosen as a numeraire. The manufacturing sector uses both labor and capital to produce a differentiated good under increasing returns to scale and monopolistic competition. Following Flam and Helpman (1987), the production of each differentiated good involves a one-time fixed cost consisting of one unit of $K$ and a per-unit-of-output cost of $a_M$ units of $L$. The implied cost function of each industrial firm is therefore given by:

$$\pi + wa_Mx,$$  \hspace{1cm} (2.1)

where $\pi$ and $w$ are the reward to capital and labor, and $x$ is the firm level output.

Physical capital can move between regions but capital owners are immobile. Thus, when pressures arise to concentrate production to one region, capital will move, but its entire reward will be repatriated to its region of origin. Labor, on the other hand, can move freely between sectors but is immobile between regions. Total supply of capital and labor in the economy is fixed, with the nation’s endowments denoted by $K_W$ and $L_W$. Since each industrial variety requires one unit of capital, the share of the nation’s
capital stock employed in a single region exactly equals the region’s share of the national manufacturing sector. Consequently, the North’s share of industry can be used, i.e., $s_N \equiv \frac{n_N}{n_N + n_S}$, where $n_N$ and $n_S$ denote the number of industrial firms in the North and the South, respectively, to represent the share of capital employed in the North, and the share of all varieties made in the North. Then clearly, $s_S = 1 - s_N \equiv \frac{n_S}{n_N + n_S}$ is the South’s share of the national manufacturing sector.

Output in the agricultural sector is traded at no cost, while interregional trade in the differentiated output is subject to an iceberg transportation cost. Hence, in order to sell one unit of the differentiated good in the other region, $\tau > 1$ units need to be shipped.

The representative consumer in each region has preferences according to:

$$U = C_M^{\mu} C_A^{1-\mu} + U(C_G),$$

(2.2)

where $\mu \in (0, 1)$, $C_A$ is consumption of the homogenous good, and $U(C_G)$ is a public utility component derived from public expenditures. Consumption of manufactures enters the utility function through the index $C_M$, which is defined by:

$$C_M \equiv \left( \int_0^{n_W} c_i^{\frac{\sigma-1}{\sigma}} d\bar{i} \right)^{\frac{\sigma}{\sigma-1}},$$

(2.3)

where $n_W = n_N + n_S$ denotes the large number of industrial varieties consumed, fixed by the nation’s total supply of capital, $c_i$ is the amount of variety $i$ consumed, and $\sigma > 1$ is the constant elasticity of substitution between any two varieties. The utility derived from public expenditures, $U(C_G)$, is not explicitly specified; it depends on the quantity consumed of a publicly provided good, $C_G$, and has a positive first and a negative second derivative. The government in region $j = N, S$, publicly provides this private good. Following Commedatore and Kubin (2008), in total $L_j C_G$ units of the good have to be produced in region $j$. The production function is:

$$L_j C_G = C_{MG}^{\nu} C_{AG}^{1-\nu},$$

(2.4)

where $C_{AG}$ represents the quantity of the agricultural good, and $C_{MG}$ denotes the quantity index of the manufactured good (again based on a substitution elasticity of $\sigma > 1$). This specification implies that the shares of government
revenues devoted to the manufacturing composite and to the agricultural commodity do not differ from the private shares given in (2.2).5

Public expenditures are financed by corporate income taxes, and the government’s budget is always balanced. Assume that corporate profits are taxed according to the source principle, that is, capital is taxed according to where firms are located.6 Further, let \( s_{Nj} (s_{Sj}) \) denote the share of capital employed in region \( j \), owned by the representative individual residing in the North (South), where \( s_{NN} + s_{SN} = 1 \) and \( s_{SS} + s_{NS} = 1 \).

Total expenditures (private and public) on manufacturing goods in region \( j \), \( E_j \), equal the share of disposable income devoted to manufactures:

\[
E_N = \mu \left( w_N L_N + \pi_N K_N - K_W \sum_j t_j s_{Nj} \pi_j \right) + \mu TR_N(t_N), \quad (2.5)
\]

and

\[
E_S = \mu \left( w_S L_S + \pi_S K_S - K_W \sum_j t_j s_{Sj} \pi_j \right) + \mu TR_S(t_S), \quad (2.6)
\]

where \( t_j \in [0, 1] \) is the average tax levied on firms or capital located in region \( j \) and paid by the representative capital owner residing in \( j \). \( K_W \sum_j t_j s_{Nj} \pi_j \) (\( K_W \sum_j t_j s_{Sj} \pi_j \)) is the total tax payments of citizens belonging to region \( N \) (\( S \)) who own capital employed in both regions \( N \) and \( S \). \( TR_N(t_N) = K_W t_N \pi_N (TR_S(t_S) = K_W t_S \pi_S) \) denotes the tax revenues of region \( N \) (\( S \));\(^7\) i.e., the public expenditure component, since the government’s budget is always balanced.

### 2.2 The Economic Equilibrium

The unit factor requirement of the homogenous good is one unit of labor \((a_A = 1)\). This good is freely traded and since it is also chosen as a numeraire,

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5This assumption is made for algebraic convenience and does not affect the results. The reason is that government revenues do not enter the location condition, that capital employed in the North must earn the same after-tax rate of return as capital in the South, and therefore do not affect the location equilibrium.

6All results hold under the assumption of taxation according to the residence principle, since this does not change the structure of the location condition.

7\( TR_N(t_N) = K_W t_N \pi_N (s_{NN} + s_{SN}) = K_W t_N \pi_N \) and \( TR_S(t_S) = K_W t_S \pi_S (s_{SS} + s_{NS}) = K_W t_S \pi_S \).
$p_A = w = 1$ in both regions.

Maximizing (2.2), subject to (2.3) and the budget constraint, to obtain the (private and public) demand function in region $j$ for variety $i$ of the differentiated good:

$$c_i = \frac{p_i^{-\sigma}E_j}{\int_{k=0}^{n_{ij}} p_{jk}^{1-\sigma} dk}, \quad (2.7)$$

where $p_i$ is the price of variety $i$. Profit maximization yields:  

$$p = \frac{\sigma}{\sigma - 1} wa_M, \quad (2.8)$$

and

$$p^* = \frac{\sigma}{\sigma - 1} \tau wa_M, \quad (2.9)$$

for each differentiated commodity sold in the home and export market, respectively. Without any loss of generality, let $a_M = \frac{\sigma - 1}{\sigma}$, then using $w = 1$ to obtain the pricing rules for firms in the manufacturing sector: $p = 1$ and $p^* = \tau$.

Since physical capital is used only in the fixed cost component of industrial production, the reward to capital is the Ricardian surplus of a typical variety, i.e., the operating profit of a variety. With a fixed capital stock and free entry, the reward to capital will be bid up to the point where the entire operating profit goes to capital. Under Dixit-Stiglitz competition, the operating profit is the value of sales divided by $\sigma$; that is, $\pi_j = \frac{x_j}{\sigma}$, where $x_j$ is the scale of production of a representative industrial firm in region $j$.  

National expenditures (private and public) on manufacturing goods, $E_{W}$, are written as:

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8Differentiating the profit function, $\Pi_i = p_i c_i(p_i) + p_i^* c_i^*(p_i^*) - \pi(1 + t) - wa_M (c_i(p_i) + c_i^*(p_i^*) \tau)$, where $c_i^*(p_i^*)$ is the foreign demand for industry $i$ products, with respect to $p_i$ and $p_i^*$, using $\sigma = \frac{\partial c_i(p_i)}{\partial p_i} \equiv \frac{\partial c_i^*(p_i^*)}{\partial p_i^*}$, solving for the optimal prices gives (2.8) and (2.9).

9This assumption is made for algebraic simplicity and does not affect any of the results, since $a_M$ does not enter the location condition. $a_M = \frac{\sigma - 1}{\sigma} < 1$, because $\sigma > 1$, implies that less than one unit of labor is needed to produce one unit of the differentiated good.

10Free entrance implies that pure profits are eliminated. Employing the profit function from note 8 for a typical manufacturing firm in region $j$, $w = 1$, $a_M = \frac{\sigma - 1}{\sigma}$, $p = 1$, $p^* = \tau$, and $x_j \equiv c_j + c_j^* \tau$ gives $\pi_j = \frac{x_j}{\sigma}$.  

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8
\[ E_W = \mu \left( L_W + \pi_W K_W - K_W \sum_j t_j \pi_j \right) + \mu \sum_j TR_j(t_j), \quad (2.10) \]

where \( \pi_W = \frac{x_W}{\sigma} = \frac{E_W}{K_W \sigma} \) (the corresponding expression in Baldwin et al. [2003], without taxation, is \( \mu E_W \)). Using this condition in (2.10), and \( K_W \sum_j t_j \pi_j = \sum_j TR_j(t_j) \),\(^{11}\) yields \( E_W = \mu \left( L_W + \frac{E_W}{\sigma} \right) \), which implies that:\(^{12}\)

\[ E_W = \frac{\sigma}{\sigma - \mu} L_W. \quad (2.11) \]

Combining \( s_{E_j} \equiv \frac{E_j}{E_W} \), where \( s_{E_j} \) represents the share of national expenditures of region \( j \), and \( s_{EN} = 1 - s_{ES} \), with (2.11) solves for the expenditures in region \( j \):

\[ E_j = s_{E_j} \frac{\sigma}{\sigma - \mu} L_W. \quad (2.12) \]

The domestic and foreign demand function for industry \( i \) products, and the optimal prices give the reward to capital or the operating profit in equilibrium:\(^{13}\)

\[ \pi_N = \frac{x_N}{\sigma} = \frac{\mu}{\sigma} \left( \frac{E_N}{n_N + \phi n_S} + \frac{\phi E_S}{\phi n_N + n_S} \right), \quad (2.13) \]

and

\[ \pi_S = \frac{x_S}{\sigma} = \frac{\mu}{\sigma} \left( \frac{\phi E_N}{n_N + \phi n_S} + \frac{E_S}{\phi n_N + n_S} \right). \quad (2.14) \]

\(^{11}\)From (2.5) and (2.6) it follows that \( K_W t_N s_N \pi_N + K_W t_S s_N \pi_S + K_W t_N s_S \pi_N \) and \( K_W t_S s_S \pi_S + K_W t_S t_S \pi_S(s_S + s_N) = K_W \sum_j t_j \pi_j \). Moreover, it can also be deduced from note 7 that \( \sum_j TR_j(t_j) = K_W t_N \pi_N(s_N + s_N) + K_W t_S \pi_S(s_S + s_N) = K_W \sum_j t_j \pi_j \). Accordingly, total tax payments and tax revenues in (2.10) cancel out, and therefore do not affect the location equilibrium.

\(^{12}\)With the residence principle, the total tax burden on capital owners residing in region \( j \) is identical to the tax revenues of \( j \). Hence, in this case, \( E_W = \mu \left( L_W + \frac{E_W}{\sigma} \right) \) and consequently (2.11) still apply.

\(^{13}\)Using the expressions for the demand functions in \( \frac{x}{\sigma} \equiv \frac{c}{\sigma} + \frac{c^*}{\sigma} \), and \( p = 1, p^* = \tau \) yields (2.13) and (2.14).
$\tau^{1-\sigma} \equiv \phi \in [0, 1]$ is a measure of the freeness of interregional trade, where 0 corresponds to infinite trade barriers and 1 represents free trade. Substituting (2.12) into (2.13) and (2.14) to obtain:

$$\pi_N = a\mu L_W \left( \frac{s_{EN}}{s_N + \phi(1 - s_N)} + \frac{\phi(1 - s_{EN})}{\phi s_N + (1 - s_N)} \right), \quad (2.15)$$

and

$$\pi_S = a\mu L_W \left( \frac{\phi s_{EN}}{s_N + \phi(1 - s_N)} + \frac{(1 - s_{EN})}{\phi s_N + (1 - s_N)} \right), \quad (2.16)$$

where $a \equiv \frac{\mu}{\sigma - \mu}$.

With perfect capital mobility and when manufacturing production takes place in both regions, the location condition requires that capital employed in the North must earn the same after-tax rate of return as capital in the South: $\pi_N (1 - t_N) = \pi_S (1 - t_S)$. Given (2.15) and (2.16), the distribution of industry solving this condition is:

$$s_N = \frac{s_{EN} (1 - \phi^2) - \phi(v - \phi)}{(1 - \phi)(v - \phi - s_{EN}(v - 1)(1 + \phi))}, \quad (2.17)$$

where $v = \frac{1-t_N}{1-t_S}$ and $v \in \mathbb{R}_+ \forall t_j \in [0, 1)$. Clearly, $0 < v < 1$ holds when capital in the South is taxed at a higher rate than capital in the North, while $v > 1$ is fulfilled when the North applies a relatively higher corporate tax rate. Let $v = 1$ (i.e., no regional tax rate difference) and differentiate (2.17) with respect to $s_{EN}$:

$$\left. \frac{\partial s_N}{\partial s_{EN}} \right|_{v=1} = \frac{1 + \phi}{1 - \phi} > 1, \quad (2.18)$$

which demonstrates the home market effect (Krugman, 1980). Thus $s_N$ increases more than proportionate to $s_{EN}$ for $\phi \in (0, 1)$, and this effect becomes stronger as trade barriers are reduced (so-called home market magnification, due to Krugman [1991]). This means that even if one region is just slightly larger than the other, it will obtain the entire manufacturing industry if transaction costs are sufficiently low.

To illustrate the effect of regional corporate taxes on the geographical equilibrium, differentiate (2.17) with respect to $v$:

$$\frac{\partial s_N}{\partial v} = -\frac{s_{EN}(1 - s_{EN})(1 + \phi)^2}{(\phi - v + s_{EN}(v - 1)(1 + \phi))^2} < 0, \quad (2.19)$$
for \( s_{EN} \in (0, 1) \). (2.19) implies that the location of manufacturing activities in the North is decreasing (increasing) in the corporate tax rate of the North (South). By differentiating (2.19) with respect to \( \phi \) and \( s_{EN} \) it is clear that the sign of the derivatives is ambiguous and depends on the relative sizes of \( s_{EN}, \phi \) and \( v \). Within this framework, like in Commedatore and Kubin (2008), there are no agglomeration rents that the larger region can tax away: holding everything else constant, a lower tax rate in region \( j \) increases the share of industry in \( j \).

The welfare of a representative individual is a function of the income devoted to manufactures and the price index prevailing in the region of residence. Given (2.2), the indirect utility functions are (see Appendix A.1 for details):

\[
V_N(v) = \ln \mu + \ln(1 + \mu a) + \frac{\mu}{\sigma - 1} \ln (s_N + (1 - s_N)\phi), \tag{2.20}
\]

\[
V_S(v) = \ln \mu + \ln(1 + \mu a) + \frac{\mu}{\sigma - 1} \ln (s_N\phi + (1 - s_N)), \tag{2.21}
\]

for an agent in the North and the South, respectively. \( V_N > V_S \) if and only if \( s_N > \frac{1}{2} \).\(^{14}\) Moreover, it is straightforward to verify that \( \frac{\partial V_N}{\partial s_N} > 0 \) and \( \frac{\partial V_S}{\partial s_N} < 0 \). Consequently, individual welfare increases in the number of firms located in the agent’s region, since this leads to a decrease in consumer prices of manufactures and saves the transportation cost facing the consumer when trade costs have to paid on a lower number of goods. The total welfare of region \( j \) can be expressed as \( W_j = L_j V_j \). This means that \( \left| \frac{\partial W_N}{\partial s_N} / \frac{\partial W_S}{\partial s_N} \right| \leq 1 \) if \( \lambda \leq \frac{s_N + (1-s_N)\phi}{s_N\phi + (1-s_N)} > 1 \), where \( \lambda = \frac{L_N}{L_S} > 1 \) since the North has relatively more labor (and capital).\(^{15}\) Thus, if the relative number of citizens in the North is lower (higher) than this threshold level, the South (North) loses (gains) more in terms of total welfare than the North (South) gains (loses) from agglomeration in the North.

It can further be established that \( \frac{\partial V_N}{\partial \nu} < 0 \) and \( \frac{\partial V_S}{\partial \nu} > 0 \). Hence, increasing the corporate tax rate of the North (South) decreases individual welfare in

\[\frac{\partial W_N}{\partial s_N} / \frac{\partial W_S}{\partial s_N} \leq 1 \text{ if } \lambda \leq \frac{s_N + (1-s_N)\phi}{s_N\phi + (1-s_N)} \]

\[\frac{\partial W_N}{\partial s_N} / \frac{\partial W_S}{\partial s_N} = \frac{s_N + (1-s_N)\phi}{s_N\phi + (1-s_N)} > 0 \text{ if } s_N + (1-s_N)\phi > s_N\phi + (1-s_N), \text{ i.e., if } s_N > \frac{1}{2}.
\]

\[\frac{\partial W_N}{\partial s_N} \leq 1 \text{ if } \lambda \leq \frac{s_N + (1-s_N)\phi}{s_N\phi + (1-s_N)} \text{, where } \frac{s_N + (1-s_N)\phi}{s_N\phi + (1-s_N)} > 1 \text{ if } s_N > \frac{1}{2}.
\]
the North (South). The reason is that a higher tax rate in region $j$ decreases the number of industrial firms in $j$, which raises the price index in this region.

With the geography model specified, it is now time to analyze how different corporate tax structures affect the location equilibrium.

3 The Tax Policy Equilibrium

Thus far, no assumption has been made on the structure of the regional tax policy instrument. In the following, industry agglomeration will be determined under progressive, proportional, and regressive corporate tax structures, where the structure itself is taken to be exogenously given. This makes it possible to derive the tax systems that prevail in equilibrium, and those that maximize regional and joint welfare.

3.1 Assumptions

Assuming that the corporate income tax function, $t_j(\pi_j)$, is continuous in $\pi_j$, the tax system is progressive (regressive) if $t_j'(\pi_j) > 0$ ($t_j'(\pi_j) < 0$). In addition, given two tax functions, $t_{1j}(\pi_j)$ and $t_{2j}(\pi_j)$, such that $t_{1j}'(\pi_j) > t_{2j}'(\pi_j) > 0$ ($t_{1j}'(\pi_j) < t_{2j}'(\pi_j) < 0$), the tax structure implied by $t_{1j}(\pi_j)$ is interpreted as more progressive (regressive) than that implied by $t_{2j}(\pi_j)$.$^{16}$

Under the proportional tax system, on the other hand, the average tax rate remains constant regardless of the profit level; hence, $t_j(\pi_j) = t_j$ and $t'_j = 0$.

Denote the progressive, proportional, and regressive tax schedule of region $j$ by $t_{P,j}(\pi_j)$, $t_{PL,j}$, $t_{R,j}(\pi_j)$, respectively, and let $\omega \equiv \frac{t_{PS}'}{t_{SN}}$ for any tax system $i = P, PL, R$.

Consider then the following two-stage game. In the first stage, firms relocate between regions in response to the level of economic integration ($\phi$) and the relative shares of total expenditures ($s_{EN}$), i.e., the relative economic strength of the North. In the second and final stage, $\pi_j$ is realized, taxes, $t_j(\pi_j)$, are exogenously set, and $v = \frac{1-t_S}{1-t_N}$ is determined. A subgame perfect

$^{16}$In reality, taxes may be locally progressive over some ranges of income and regressive over others. The degree to which such taxes could then be deemed to be globally progressive or regressive over the whole domain of the income distribution needs to depend on the relative weights respectively attached to local progressiveness or regressiveness over the various income ranges. For simplicity, corporate taxes are assumed to be globally progressive or regressive.
equilibrium of this game is solved by backward induction. In the final stage, the distribution of industry, \( s_N \), is exogenously given by the equilibrium in the first stage; that is, profits are a function \( \pi_j(s_{EN}, \phi, s_N) \), where \( s_N \) is exogenous.

From the perspective of the first stage, and through backward induction, firms in region \( j \) must take the second-stage equilibrium into account when relocation decisions are made. Hence, the first-stage equilibrium is affected by the realization of \( \nu \). Consequently, (2.17) can be reproduced as:

\[
s_N = f(s_{EN}, \phi, \nu), \tag{3.1}
\]

where \( \nu = f(t_N(\pi_N), t_S(\pi_S)) \), \( \nu = f(t_N, t_S(\pi_S)) \), \( \nu = f(t_N(\pi_N), t_S) \) or \( \nu = f(t_N, t_S) \) depending on the tax structures.

### 3.2 Tax Structure and Increased Spending

Taking the total derivative of (3.1) with respect to \( s_{EN} \) to obtain:

\[
\frac{\partial s_N}{\partial s_{EN}} = \frac{\partial f}{\partial s_{EN}} + \frac{\partial f}{\partial \nu} \left( \frac{\partial \nu}{\partial t_N} \frac{\partial \pi_N}{\partial s_{EN}} + \frac{\partial \nu}{\partial t_S} \frac{\partial \pi_S}{\partial s_{EN}} \right), \tag{3.2}
\]

where \( \frac{\partial f}{\partial s_{EN}} > 0 \) if firms relocate to the larger region as the share of national expenditures of this region increases (which is assumed throughout the rest of the paper), \( \frac{\partial f}{\partial \nu} < 0 \) by (2.19), \( \frac{\partial \nu}{\partial t_N} = \frac{1-t_S}{(t_N-1)} > 0 \), \( \frac{\partial \nu}{\partial t_S} = -\frac{1}{1-t_N} < 0 \), \( \frac{\partial \pi_N}{\partial s_{EN}} = a \mu L \left( \frac{1}{s_N + \phi(1-s_N)} - \frac{\phi}{\phi s_N + (1-s_N)} \right) > 0 \) and \( \frac{\partial \pi_S}{\partial s_{EN}} = a \mu L \left( \frac{1}{s_N + \phi(1-s_N)} - \frac{\phi}{\phi s_N + (1-s_N)} \right) < 0 \). Thus the sign of the second term on the right-hand side of (3.2) is solely determined by the sign of \( \frac{\partial \nu}{\partial t_N} \equiv t'_N(\pi_N) \) and \( \frac{\partial \nu}{\partial \pi_N} \equiv t'_N(\pi_S) \); i.e., whether the tax system in region \( j \) is progressive, proportional or regressive. It can be shown that

\[
\frac{\partial \nu}{\partial t_N} < 0, \quad \frac{\partial \nu}{\partial t_S} < 0, \quad \frac{\partial \nu}{\partial \pi_N} < 0, \quad \frac{\partial \nu}{\partial \pi_S} < 0.
\]

**Progressive Taxation in the North and the South**

Assuming that both regions apply progressive taxation, noting that \( t'_{P,j}(\pi_j) > 0 \), the second term on the right-hand side of (3.2) becomes negative, and therefore \( \frac{\partial s_N}{\partial s_{EN}} < \frac{\partial f}{\partial s_{EN}} \). Hence, introducing symmetric progressive taxation

\[
\frac{\partial \nu}{\partial t_N} \frac{\partial \pi_N}{\partial s_{EN}} + \frac{\partial \nu}{\partial t_S} \frac{\partial \pi_S}{\partial s_{EN}} = a \mu L \left( 1-\phi^2 \right) \left( t'_N(\pi_N)(1-t_N)s_N + t'_N(\pi_N)(1-t_S)(1-s_N) \right) \geq 0 \text{ if } t'_S(\pi_S)(1-t_N)s_N + t'_N(\pi_N)(1-t_S)(1-s_N) \geq 0; \text{ that is, if } t'_S(\pi_S)s_N + t'_N(\pi_N)(1-s_N)u \geq 0.
\]
slows down the agglomeration process in the North, as compared to the location equilibrium without corporate tax structure. On the one hand, the equilibrium location of manufacturing production in the North is increasing in its share of national expenditures; this direct effect is captured by the first term, \( \frac{\partial f}{\partial s_{EN}} \), on the right-hand side of (3.2) and represents the agglomeration effect without corporate tax systems. On the other hand, as \( s_{EN} \) increases, \( \pi_N \) increases while \( \pi_S \) decreases. This increases \( t_N \) and decreases \( t_S \) with tax progression in both regions, in turn increasing \( v \), thereby making it relatively less beneficial to produce in the larger region. Consequently, \( s_N \) decreases by (2.19), so the agglomeration of firms in the North slows down compared to the equilibrium without corporate tax structure. This indirect location effect is associated with the second term on the right-hand side of (3.2), and captures the agglomeration effect from the regions’ tax regimes.

**Regressive Taxation in the North and the South**

Consider instead the case where both regions set regressive taxes: \( t'_{R,j}(\pi_j) < 0 \). Under this combination of tax regimes, the second term on the right-hand side of (3.2) becomes positive, implying higher agglomeration economies in the larger region compared to the location equilibrium without corporate tax structure, i.e., \( \frac{\partial s_N}{\partial s_{EN}} > \frac{\partial f}{\partial s_{EN}} \). As the North’s share of total expenditures increases, \( \pi_N \) increases and \( \pi_S \) decreases. With regressive taxation, \( t_N \) decreases while \( t_S \) increases, which decreases \( v \), thus increasing \( s_N \) by (2.19). In other words, higher expenditures in the North, via a higher profit of the representative North firm and a lower profit of the typical South firm, increases the relative corporate taxation in the South under a symmetric regressive tax system. This increases the agglomeration of firms in the North over and above what is captured by the term \( \frac{\partial f}{\partial s_{EN}} \).

**Proportional Taxation in the North and the South**

In contrast, if both regions apply a proportional tax system, where \( t'_{PL,j} = 0 \), the effects of expenditures on the location equilibrium, through profits and taxation, are eliminated. Accordingly, in this case, equation (3.2) reduces to \( \frac{\partial s_N}{\partial s_{EN}} = \frac{\partial f}{\partial s_{EN}} \), i.e., the location equilibrium without tax structure.
Progressive Taxation in the North and Regressive Taxation in the South

Suppose that the regions are subject to different tax regimes, where initially the North has a progressive system, i.e., \( t'_{P,N}(\pi_N) > 0 \), while the South applies a regressive scheme, that is, \( t'_{R,S}(\pi_S) < 0 \). Then the second term on the right-hand side of (3.2) becomes positive, implying that \( \frac{\partial s_N}{\partial s_{EN}} > \frac{\partial f}{\partial s_{EN}} \), if \( t'_{R,S}(\pi_S)s_N + t'_{P,N}(\pi_N)(1 - s_N)\nu < 0 \); in other words, if \( \frac{s_N}{1 - s_N} > -\frac{\nu}{\omega} \). The intuition behind this inequality can be explained in the following way. First, for a low tax progression in the North and a high tax regressiveness in the South—i.e., for a high absolute value of \( \omega \)—the progressive tax rate in the North increases by a relatively smaller factor in response to a higher \( \pi_N \) when \( s_{EN} \) increases compared to the increase in \( t'_{R,S}(\pi_S) \) following from a lower \( \pi_S \). This increases the relative incentives to relocate production to the North.

Second, the location effect of increased taxation in the North is diminished by a higher \( s_N \), that is, as the market-access advantage of producing in the larger region increases. For higher \( s_N \), the rate decreases at which profits in the North increase relative to the fall in profits in the South, following higher spending. This further reduces the increase in the North’s tax rate, and makes it even more beneficial to relocate production to the North. To see this, note that \( \frac{\partial \pi_N}{\partial s_{EN}} / \frac{\partial \pi_S}{\partial s_{EN}} = -\frac{1 - s_N}{s_N} \omega \), which decreases in absolute value as \( s_N \) increases.

Third, the location effect of higher taxes in the North is also reduced by a low \( \nu \). For lower \( t_{P,N}(\pi_N) \) and higher \( t_{R,S}(\pi_S) \) (a low \( \nu \)), higher taxes in the North affect \( \nu \) and ultimately \( s_N \) at a relatively slower rate. This can be seen by observing that \( \frac{\partial \nu}{\partial t_{P,N}(\pi_N)} / \frac{\partial \nu}{\partial t_{R,S}(\pi_S)} = -\frac{1 - t_{R,S}(\pi_S)}{1 - t_{P,N}(\pi_N)} = -\nu \), which decreases in absolute value when \( t_{P,N}(\pi_N) \) is lower and \( t_{R,S}(\pi_S) \) higher. Therefore, \( \frac{\partial s_N}{\partial s_{EN}} > \frac{\partial f}{\partial s_{EN}} \) is satisfied for a high absolute value of \( \omega \), high \( s_N \), and low \( \nu \).

Regressive Taxation in the North and Progressive Taxation in the South

Assume instead that the North has a regressive tax structure (where \( t'_{R,N}(\pi_N) < 0 \)) and the South has a progressive system (that is, \( t'_{P,S}(\pi_S) > 0 \)). Then \( \frac{\partial s_N}{\partial s_{EN}} > \frac{\partial f}{\partial s_{EN}} \) if \( \frac{s_N}{1 - s_N} < -\frac{\nu}{\omega} \). That is, the North attracts relatively more production compared to the location equilibrium without tax structure for low \( s_N \), high \( \nu \) and low \( |\omega| \). As intuition might suggest, for a highly regressive
taxation structure in the North and a low progressiveness in the South (i.e., for low $|\omega|$), the regressive tax rate in the North decreases by a large amount when $\pi_N$ increases in response to a higher $s_{EN}$, while the progressive tax rate of the South falls relatively less as $\pi_S$ decreases following higher $s_{EN}$. This induces firms to relocate from the South to the North at a higher rate than what is captured by the term $\frac{\partial f}{\partial s_{EN}}$.

The location effect is reinforced by the relative regional size ($s_N$). An equilibrium with a less capital-abundant North (low $s_N$) increases the rate at which the profit of the representative Northern firm increases relative to the decrease in the operating profit of the typical Southern firm, as a result of the shift in $s_{EN}$. This accelerates the relative fall in the North’s tax rate, and increases the relocation from the South to the North. Again, this is seen by noting that $\frac{\partial \pi_N}{\partial s_{EN}} / \frac{\partial \pi_S}{\partial s_{EN}} = -\frac{1-s_N}{s_N}$, which increases in absolute value as $s_N$ decreases.

Moreover, the location effect is also strengthened by a relatively higher taxation in the North (high $\nu$). Intuitively, a higher $t_{R,N}(\pi_N)$ relative to $t_{P,S}(\pi_S)$ increases the rate at which lower taxes in the North affect $\nu$ and ultimately $s_N$, compared to the effect of taxation in the South on $\nu$ and $s_N$. This is evident since $\frac{\partial \nu}{\partial R,N(\pi_N)} / \frac{\partial \nu}{\partial P,S(\pi_S)} = -\frac{1-t_{P,S}(\pi_S)}{1-t_{R,N}(\pi_N)} = -\nu$, which increases in absolute value when $t_{R,N}(\pi_N)$ increases relative to $t_{P,S}(\pi_S)$.

**Proportional Taxation in the North and Progressive Taxation in the South**

Consider then the case where the North applies a proportional tax system (i.e., $t'_{PL,N} = 0$) and the South sets progressive taxes (where $t'_{P,S}(\pi_S) > 0$). Clearly, the second term on the right-hand side of (3.2) reduces to $\frac{\partial f}{\partial \nu} \frac{\partial \nu}{\partial s_{EN}} < 0$, which implies that $\frac{\partial \nu}{\partial s_{EN}} < 0$. Since $t'_{PL,N} = 0$, the effect of expenditures on the location equilibrium, channeled through Northern profits and taxation, is eliminated. By contrast, increasing $s_{EN}$ decreases the operating profit of the representative Southern firm, which in turn, with tax progression, decreases the corporate tax rate levied on firms located in the smaller region. As a result, firms will relocate to the North at a relatively slower rate as $s_{EN}$ increases.
Proportional Taxation in the North and Regressive Taxation in the South

Again, let the North set proportional taxes \( t'_{PL,N} = 0 \), while the South levies regressive taxes \( t'_{R,S}(\pi_S) < 0 \). Thus the second term on the right-hand side of (3.2) becomes \( \frac{\partial f}{\partial \nu} \frac{\partial v}{\partial S} \frac{\partial \pi_S}{\partial \pi_S} > 0 \), and it follows that \( \frac{\partial s_N}{\partial s_{EN}} > \frac{\partial f}{\partial s_{EN}} \). \( t'_{PL,N} = 0 \) eliminates the effect of \( s_{EN} \) on \( s_N \) via \( \pi_N \) and \( t_{PL,N} \). However, as the North’s share of expenditures increases, \( \pi_N \) falls, increasing regressive taxation in the South which increases the number of firms relocating from the South to the North.

Progressive Taxation in the North and Proportional Taxation in the South

Under the assumption of progressive taxation in the North \( t'_{P,N}(\pi_N) > 0 \) and a proportional system in the South \( t'_{PL,S} = 0 \), the second term on the right-hand side of (3.2) equals \( \frac{\partial f}{\partial \nu} \frac{\partial v}{\partial \pi_N} \frac{\partial \pi_N}{\partial s_{EN}} < 0 \), hence \( \frac{\partial s_N}{\partial s_{EN}} < \frac{\partial f}{\partial s_{EN}} \). \( t'_{PL,S} = 0 \) implies that taxation of capital in the South does not affect the location of industry as \( s_{EN} \) increases. Since \( t'_{P,N}(\pi_N) > 0 \), corporate taxes in the North are raised through higher operating profits of Northern firms as \( s_{EN} \) increases. This decreases the agglomeration benefit of producing in the North and slows down the relocation of firms to the larger region, relative to the case without corporate tax systems.

Regressive Taxation in the North and Proportional Taxation in the South

Let the North set regressive taxes (i.e., \( t'_{R,N}(\pi_N) < 0 \)) and the South adopt a proportional tax regime \( t'_{PL,S} = 0 \). Then the second term on the right-hand side of (3.2) becomes \( \frac{\partial f}{\partial \nu} \frac{\partial v}{\partial \pi_N} \frac{\partial \pi_N}{\partial s_{EN}} > 0 \), and \( \frac{\partial s_N}{\partial s_{EN}} > \frac{\partial f}{\partial s_{EN}} \). Proportional taxation in the South does not affect the location of industry as \( s_{EN} \) increases. The regressive tax regime in the North, however, implies that \( t_{R,N}(\pi_N) \) decreases as \( \pi_N \) increases when expenditures of the North rise, leading to higher agglomeration economies in the larger region.
Equilibrium Tax Regimes

It is straightforward to show that the North always attracts more industry by setting a regressive tax schedule, while the South prefers a progressive system. Intuitively, both regions gain industry by applying the tax regime that produces the lowest tax level. As \( s_{EN} \) increases, \( \pi_N \) and \( \pi_S \) increases and decreases, respectively. This decreases both \( t_{R,N}(\pi_N) \) and \( t_{P,S}(\pi_S) \), since \( t_{R,N}'(\pi_N) < 0 \) and \( t_{P,S}'(\pi_S) > 0 \). The lower tax rate in the North (South) in turn gives rise to higher (lower) agglomeration economies in the North, and therefore makes it more (less) profitable to relocate to this region. Hence, the unique Nash equilibrium in pure strategies has the North applying a regressive tax regime and the South choosing a progressive schedule. As noted above, this outcome has \( \frac{\partial s_{EN}}{\partial s_{EN}} \geq \frac{\partial f}{\partial s_{EN}} \) if \( \frac{s_N}{1-s_N} \leq -\frac{\nu}{\omega} \). Thus, the tax policy equilibrium may imply higher as well as lower agglomeration economies compared to the location equilibrium with no tax structure. Which region attracts relatively more industrial production depends on the tax regressiveness of the North relative to the tax progressiveness of the South (\( \omega \)), on the relative regional tax level (\( \nu \)), and on the relative capital abundance of the North (\( s_N \)).

However, the North (South) gains relatively more production if both regions set a regressive (progressive) tax system, than if the North (South) solely applies regressive (progressive) taxation. The reason is that a regressive (progressive) taxation in the South (North) increases (decreases) the agglomeration economies in the North over and above what is implied by the Nash equilibrium. The intuition for this is as follows. As \( s_{EN} \) increases, \( \pi_S \) (\( \pi_N \)) decreases (increases) which, with regressive (progressive) taxation in the South (North), leads to a higher relative tax rate in the South (North) than in the Nash equilibrium. This increases (decreases) the benefit of establishing production in the North, compared to the case where the North (South) solely sets regressive (progressive) taxes. Mathematically, the location effect of both regions applying regressive (progressive) taxation is captured by the second term on the right-hand side of (3.2), which always is positive (negative) when the regions adopt a regressive (progressive) tax system; hence \( \frac{\partial s_{EN}}{\partial s_{EN}} > \frac{\partial f}{\partial s_{EN}} \)

\[
\left( \frac{\partial s_{EN}}{\partial s_{EN}} < \frac{\partial f}{\partial s_{EN}} \right) \text{ for all parameter values. In contrast, in the Nash equilibrium, the second term on the right-hand side is positive (negative) and } \frac{\partial s_{EN}}{\partial s_{EN}} > \frac{\partial f}{\partial s_{EN}}; \text{ if and only if } \frac{s_N}{1-s_N} < -\frac{\nu}{\omega} \left( \frac{s_N}{1-s_N} > -\frac{\nu}{\omega} \right). \text{ If the North (South) }
\]
sets regressive (progressive) taxes, while the South (North) adopts proportional taxation, this term reduces to
\[ \frac{\partial f}{\partial v} \frac{\partial \pi_N}{\partial t} \frac{\partial \pi_N}{\partial s_{EN}} > 0 \left( \frac{\partial f}{\partial v} \frac{\partial \pi_S}{\partial t} \frac{\partial \pi_S}{\partial s_{EN}} < 0 \right), \]
which clearly is lower (larger) than the second term on the right-hand side of (3.2) if both regions would apply regressive (progressive) taxation.

As shown in Subsection 2.2, individual welfare of region \( j \) increases in the number of firms located to \( j \) (i.e., \( \frac{\partial V_N}{\partial s_{EN}} > 0 \) and \( \frac{\partial V_S}{\partial s_{EN}} < 0 \)), since more industrial production leads to a decrease in consumer prices of manufactures, and saves the transportation cost facing the consumers residing in \( j \). Accordingly, as \( s_{EN} \) increases, the unilateral welfare-maximizing tax structure in the North (South) is the regressive (progressive) system. Welfare of the North (South) is, however, maximized if both regions adopt a regressive (progressive) tax structure, since this gives rise to the largest (smallest) agglomeration in the North.

Note that \( \left| \frac{\partial W_N}{\partial s_{EN}} / \frac{\partial W_S}{\partial s_{EN}} \right| \leq 1 \) if \( \lambda \leq \frac{s_N + (1-s_N)\phi}{s_N \phi + (1-s_N)} \). This means that the South (North) loses (gains) more in terms of total welfare than the North (South) gains (loses) from agglomeration in the North, provided that the relative number of citizens in the North is lower (higher) than a critical value. Therefore, absent compensating interregional transfer payments, if the relative endowment of labor of the North is lower (higher) than this threshold, any combination of tax systems that maximizes the location of firms in the smaller (larger) region represents a joint efficient outcome. Hence, progressive (regressive) taxation in both regions represents a joint welfare maximizing outcome, because this maximally increases the location of firms to the smaller (larger) region relative to the equilibrium without tax systems.

The main results so far are summarized in:

**Result 1:** As the share of expenditures in the North increases, the Nash equilibrium has this region adopting a regressive tax system, while the South applies a progressive tax schedule. The unilateral welfare-maximizing tax structure in the North (South) is the regressive (progressive) system, when expenditures in the North increase. Welfare of the North (South) is maximized if both regions set regressive (progressive) taxes.

Consider next how the location of industry is affected by regional integration under the progressive, proportional, and regressive tax systems.
3.3 Tax Structure and Liberalized Trade

Taking the total derivative of (3.1) with respect to \( \phi \) to obtain:

\[
\frac{\partial s_N}{\partial \phi} = \frac{\partial f}{\partial \phi} + \frac{\partial f}{\partial \upsilon} \left( \frac{\partial \nu}{\partial t_N} \frac{\partial \pi_N}{\partial \phi} + \frac{\partial \nu}{\partial t_S} \frac{\partial \pi_S}{\partial \phi} \right),
\]

(3.3)

where \( \frac{\partial f}{\partial \phi} > 0 \) if firms relocate to the larger region as trade is liberalized (which is assumed throughout the rest of the paper), \( \frac{\partial f}{\partial \upsilon} < 0 \) by (2.19), \( \frac{\partial \nu}{\partial t_N} = \frac{1-t_N}{(t_N-1)^2} > 0 \), and \( \frac{\partial \nu}{\partial t_S} = -\frac{1}{1-t_N} < 0 \).\( \frac{\partial \pi_N}{\partial \phi} = -a\mu L_W(1-s_N) \left( \frac{s_{E_N}}{(s_N+\phi(1-s_N))^2} - \frac{1-s_{E_N}}{(\phi s_N+(1-s_N))^2} \right) \geq 0 \) and \( \frac{\partial \pi_S}{\partial \phi} = a\mu L_W s_N \left( \frac{s_{E_N}}{(s_N+\phi(1-s_N))^2} - \frac{1-s_{E_N}}{(\phi s_N+(1-s_N))^2} \right) \leq 0 \) if \( s_{E_N} \leq \frac{(s_N(1-\phi)+\phi)^2}{(s_N(1-\phi)+\phi)^2+(1-s_N(1-\phi))^2} > 1/2.18 \) The relation \( \frac{\partial \pi_N}{\partial \phi} > 0 \) and \( \frac{\partial \pi_S}{\partial \phi} < 0 \) is the main focus of the following analysis; all results are however reported and discussed for the case \( \frac{\partial \pi_N}{\partial \phi} < 0 \) and \( \frac{\partial \pi_S}{\partial \phi} > 0 \). Assuming that \( \frac{\partial \pi_N}{\partial \phi} > 0 \) and \( \frac{\partial \pi_S}{\partial \phi} < 0 \), \( \frac{\partial \nu}{\partial t_N} \frac{\partial \pi_N}{\partial \phi} + \frac{\partial \nu}{\partial t_S} \frac{\partial \pi_S}{\partial \phi} \geq 0 \), that is, \( t_S'(s)(1-t_N)s_N + t_N'(s)(1-t_S)s_N(1-t_N) \geq 0 \).\( \frac{t_S'(s)(s_{E_N})}{t_{i,N}(s_{E_N})} \) for any tax system \( i = P, PL, R \).

Progressive Taxation in the North and the South

Let both regions adopt progressive taxation \( (t_{P,j}(s) > 0) \), which means that the second term on the right-hand side of (3.3) becomes negative, implying \( \frac{\partial s_N}{\partial \phi} < \frac{\partial f}{\partial \phi} \) and lower agglomeration economies in the North compared to the equilibrium without tax structure. The direct effect of trade liberalization is captured by the first term on the right-hand side of (3.3), \( \frac{\partial f}{\partial \phi} \). This term is assumed to be positive, which implies that firms relocate to the North as trade costs are lowered, an effect mediated via the home market magnification effect. The second term on the right-hand side represents the indirect effect of trade liberalization, channeled through profits and taxation. Intuitively, as trade is liberalized, \( \pi_N \) increases while \( \pi_S \) decreases, which increases the progressive corporate tax rate in North and reduces the progressive taxation in the South, making it relatively less profitable to locate to the North. In contrast, if \( \frac{\partial \pi_N}{\partial \phi} < 0 \) and \( \frac{\partial \pi_S}{\partial \phi} > 0 \), the second term
on the right-hand side becomes positive, and consequently $\frac{\partial s_N}{\partial \phi} > \frac{\partial f}{\partial \phi}$, so the agglomeration economies in the North instead increase. Hence, the share of expenditures of the North, that is, how operating profits respond to trade liberalization—$\frac{\partial \pi_N}{\partial \phi} > 0$ and $\frac{\partial \pi_S}{\partial \phi} < 0$ or $\frac{\partial \pi_N}{\partial \phi} < 0$ and $\frac{\partial \pi_S}{\partial \phi} > 0$—is essential to the analysis of how lower trade costs affect taxation and the location of industry.

**Regressive Taxation in the North and the South**

Suppose that the regions set regressive taxes, where $t_{R,j}'(\pi_j) < 0$. Then the second term on the right-hand side of (3.3) becomes positive, and consequently $\frac{\partial s_N}{\partial \phi} > \frac{\partial f}{\partial \phi}$. When $\phi$ increases $t_{R,N}(\pi_N)$ and $t_{R,S}(\pi_S)$ decreases and increases, respectively, via higher $\pi_N$ and lower $\pi_S$. Lower relative regional taxation in the North makes it more beneficial to produce in the larger region, increasing the location of industry in the North over and above what is captured by the term $\frac{\partial f}{\partial \phi}$. On the other hand, given that $\frac{\partial \pi_N}{\partial \phi} < 0$ and $\frac{\partial \pi_S}{\partial \phi} > 0$, the second term on the right-hand side instead becomes negative, and hence $\frac{\partial s_N}{\partial \phi} < \frac{\partial f}{\partial \phi}$ holds.

**Proportional Taxation in the North and the South**

Consider proportional taxes in both regions ($t_{PL,j}' = 0$). In this case, (3.3) reduces to $\frac{\partial s_N}{\partial \phi} = \frac{\partial f}{\partial \phi}$. The reason is that symmetric proportional taxation eliminates the effects of trade liberalization on the location of industry by shutting down the channels through which profits affect the regions’ tax rates. Clearly, this also holds for $\frac{\partial \pi_N}{\partial \phi} < 0$ and $\frac{\partial \pi_S}{\partial \phi} > 0$.

**Progressive Taxation in the North and Regressive Taxation in the South**

Let the regions be subject to different tax regimes. Specifically, let the North apply progressive taxation, $t_{P,N}'(\pi_N) > 0$, while the South sets regressive taxes, $t_{R,S}'(\pi_S) < 0$. Then the sign of the second term on the right-hand side of (3.3) is positive, and consequently $\frac{\partial s_N}{\partial \phi} > \frac{\partial f}{\partial \phi}$, if $t_{R,S}'(\pi_S)s_N + t_{P,N}'(\pi_N)(1 - s_N)v < 0$. Thus $\frac{\partial s_N}{\partial \phi} > \frac{\partial f}{\partial \phi}$ if $\frac{s_N}{1 - s_N} > -\frac{v}{\omega}$. The rationale for this inequality is laid out in Subsection 3.2. Reiterating the arguments developed there: for

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20Note that this holds under the assumption of $\frac{\partial \pi_N}{\partial \phi} > 0$ and $\frac{\partial \pi_S}{\partial \phi} < 0$. Otherwise, if $\frac{\partial \pi_N}{\partial \phi} < 0$ and $\frac{\partial \pi_S}{\partial \phi} > 0$, then $\frac{\partial s_N}{\partial \phi} > \frac{\partial f}{\partial \phi}$ if $\frac{s_N}{1 - s_N} < -\frac{v}{\omega}$.
a low progressiveness in the North, and a highly regressive taxation structure in the South (i.e., for a high $|\omega|$), the tax rate in the North increases by a relatively smaller factor when $\pi_N$ increases in response to a higher $\phi$, compared to the increase in the tax rate of the South as $\pi_S$ decreases when trade costs fall. This induces firms to establish more production in the North than what is captured by $\frac{\partial f}{\partial \phi}$. The effect is reinforced by the relative regional size of the North ($s_N$), since a higher $s_N$ decreases the rate at which profits of North firms increase relative to the decrease in the profits of South firms as trade is liberalized; this slows down the relative increase in the North’s tax rate, and further increases the relocation of firms from the South to the North. This can be seen by noting that $\frac{\partial \pi_N}{\partial \phi} / \frac{\partial \pi_S}{\partial \phi} = -\frac{1-s_N}{s_N}$, which decreases in absolute value as $s_N$ increases. The location effect is also strengthened by a relatively lower taxation in the North (a low $\upsilon$), because a lower $t_{P,N}(\pi_N)$ relative to $t_{R,S}(\pi_S)$ decreases the rate at which higher taxation in the North affects $\upsilon$ and ultimately $s_N$, compared to the effect of higher taxation in the South on $\upsilon$ and $s_N$. Once again, this becomes clear by observing that $\frac{\partial \pi_N}{\partial \phi} / \frac{\partial \pi_S}{\partial \phi} = -\frac{1-t_{R,S}(\pi_S)}{1-t_{P,N}(\pi_N)} = -\frac{1}{\upsilon}$, which decreases in absolute value as $t_{P,N}(\pi_N)$ decreases relative to $t_{R,S}(\pi_S)$. To summarize, $\frac{\partial s_N}{\partial \phi} > \frac{\partial f}{\partial \phi}$ is satisfied for high $|\omega|$, high $s_N$, and low $\upsilon$, provided that $\frac{\partial \pi_N}{\partial \phi} > 0$ and $\frac{\partial \pi_S}{\partial \phi} < 0$ holds.21

Regressive Taxation in the North and Progressive Taxation in the South

Assume that the North adopts a regressive tax structure (i.e., $t'_{R,N}(\pi_N) < 0$), whereas the South sets progressive taxes ($t'_{P,S}(\pi_S) > 0$), leading to the second term on the right-hand side of (3.3) becoming positive, and $\frac{\partial s_N}{\partial \phi} > \frac{\partial f}{\partial \phi}$, if $\frac{s_N}{1-s_N} > -\frac{\upsilon}{\omega}$.22 In other words, the North attracts more production compared to the equilibrium without tax structure if $s_N$ is low, $\upsilon$ is high and $|\omega|$ low. Following the arguments made in Subsection 3.2, when the regressiveness of the tax system in the North increases and the progressiveness of taxes in the South decreases (for low $|\omega|$), $t_{R,N}(\pi_N)$ decreases relatively more when $\pi_N$ increases following a higher $\phi$, compared to the fall in $t_{P,S}(\pi_S)$ as $\pi_S$ decreases when trade costs become lower. The relatively lower tax rate in

\[ \frac{\partial \upsilon}{\partial t_{P,N}(\pi_N)} / \frac{\partial \upsilon}{\partial t_{R,S}(\pi_S)} = -\frac{1-t_{R,S}(\pi_S)}{1-t_{P,N}(\pi_N)} = -\upsilon, \]

which decreases in absolute value as $t_{P,N}(\pi_N)$ decreases relative to $t_{R,S}(\pi_S)$. To summarize, $\frac{\partial s_N}{\partial \phi} > \frac{\partial f}{\partial \phi}$ is satisfied for high $|\omega|$, high $s_N$, and low $\upsilon$, provided that $\frac{\partial \pi_N}{\partial \phi} > 0$ and $\frac{\partial \pi_S}{\partial \phi} < 0$ holds.21

21 If, on the other hand, $\frac{\partial \pi_N}{\partial \phi} < 0$ and $\frac{\partial \pi_S}{\partial \phi} < 0$ is satisfied, then $\frac{\partial \upsilon}{\partial \phi} > \frac{\partial f}{\partial \phi}$ holds for low $|\omega|$, low $s_N$, and high $\upsilon$.

22 This holds under the assumption that $\frac{\partial \pi_N}{\partial \phi} > 0$ and $\frac{\partial \pi_S}{\partial \phi} < 0$. If $\frac{\partial \pi_N}{\partial \phi} < 0$ and $\frac{\partial \pi_S}{\partial \phi} > 0$, then $\frac{\partial \upsilon}{\partial \phi} > \frac{\partial f}{\partial \phi}$ if $\frac{s_N}{1-s_N} > -\frac{\upsilon}{\omega}$. 

22
the North increases the relocation of firms from the South to the North by more than what is implied by the equilibrium without tax systems; hence \( \frac{\partial s_N}{\partial \phi} > \frac{\partial f}{\partial \phi} \). This location effect becomes stronger for smaller \( s_N \), because a less capital-abundant North (low \( s_N \)), increases the rate at which profits of Northern firms increase relative to the decrease in the operating profit of a typical Southern firm in response to a shift in \( \phi \); this accelerates the relative fall in \( t_{R,N}(\pi_N) \) and increases the relocation of production to the North. To see this, note that \( \frac{\partial \pi_N}{\partial \phi} / \frac{\partial \pi_S}{\partial \phi} = -\frac{1-s_N}{s_N} \), which increases in absolute value as \( s_N \) falls. This location effect is also reinforced by a relatively higher taxation in the North (high \( \upsilon \)): higher \( t_{R,N}(\pi_N) \) relative to \( t_{P,S}(\pi_S) \) increases the rate at which taxes in the North affect \( \upsilon \) and ultimately \( s_N \), compared to the effect of taxation in the South on \( \upsilon \) and \( s_N \). As in Subsection 3.2, this can be illustrated by observing that \( \frac{\partial \upsilon}{\partial t_{R,N}(\pi_N)} / \frac{\partial \upsilon}{\partial t_{P,S}(\pi_S)} = -\frac{1-t_{P,S}(\pi_S)}{1-t_{R,N}(\pi_N)} = -\upsilon \), which increases in absolute value when \( t_{R,N}(\pi_N) \) increases relative to \( t_{P,S}(\pi_S) \).

Hence, provided that \( \frac{\partial \pi_N}{\partial \phi} > 0 \) and \( \frac{\partial \pi_S}{\partial \phi} < 0 \) holds, \( \frac{\partial s_N}{\partial \phi} > \frac{\partial f}{\partial \phi} \) is satisfied for low \( |\omega| \), low \( s_N \), and high \( \upsilon \).23

Proportional Taxation in the North and Progressive Taxation in the South

Let the North apply proportional taxation (i.e, \( t'_{PL,N} = 0 \)) and the South set progressive taxes (where \( t'_{P,S}(\pi_S) > 0 \)). This means that the second term on the right-hand side of (3.3) reduces to \( \frac{\partial f}{\partial \upsilon} \frac{\partial \upsilon}{\partial t_{P,S}(\pi_S)} < 0 \) and, as a result, \( \frac{\partial s_N}{\partial \phi} < \frac{\partial f}{\partial \phi} \). The proportional tax system of the North eliminates the effects of trade liberalization on \( s_N \) mediated through the operating profit and the tax rate of the North. In contrast, lowering trade costs decreases the South firms’ profits, in turn decreasing the progressive tax rate in the South, which slows down the agglomeration activity in the larger region. If, on the other hand, \( \frac{\partial \pi_N}{\partial \phi} < 0 \) and \( \frac{\partial \pi_S}{\partial \phi} > 0 \) holds, then \( \frac{\partial f}{\partial \upsilon} \frac{\partial \upsilon}{\partial t_{R,N}(\pi_N)} > 0 \) and thus \( \frac{\partial s_N}{\partial \phi} > \frac{\partial f}{\partial \phi} \).

Proportional Taxation in the North and Regressive Taxation in the South

Suppose again that the North is subject to proportional taxation (\( t'_{PL,N} = 0 \)), while the South levies regressive taxes (\( t'_{R,S}(\pi_S) < 0 \)). This implies that the second term on the right-hand side of (3.3) reduces to \( \frac{\partial f}{\partial \upsilon} \frac{\partial \upsilon}{\partial t_{R,S}(\pi_S)} \frac{\partial \pi_S}{\partial \phi} > 0 \),
so \( \frac{\partial s_N}{\partial \phi} > \frac{\partial f}{\partial \phi} \). Since \( t'_{PL,N} = 0 \), taxation in the North does not affect \( s_N \) as \( \phi \) increases. However, \( t'_{R,S}(\pi_S) < 0 \) raises the tax rate in the South as \( \pi_S \) decreases when \( \phi \) increases, and this makes it more attractive for firms to establish production in North. If \( \frac{\partial \pi_N}{\partial \phi} < 0 \) and \( \frac{\partial \pi_S}{\partial \phi} > 0 \) is satisfied, \( \frac{\partial f}{\partial \phi} \). \( \frac{\partial f}{\partial \phi} \).

**Progressive Taxation in the North and Proportional Taxation in the South**

Consider progressive taxation in the North \((t'_{P,N}(\pi_N) > 0)\) and a proportional tax system in the South \((t'_{PL,S} = 0)\), such that the second term on the right-hand side of (3.3) becomes negative, \( \frac{\partial f}{\partial \phi} \). Clearly, in this case, as trade costs become lower, taxation in South does not affect the location of industry. Still, as profits in the North increase when trade is liberalized, \( t_{P,N}(\pi_N) \) increases, thereby decreasing the agglomeration economies in the North. If \( \frac{\partial \pi_N}{\partial \phi} < 0 \) and \( \frac{\partial \pi_S}{\partial \phi} > 0 \) holds, \( \frac{\partial f}{\partial \phi} \). \( \frac{\partial f}{\partial \phi} \).

**Regressive Taxation in the North and Proportional Taxation in the South**

Assume that the North adopts a regressive tax regime \((t'_{R,N}(\pi_N) < 0)\) and the South applies proportional taxation \((t'_{PL,S} = 0)\), thereby reducing the second term on the right-hand side of (3.3) to \( \frac{\partial f}{\partial \phi} \). The proportional tax rate of the South eliminates the location effect of trade liberalization, channeled through operating profits and taxation in the South. Regressive taxation in the North decreases \( t_{R,N}(\pi_N) \) via higher \( \pi_N \) as \( \phi \) increases, which increases the location of firms to the North over and above what is captured by the term \( \frac{\partial f}{\partial \phi} \). If \( \frac{\partial \pi_N}{\partial \phi} < 0 \) and \( \frac{\partial \pi_S}{\partial \phi} > 0 \), \( \frac{\partial f}{\partial \phi} \) and \( \frac{\partial s_N}{\partial \phi} < \frac{\partial f}{\partial \phi} \).

**Equilibrium Tax Regimes**

Again it can be verified that the North attracts more industrial production by setting regressive taxes, while the South receives more industry from levying progressive taxes. As pointed out in Subsection 3.2, both regions optimally adopt tax regimes that give rise to the lowest tax level. When trade costs
become lower, the profit of the representative North firm increases and the profit of the typical South firm decreases. With a regressive (progressive) system, this reduces taxation in the North (South) and increases (decreases) the agglomeration of firms in the North, compared to the equilibrium without tax structure. Therefore, when trade is liberalized, the Nash equilibrium in pure strategies prescribes the North applying a regressive tax schedule, while the South adopts a progressive system. As shown in Subsection 3.2, this equilibrium has \( \frac{\partial s_N}{\partial \phi} \geq \frac{\partial f}{\partial \phi} \) if \( \frac{s_N}{1-s_N} \leq -\frac{\nu}{\omega} \). Accordingly, the direction of the agglomeration relative to the location outcome with no tax structure, depends on the degree of tax regressiveness of the North in relation to the progressiveness of taxes set in the South (captured by \( \omega \)), the relative regional taxation level (\( \nu \)), and the relative capital abundance of the North (\( s_N \)).

The North (South) attracts more industry given that the South (North) also applies regressive (progressive) taxation. Given regressive (progressive) taxes in the South (North), as \( \phi \) increases, \( \pi_S (\pi_N) \) decreases (increases). This in turn increases taxation in the South (North), making it relatively more beneficial to establish production in the North (South), compared to the case where the North (South) solely sets regressive (progressive) taxes. Formally, this location effect is captured by the second term on the right-hand side of (3.3), which always is positive (negative) when both regions set regressive (progressive) taxes, implying that \( \frac{\partial s_N}{\partial \phi} > \frac{\partial f}{\partial \phi} \left( \frac{\partial s_N}{\partial \phi} < \frac{\partial f}{\partial \phi} \right) \) for all parameter values. Given the Nash strategies followed by the two regions, the second term on the right-hand side is positive (negative) if and only if \( \frac{s_N}{1-s_N} < -\frac{\nu}{\omega} \left( \frac{s_N}{1-s_N} > -\frac{\nu}{\omega} \right) \). If the North (South) adopts regressive (progressive) taxation, and the South (North) sets proportional taxes, the right-hand side of (3.3) reduces to \( \frac{\partial f}{\partial \nu} \frac{\partial \pi_N}{\partial \pi_S} \frac{\partial \pi_N}{\partial \phi} > 0 \left( \frac{\partial f}{\partial \nu} \frac{\partial \pi_S}{\partial \pi_S} \frac{\partial \pi_S}{\partial \phi} < 0 \right) \), which is lower (larger) than the right-hand side of (3.3) if both regions would apply regressive (progressive) taxation. By contrast, if \( \frac{\partial \pi_N}{\partial \phi} < 0 \) and \( \frac{\partial \pi_S}{\partial \phi} > 0 \), it is straightforward to show that the North always attracts more industry by adopting a progressive tax regime, and the South prefers the regressive tax system. In this case, the Nash equilibrium has the North setting progressive taxes and the South levying regressive taxes, while the optimal outcome for the North (South) is having both regions applying progressive (regressive) taxation.

Since the welfare of region \( j \) is increasing in the number of firms located in \( j \), the unilateral welfare-maximizing tax structure of the North (South),
as trade is liberalized, is the regressive (progressive) system. Welfare of
the North (South) is however maximized if both regions apply regressive
(progressive) taxation. Given that \( \frac{\partial \pi_N}{\partial \phi} < 0 \) and \( \frac{\partial \pi_S}{\partial \phi} > 0 \), as trade costs
become lower, the unilateral welfare-maximizing tax regime of the North
(South) is the progressive (regressive) system. And the North’s (South’s)
welfare is maximized if both regions set progressive (regressive) taxes.

\[ \left| \frac{\partial W_N}{\partial s_N} / \frac{\partial W_S}{\partial s_N} \right| \leq 1 \text{ if } \lambda \leq s_N + (1-s_N) \phi \text{, which implies that the South (North)
loses (gains) more in terms of total welfare than the North (South) gains
(loses) from agglomeration in the North, provided that the relative number
of citizens in the North is lower (higher) than the threshold. Therefore, as
in Subsection 3.2, in the absence of interregional transfers, if the relative
endowment of labor of the North is lower than the critical value, a tax com-
bination that maximizes production in the South (North) is a joint efficient
outcome. Hence, progressive (regressive) taxation in both regions represents
a joint welfare maximizing outcome. The reason is that this combination
of tax systems maximizes (minimizes) the agglomeration economies in the
larger region.

The main results of Subsection 3.3 can be summarized in the following
way:

**Result 2:** As trade is liberalized, the Nash equilibrium has the North adopting
a regressive tax system, while the South sets progressive taxes. As trade costs
fall, the unilateral welfare-maximizing tax structure in the North (South) is
the regressive (progressive) system. The North’s (South’s) welfare is, how-
ever, increased if the South (North) also adopts regressive (progressive) tax-
ation. This holds if \( \frac{\partial \pi_N}{\partial \phi} > 0 \) and \( \frac{\partial \pi_S}{\partial \phi} < 0 \). If \( \frac{\partial \pi_N}{\partial \phi} < 0 \) and \( \frac{\partial \pi_S}{\partial \phi} > 0 \), the Nash
equilibrium prescribes progressive taxes in the North, while the South sets
progressive taxes. The unilateral welfare-maximizing tax system in the North
(South) is the progressive (regressive) regime as trade is liberalized. Welfare
of the North (South) is, however, maximized if both regions apply progressive
(regressive) taxation.

Proportional taxation is never an equilibrium, neither as regional spend-
ing changes, nor as trade is liberalized. If both regions set proportional taxes,
the agglomeration and welfare effects of higher spending and liberalized trade
are eliminated since, in this case, taxes are invariant to changes in profits,
which channel the effects of expenditures and trade costs on taxes and the
distribution of industry. This means that the tax policy equilibrium reduces to the location equilibrium without regional tax regimes, i.e., \( \frac{\partial s_N}{\partial s_{EN}} = \frac{\partial f}{\partial s_{EN}} \)
and \( \frac{\partial s_N}{\partial \phi} = \frac{\partial f}{\partial \phi} \). That is, symmetric proportional regional taxation implies that the tax structure does not contribute to the location of production over and above what is given by the market-access advantage, transportation costs, and the relative regional tax level.

4 Conclusions

The tax competition literature focuses on how tax levels affect the location of factors of production. In doing so, this line of research assumes that the average tax rate is independent of the tax base; that is, taxes are taken to be proportional. Hence, these studies do not take into account how the economic geography might vary with the specific tax structure. In filling that gap, this paper has analyzed how different corporate tax systems affect the location of industry, recognizing that the system itself has important implications for the agglomeration of firms.

Using a standard model of new economic geography with asymmetric-sized regions, both regions gain industry by adopting the tax regime that produces the lowest tax level. This implies that in equilibrium, the regions apply different tax regimes, and such a combination does not coincide with the joint welfare maximizing outcome.

As the share of expenditures in the North increases, the North optimally sets regressive taxes, while the South adopts a progressive tax system. The North (South) gains relatively more production if both regions set regressive (progressive) taxes, compared to if the North (South) solely applies regressive (progressive) taxation. As trade is liberalized, and provided that the North’s share of regional expenditures is lower (higher) than a certain threshold value, the North optimally sets regressive (progressive) taxes, while the South prefers to apply a progressive (regressive) tax scheme. A combination of tax schedules that maximizes the location of production in the South (North) is a joint efficient outcome provided that the relative endowment of labor of the North is below (above) a critical level. Furthermore, proportional taxation is never an equilibrium; the proportional tax policy outcome replicates the location equilibrium without tax regimes.
References


A Appendix

A.1 Derivation of (2.20) and (2.21)

The indirect utility function of a representative agent in region \( j \) with preferences given by (2.2) is:

\[
E_j^r = \left( \int_{k=0}^{nW} p_{jk}^{1-\sigma} dk \right)^{\frac{-\mu}{\sigma - 1}}. 
\]

Using \( w = 1, \pi_W K_W = \frac{z_W K_W}{\sigma} \) and normalizing \( L_W \) to unity without loss of generality to obtain the representative individual’s net earnings devoted to manufactures:

\[
E_j = \mu (1 + \pi_K K) = \mu (1 + \frac{z_W K_W}{\sigma}) = \pi_N n_N + \pi_S n_S. 
\]

Substituting (2.12) into (2.13) and (2.14) gives:

\[
\begin{align*}
\pi_N n_N + \pi_S n_S &= \mu \left( \frac{a s_E n_{N + n_S \phi}}{(n_N + n_S \phi)} + \frac{a(1 - s_E n)(n_N \phi + n_S)}{(n_N \phi + n_S)} \right) = \mu a, \text{ which yields:} \\
E_j &= \mu (1 + \mu a). 
\end{align*}
\]

Solving for the price index prevailing in the North and the South, respectively:

\[
\left( \int_{k=0}^{nW} p_{Nk}^{1-\sigma} dk \right)^{\frac{-\mu}{\sigma - 1}} = (n_N + n_S \phi)^{\frac{-\mu}{\sigma - 1}} = (s_N + (1 - s_N) \phi)^{\frac{-\mu}{\sigma - 1}}, \tag{A.2} 
\]

and

\[
\left( \int_{k=0}^{nW} p_{Sk}^{1-\sigma} dk \right)^{\frac{-\mu}{\sigma - 1}} = (n_N \phi + n_S)^{\frac{-\mu}{\sigma - 1}} = (s_N \phi + (1 - s_N))^\frac{-\mu}{\sigma - 1}. \tag{A.3} 
\]

Substituting (A.1), (A.2) and (A.3) into the indirect utility function and taking the logarithm of the resulting expression to obtain (2.20) and (2.21).