DISTORTIONARY TAX INSTRUMENTS AND IMPLEMENTABLE MONETARY POLICY

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Abstract. I introduce distortionary taxes on consumption, labor and capital income into a New Keynesian model with Calvo pricing and nominal bonds. I study the relation between tax instruments and optimal monetary policy by computing simple rules for monetary and fiscal policy when one tax instrument at a time varies, while the other two are fixed at their steady-state level. The optimal rules maximize the second-order approximation to intertemporal utility. Three results emerge: (a) when prices are sticky, perfect inflation stabilization is optimal independently from the tax instrument adopted; (b) the optimal degree of responsiveness of monetary policy to output varies depending on which tax instrument induces fluctuations in the average tax rate; (c) when prices are flexible, fiscal rules that prescribe unexpected variations in the price level to support debt changes are always welfare-maximizing.

Keywords: Nominal rigidities, distortionary taxation, monetary-policy rules
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“... (A) central bank charged with maintaining price stability cannot be indifferent as to how fiscal policy is determined.”

Woodford (2001)

1. INTRODUCTION

The literature on the quantitative aspects of fiscal and monetary policy has recently evolved along two paths. On the one hand, the landmark contribution of Schmitt-Grohé and Uribe (2005a) computes the Ramsey policy of a realistic macroeconomic model of the U.S. economy with taxes on both capital and labor income. They propose an implementation scheme that achieves the Ramsey allocations through simple policy rules. Another strand of literature includes the works of Marzo (2003), Kollmann (2006) and Schmitt-Grohé and Uribe (2006). These papers characterize policy rules that maximize measures of household’s intertemporal utility. The feedback policy rules for monetary and fiscal policy are designed to respond to the deviations of selected macroeconomic aggregates from their distorted long-run equilibria.

Both the studies on Ramsey optimal and on simple feedback rules assume that cyclical fiscal policy takes place through changes to an average distortionary tax rate. Hence, the literature ignores the fact that national governments have multiple instruments at hand in the implementation of fiscal policy, such as tax rates on consumption, labor and capital income. This suggests that the average tax rate can be a poor indicator of the underlying composition of the tax burden. For instance, a government can change the relative tax rates on the incomes from capital and labor while leaving the average rate unchanged over time.

The relevance of the composition of taxes for the setting of monetary policy is supported by a number of results on Ramsey optimal policy. Correia, Nicolini, and Teles (2003) provide equivalence theorems on the mix between consumption and labor-income taxes as a substitute for state-contingent debt in the delivery of Ramsey allocations. However, these results are typically based on stylized models. In the framework used by Correia, Nicolini, and Teles (2003), there is no capital accumulation. Moreover, it is unclear how the proposed allocations should be implemented for the purpose of stabilization around distorted steady states.

The aim of this paper is to fill the gap in the literature by deriving benchmark results about the impact of alternative sources of distortionary taxation on monetary policy. I introduce taxes on consumption, labor and capital income into a standard New Keynesian model with Calvo pricing and nominal bonds. The pricing mechanism proposed by Calvo (1983) assumes that there is a fixed proportion of firms that are allowed to change prices in every period. I compute optimized simple rules for monetary and fiscal policy when one tax instrument at a time varies, while keeping the other two distortionary tax rates
fixed at their steady-state levels. The optimal rules maximize a measure of intertemporal (conditional) utility, in order to account fully for the transitional effects of alternative policy arrangements. To that end, I approximate the solution to the system of optimality conditions through the second-order Taylor approximation around the distorted steady states suggested by Schmitt-Grohé and Uribe (2006).

The results can be summarized as follows. When prices are sticky, inflation stabilization is optimal independently from the tax instrument considered. As a result, optimal fiscal policy is ‘passive’ in the sense that the fluctuations of government liabilities require no adjustment to the price level in order to sustain fiscal solvency (see Leeper, 1991). This is explained by the fact that, with Calvo pricing, movements in the inflation rate generate an inefficient (welfare-reducing) dispersion of markups between the firms that change prices and those that cannot. Differently from Schmitt-Grohé and Uribe (2006), I find that the optimal degree of responsiveness of monetary policy to output can depend on the instrument for tax policy. In particular, when labor-income taxes follow a simple rule, acyclical monetary policy generates indeterminacy, i.e. it is consistent with multiple equilibria. This means that the combination between rules for monetary and fiscal policy is incapable of pinning down a unique desired macroeconomic outcome. However, the choice of the optimal policy mix satisfies the logic outlined in Leeper (1991), and minimizes the welfare effects of the dispersion in markups.

Finally, when prices are flexible, fiscal-policy rules that prescribe unexpected variations in the price level are optimal independently from the tax instrument considered. Interestingly though, the quantitative findings indicate that an fiscal policy ‘active’ in the sense of Leeper (1991) need not arise from a strong reaction of taxes to changes in government liabilities. The reason is that, in Leeper (1991)’s framework, taxes are lump-sum. Distortionary taxes instead affect real allocations both directly through their impact on equilibrium choices, and indirectly through inflation expectations. Thus, although the logic of Leeper (1991) applies, the quantitative dynamics is different from the baseline setting with lump-sum taxes.

This paper is organized in the following way. Section 2 describes the New Keynesian model with price rigidity on which the main results are based. Section 3 introduces the simple rules for monetary and fiscal policy. Aggregation and equilibrium conditions are discussed in section 4. Section 5 presents the calibration strategy. Section 6 discusses both the framework for welfare evaluation, and the conditions for the local validity of the approximate solutions. The results are detailed in section 7. Section 8 considers the economy with flexible prices and nominal bonds. Section 9 presents some concluding remarks.
2. The model

The structure of the model follows the New Keynesian tradition. It combines nominal price rigidities in the form advocated by Calvo (1983) with real distortions due to monopolistic competition in intermediate-product markets. The novel element is the introduction of three forms of distortionary taxation — on consumption, labor and capital income — in the household’s budget constraint. There are exogenous shocks to both productivity and government consumption.

2.1. The final-good sector

The model includes a perfectly-competitive market where a representative firm sells a final product. The firm purchases intermediate goods and re-packages them through the Dixit-Stiglitz technology

\[ y_t \leq \left[ \int_{i \in \varpi} y_{it}^{\frac{\theta - 1}{\theta}} \right]^{\frac{\theta}{\theta - 1}}, \tag{1} \]

where \( \iota \) indexes the inputs over the measure \( \varpi \) of intermediate firms. The demand for each intermediate good \( y_{it} \) follows from the static profit maximization problem

\[ \max_{(y_{it}), i \in \varpi} P_t \left[ \int_{i \in \varpi} y_{it}^{\frac{\theta - 1}{\theta}} dt \right]^{\frac{\theta}{\theta - 1}} - \int_{i \in \varpi} P_{it} y_{it} dt, \tag{2} \]

and takes the form

\[ y_{it} = \left[ \frac{P_{it}}{P_t} \right]^{-\theta} y_t. \tag{3} \]

At a zero-profit equilibrium, the following price index of final goods can be derived

\[ P_t = \left[ \int_{i \in \varpi} P_{it}^{1-\theta} dt \right]^{\frac{1}{1-\theta}}. \tag{4} \]

2.2. The intermediate-good sector

In the intermediate sector, firm \( i \in \varpi \) uses capital and labor as production inputs according to a constant returns-to-scale technology

\[ y_{it} \leq z_t k_{it}^{\alpha} (l_{it})^{1-\alpha}. \tag{5} \]

where \( z_t \) is an exogenous productivity shock common to all firms

\[ \ln[z_{t+1}] = \rho_z \ln[z_t] + \sigma_z \epsilon_{z+1}. \tag{6} \]
and \( \epsilon_t \sim N(0, 1) \). Capital services are rented from centralized markets, and are perfectly mobile across firms.

Each firm chooses \( k_{it} \) and \( \ell_{it} \) taking their rental rates as given. The allocation problem for production factors is

\[
\max_{\{\ell_{it+n}, k_{it+n}\}} \sum_{n=0}^{\infty} E_0 \left[ \sum_{t+n}^{\infty} \Xi_{t+n}\left( \frac{P_{t+n}}{P_{t+n}} y_{t+n} - w_{t+n} \ell_{t+n} - r_{t+n} k_{t+n} \right) \right],
\]

subject to the constraints 3 and 5. The stochastic discount factor \( \Xi_{t+n} \) collects the prices of the claims that pay each one unit of money for a given state of nature at \( t + n \), normalized by the probability of the state.

Sticky prices arise from staggered price contracts in the tradition of Calvo (1983). Each firm is allowed to change the price of its intermediate good with a fixed probability \( 1 - \phi_p \). A price that is not negotiable in the current period increases at the steady-state rate of inflation \( \bar{\pi} \). Along with the assumption of monopolistically-competitive markets, this mechanism implies that firms are willing to satisfy unexpected fluctuations in demand even if they cannot change their prices. When a re-optimization takes place, the price-setting decision of firm \( i \) in period \( t \) involves choosing a contingent plan for \( \hat{P}_{it} \) such that

\[
\max_{\hat{P}_{it}} \sum_{n=0}^{\infty} E_0 \left[ \sum_{t+n}^{\infty} \beta_t u \left( c_{t+n}, \ell_{t+n} \right) \right]
\]

The demand side of the model economy is populated by a representative infinitely-lived consumer. The agent enjoys utility from current consumption \( c_t \) and disutility from hours worked \( \ell_t \). The history of events \( s^t = \{s_0, \ldots, s_t\} \) up to date \( t \) is assigned a time-0 probability mass \( \mu(s^t) \). The uncertainty in the choice process is summarized by the conditional-expectation operator \( E_0[\cdot] := \sum_{s_{t+1}} \mu(s_{t+1}|s^t) \). Given this structure, the household’s allocation problem takes the form

\[
\max_{\{c_t, \ell_t, \ell_{t+1}, A_{b,t}, A_{b,t+1}\}} \sum_{t=0}^{\infty} \beta^t u \left( c_t, \ell_t \right)
\]

s. t. \( (1 + \tau^t_C) C_t + \eta_t A_{b,t} + I_t + \leq (1 - \tau^t_C) W_t \ell_t + \left[ (1 - \tau^t_r) r_t + \tau^t_r \delta \right] K_t + A_{b,t-1} + TR + \Omega_t. \)
The portfolio of financial assets includes one-period riskless nominal bonds \( A_{b,t} \) with price \( \eta_t \). The representative household also owns the claims to the profit \( \omega_\iota \) of the monopolistically-competitive firm \( \iota \). The gross interest rate on bonds is denoted as \( R_t \). Let \( \Omega_{\iota t} \) denote the dividend stream generated by firm \( \iota \) and appropriated by household. The total dividend payment to household is
\[
\Omega_t := \int_{\iota \in \mathcal{I}} \omega_\iota \Omega_{\iota t} dt. \tag{10}
\]
For the purpose of analytical simplicity, I assume that the allocation of ownership shares across agents is constant, and beyond the control of households.

The representative consumer controls the evolution of the real capital stock \( k_t \) through the individual decision on investment \( i_t \). Capital services are rented to the firms of the intermediate-good sector at the rate \( r_t \). Capital accumulation follows a linear law of motion
\[
k_{t+1} = i_t + (1 - \delta) k_t. \tag{11}
\]

Three types of distortionary taxes enter the consumer’s budget constraint. There are taxes on consumption, labor income and capital income at the average rates \( \tau^c_t \), \( \tau^\ell_t \) and \( \tau^k_t \) respectively. Capital taxes are imposed on the nominal return of capital. Following Kim and Kim (2003b), I introduce a depreciation allowance on capital taxation, with \( r_t \) as the rental rate of capital. Households also enjoy a nominal flow \( \bar{T}R \) of real government transfers. These transfers are fixed at the steady-state level, and are introduced to improve the calibration of the model.

3. Fiscal and Monetary Policy Rules

The government faces the flow budget constraint
\[
D_t + P_t \tau_t = R_{t-1} D_{t-1} + P_t g^c_t + P_t \bar{T}R. \tag{12}
\]
Real total taxation is denoted as \( \tau_t \), and \( g_t \) indicates total government spending. The government issues one-period riskless (non-state contingent) nominal bonds denoted by \( D_t \). The total revenues from taxation are decomposed into consumption taxes \( \tau^c_t \), capital taxes \( \tau^k_t \) and labor taxes \( \tau^\ell_t \)
\[
\tau_t := \tau^c_t c_t + \tau^k_t (r_t - \delta) k_t + \tau^\ell_t w_t \ell_t. \tag{13}
\]
Public spending is an exogenous process

$$\ln[g_{t+1}^g] = \rho_g \ln[g_t^g] + (1 - \rho_g) \ln [\bar{g}^g] + \sigma_g \epsilon_{t+1}^g.$$

(14)

with $\epsilon_{t}^g \sim N(0,1)$. There are also transfers $\bar{r}$ to households that are fixed to their steady-state level. Government transfers are introduced for the purpose of achieving a realistic calibration of the steady-state ratio between public debt and output. The intertemporal budget constraint of the government is written as

$$R_t D_t \leq \sum_{p=0}^{\infty} E_{t+p} \left( \frac{1}{R_{t+p}} \right)^p \left[ P_{t+p+1} P_{t+p} g_t^c + P_{t+p+1} P_{t+p} g_t^g - P_{t+p} g_t^g \right].$$

(15)

with total public spending $g_t := g_t^c + \bar{r}$. This amounts to saying that the maximum level of outstanding debt in every period should not exceed the discounted sum of future primary surpluses.

The literature on public finance provides plenty of results of equivalence between different types of taxation in terms of welfare impact. In these cases, the allocation achieved under a given tax structure can be replicated through alternative structures where the redundant tax rates are removed. Renström (2006) shows that the tax equivalence proposition breaks down in a dynamic framework where the consumption plans of households can be changed at a frequency higher than tax rates. Correia, Nicolini, and Teles (2003) provide equivalence theorems on the mix between consumption and labor-income taxes as a substitute for state-contingent debt in the delivery of Ramsey optimal allocations. Like in the seminal contribution of Ramsey (1927), these equivalence theorems arise in simplified economies that include only a limited number of frictions. This provides the motivation for including proportional tax rates on consumption, labor and capital income in the model.

From an operational point of view, the tax rates on consumption, labor and capital income are the three instruments that the government can employ for cyclical fiscal policy. In this paper, I impose a fiscal feedback rule that makes one of the tax rates change, while holding the other two constant at their steady state values. Following Schmitt-Grohé and Uribe (2006), I define the total amount of government liabilities $l_t$ in equilibrium $l_t := R_t d_t$. Hence, the flow government budget constraint in equilibrium can be expressed in terms of total liabilities

$$l_t = \frac{R_t l_{t-1}}{\pi_t} + R_t (g_t - \tau_t).$$

(16)

The evolution of total distortionary taxes is linked to the outstanding value of government
liabilities through the simple rule:

\[ \tau_t = \psi_0 + \psi_1 (l_{t-1} - \bar{l}) + \psi_2 \left[ g_t + \left( \frac{R_{t-1} - 1}{R_{t-1}} \right) \left( \frac{l_{t-1}}{\pi_t} \right) \right]. \]  

(17)

where \( \bar{l} \) denotes the deterministic steady state of government liabilities. Suppose that the tax rate on consumption \( \tau_t^c \) is the instrument for fiscal policy. From equation (17), I get

\[ \tau_t^c = \psi_0 / c_t - \left[ \bar{\tau}_k (r_t - \delta) k_t + \bar{\tau}_\ell w_t \ell_t \right] / c_t + \psi_1 (l_{t-1} - \bar{l}) / c_t + \psi_2 \left[ g_t + \left( \frac{R_{t-1} - 1}{R_{t-1}} \right) \left( \frac{l_{t-1}}{\pi_t} \right) \right] / c_t. \]  

(18)

Similar expressions can be derived for \( \tau_t^k \) and \( \tau_t^\ell \).

The fiscal rule 17 allows us to distinguish between two kinds of liability stabilization. A simple fiscal feedback rule à la Leeper (1991) is obtained by setting \( \psi_2 = 0 \), whereas a balanced budget rule holds when \( \psi_1 = 0 \) and \( \psi_2 = 1 \). In the case of liability targeting, Leeper (1991) distinguishes between two policy regimes. With ‘active’ fiscal policy, the evolution of government liabilities plays an important role in the determination of the price level. A ‘passive’ fiscal policy, instead, is such that tax policy does not constrain the path of the inflation rate. Combining the fiscal rule for liability targeting with the flow budget constraint, a linear difference equation can be obtained:

\[ l_t = (R_t / \pi_t) (1 - \psi_1 \pi_t) l_{t-1} + \text{rest}. \]

An active fiscal policy requires: \( |1 - \psi_1 \pi_t| > 1 \). This condition implies that government liabilities grow at a rate higher than the real interest rate. In order to make the problem stationary, the initial price level should adjust accordingly.

Finally, I assume that the central bank sets policy rates according to a simple feedback rule

\[ \ln \left[ \frac{R_t}{\bar{R}} \right] = \alpha_\pi \ln \left[ \frac{\pi_t}{\bar{\pi}} \right] + \alpha_y \ln \left[ \frac{y_t}{\bar{y}} \right] + \alpha_R \ln \left[ \frac{R_{t-1}}{\bar{R}} \right]. \]  

(19)

This formulation has become standard since the work of Taylor (1993).

4. AGGREGATION AND EQUILIBRIUM

The Calvo model of price rigidity introduces heterogeneity in price setting. This section presents the assumptions for aggregation and the equilibrium conditions.

ASSUMPTION 1: All the firms that can change their idiosyncratic price contracts choose, respectively, the same new prices. Hence, the average price level can be written as

\[ (P_t)^{1-\theta} = \phi_p (\pi P_{t-1})^{1-\theta} + (1 - \phi_p) \left( \tilde{P}_t \right)^{1-\theta}, \]
which can be re-written as
\[ 1 = \phi_p \left( \frac{\tilde{\pi}}{\pi_t} \right)^{1-\theta} + (1 - \phi_p) (\tilde{p}_t)^{1-\theta}, \]
with \( \tilde{p}_t := \frac{\tilde{P}_t}{P_{t-1}} \) and \( \pi_t := \frac{P_t}{\tilde{P}_{t-1}} \).

**Proposition 1:** Equilibria of this set of economies are stationary sequences of prices \( \{P_t\}_{t=0}^{\infty} := \{P^*_t, \tilde{P}_t, R_t^*, \eta^*_t, w^*_t, r^*_t, s^*_t\}_{t=0}^{\infty} \), quantities \( \{Q_t\}_{t=0}^{\infty} := \{\{Q^h_t\}_{t=0}^{\infty}, \{Q^f_t\}_{t=0}^{\infty}, \{Q^g_t\}_{t=0}^{\infty}\} \) with \( \{Q^h_t\}_{t=0}^{\infty} := \{c^*_t, \ell^*_t, k^*_t, \eta^*_t, a^*_t\}_{t=0}^{\infty} \), \( \{Q^f_t\}_{t=0}^{\infty} := \{y^*_t, k^*_t, \ell^*_t\}_{t=0}^{\infty} \), \( \{Q^g_t\}_{t=0}^{\infty} := \{g^*_t, \bar{r}^*, \tau^*_t, \tau^*_k, \tau^*_f, d^*_t\}_{t=0}^{\infty} \), and stochastic shocks \( \{\varepsilon_t\}_{t=0}^{\infty} := \{\varepsilon^*_t, \varepsilon^g_t\}_{t=0}^{\infty} \) that are bound in a neighborhood of the deterministic steady state, and such that:

(i) **given prices** \( \{P_t\}_{t=0}^{\infty} \) and shocks \( \{\varepsilon_t\}_{t=0}^{\infty} \), \( \{Q^h_t\}_{t=0}^{\infty} \) is a solution to the representative household’s problem;

(ii) **given prices** \( \{P_t\}_{t=0}^{\infty} \) and shocks \( \{\varepsilon_t\}_{t=0}^{\infty} \), \( \{Q^f_t\}_{t=0}^{\infty} \) is a solution to the representative problem of the firm;

(iii) **given quantities** \( \{Q_t\}_{t=0}^{\infty} \) and shocks \( \{\varepsilon_t\}_{t=0}^{\infty} \), \( \{P_t\}_{t=0}^{\infty} \) clears the markets for both goods and factors of production
\[ y^*_t = [c^*_t + i^*_t + g^*_t + \bar{r}^*] s^*_t, \]
\[ s^*_t = (1 - \phi_p) \left( \frac{\tilde{p}_t}{\tilde{p}^*_t} \right)^{\theta} + \phi_p \left( \frac{\pi_t}{\tilde{\pi}} \right)^{\theta} s^*_{t-1}, \]
and the markets for bonds
\[ R^*_t = \frac{1}{\eta^*_t}, \]
\[ a^*_t = d^*_t. \]

(iv) **given quantities** \( \{Q_t\}_{t=0}^{\infty} \), **prices** \( \{P_t\}_{t=0}^{\infty} \) and shocks \( \{\varepsilon_t\}_{t=0}^{\infty} \), \( \{Q^g_t\}_{t=0}^{\infty} \) satisfies both the government flow and the intertemporal budget constraint;

(v) fiscal and monetary policies are set according to the simple rules outlined in section 3.

5. Calibration

I calibrate the model on quarterly data for an average G-7 economy. The parameter values are reported in Table I. The long-run inflation rate is set to 4% a year. The intertemporal discount factor \( \beta \) equals 0.9949. Households devote a steady-state share of time to market activity equal to 0.4.

I choose an investment-output ratio of 0.24 at the steady state. The quarterly depreciation rate is consistent with an annual rate of 10%. The resulting capital-output ratio
(9.95) is close to what data for the U.S. suggest (see Christiano, Eichenbaum, and Evans, 2005). The price-marginal cost markup factor is set at \( \frac{\theta}{(\theta - 1)} = 1.25 \), as suggested by Bayoumi, Laxton, and Pesenti (2004). Following Pappa (2004), I choose \( \phi_p = 2/3 \), which implies an average contract duration of \( 1/(1 - 2/3) = 3 \) quarters. The capital elasticity of output \( \alpha \) is 0.4 (see Kim and Kim, 2003b).

The steady-state ratio between public debt and output is calibrated at 70%, with government consumption representing a share of 15% of output. The resulting ratio between public transfers and output is approximately 0.10. The effective tax rates on consumption, labor and capital income match the G-7 averages from Kim and Kim (2003b). In the model with lump-sum taxes, I adjust the ratio between public transfers and output to achieve a public-debt share of output of 70%.

6. Computational aspects

6.1. Welfare evaluation

The coefficients of the policy rules for both monetary and fiscal policy are chosen to maximize a utility-based welfare function. In this paper, I consider a measure of conditional household welfare

\[
W_0 := E \left[ \sum_{t=0}^{\infty} \beta^t (\log(c_t) - \gamma \ell_t) \left| \tilde{s} \sim (s, \Omega) \right. \right]
\]

that takes into account the transitional costs of moving from the initial state \( \tilde{s} \) to the stochastic steady state of the model, with \( s \) and \( \Omega \) as the mean and the covariance matrix of the distribution of the initial state.

An important factor in the evaluation of welfare is that the sources of distortions that lead the deterministic steady state away from the first best are not switched off in this paper. Woodford (2003) points out that this requires approximating the welfare criteria through a second-order Taylor expansion around the distorted steady state. Omitting second-order terms could generate spurious welfare reversals as it would not take into account the transitional welfare costs of policies (see Kim and Kim, 2003a). A first-order approximation to welfare is accurate only if the deterministic steady state coincides with the first-best equilibrium. Given the structure of the welfare-evaluation problem, I solve the models through the second-order Taylor expansion method of Schmitt-Grohé and Uribe (2004). The approximate solution is then used to compute the Taylor expansions of both \( W_0 \) and \( W \).

In order to compare the outcomes of different policies, I compute the permanent change in consumption, relative to the model’s steady state, that yields the expected utility level of the distorted economy. Given steady states of consumption \( \bar{c}^t \) and hours
worked $\bar{\ell}$ of model $\iota$, this translates into the number $\Phi^{c}_{\iota}$ such that

$$
\sum_{t=0}^{\infty} \beta^t u \left( \left[ 1 - \Phi^{c}_{\iota} \right] \bar{c}^t, \bar{\ell} \right) = W_{0}^{0}.
$$

(25)

Four elements determine the size of the welfare metric. On the right-hand side of the equality, there are the deterministic steady state, its stochastic counterpart, and the transition from the deterministic to the stochastic long-run equilibrium of $\iota$. On the left-hand side, instead, there are the deterministic steady states of the benchmark model with respect to which the current distorted economy is compared. In expression 25, I follow the standard practice of using the model’s own steady states as the benchmark.

Following Kollmann (2006), I decompose the conditional welfare cost $\Delta^{c}_{\iota}$ into two components denoted as $\Delta^{E}_{\iota}$ and $\Delta^{V}_{\iota}$. Denoting by hats the log-deviations from the deterministic steady state, the following approximation holds:

$$
u \left( \left[ 1 - \Delta^{c}_{\iota} \right] \bar{c}, \bar{\ell} \right) \approx u \left( \bar{c}, \bar{\ell} \right) + (1 - \beta) \sum_{t=0}^{\infty} \beta^t \left( \mathbb{E} \left[ \hat{c}_t - \tilde{\ell} \hat{\ell}_t | s_0 \right] - \frac{1}{2} \text{VAR} \left[ \hat{c}_t | s_0 \right] \right).$$

(26)

I compute the change in mean consumption $\Delta^{c}_{\iota}$ that the household faces while giving up the total fraction of certainty-equivalent consumption $\Delta^{c}_{\iota}$

$$
u \left( \left[ 1 - \Delta^{E}_{\iota} \right] \bar{c}, \bar{\ell} \right) = u \left( \bar{c}, \bar{\ell} \right) + (1 - \beta) \sum_{t=0}^{\infty} \beta^t \left( \mathbb{E} \left[ \hat{c}_t | s_0 \right] - \mathbb{E} \left[ \tilde{\ell} \hat{\ell}_t | s_0 \right] \right).$$

(27)

Since the solution method is non-certainty equivalent, I can also calculate the change in conditional variance of consumption that is consistent with the total welfare cost of policies

$$
u \left( \left[ 1 - \Delta^{V}_{\iota} \right] \bar{c}, \bar{\ell} \right) = u \left( \bar{c}, \bar{\ell} \right) - (1 - \beta) \frac{1}{2} \sum_{t=0}^{\infty} \beta^t \text{VAR} \left[ \hat{c}_t | s_0 \right],$$

(28)

where hats denote log-deviations from the deterministic steady states. It can be shown that the three measures of welfare are linked in the following way:

$$(1 - \Delta^{c}_{\iota}) = (1 - \Delta^{E}_{\iota}) (1 - \Delta^{V}_{\iota}).$$

(29)

Since there are no closed-form solutions for the infinite summations in the expressions for $\Delta^{V}_{\iota}$ and $\Delta^{E}_{\iota}$, I simulate the conditional moments for 10000 periods and compute the discounted sum. The (finite-horizon) conditional moments are computed through the analytical formulas presented in Appendix 5.
6.2. Local validity of approximate solutions

Second-order perturbations are defined only within small neighborhoods around the approximation points, unless the function to be evaluated is globally analytic (see Anderson, Levin, and Swanson, 2004). Since the conditions for an analytic form of the policy function are hardly establishable, the problem of validity of the Taylor expansion remains. I approach this issue at different levels. First, I calibrate the processes for exogenous shocks in such a way that their fluctuations are constrained within small intervals, like Schmitt-Grohé and Uribe (2006). However, given the large degree of inertia in public debt fluctuations induced by the fiscal rules, this might not be enough to guarantee the local validity of the approximation.

The second step consists in imposing an ad hoc bound that restricts the stochastic steady state of each variable to be arbitrarily close to its deterministic counterpart. For a variable \( x \), the constraint takes the form

\[
|E[\hat{x}]| < \kappa_1.
\]  

(30)

This condition is verified after computing the second-order solution.\(^1\) All the rational-expectations equilibria that do not comply with 30 are not considered. The reason for introducing this type of constraint relates to the large persistence induced by the fiscal-policy rule, which could make the stochastic steady states drift far apart from the deterministic steady states (see Kollmann, 2006). In that case, the Taylor approximation to the system of first-order conditions would not be locally valid any longer. The parameter \( \kappa_1 \) is calibrated equal to 0.01.\(^2\)

7. Main results with price rigidity

In this section, I present the results with simple rules for a policy mix in the model with price rigidity that: (a) generate unique rational-expectations equilibria; (b) achieve the highest level of conditional welfare in the admissible region of parameters for the policy rules; (c) comply with the constraint 30. I compute the welfare-maximizing values of the parameters for the fiscal and monetary policy rules over a grid. The parameter \( \psi_1 \) is allowed to vary within \([0, 4]\). The inflation coefficient \( \alpha_\pi \) takes values in \([0, 2]\). The parameters \( \alpha_y \) and \( \alpha_R \) are bound over \([0, 1]\).

\(^1\)Kollmann (2006) imposes the bound 30 only on real public debt. However, it is unclear why only one variable should be restricted.

\(^2\)Numerical experiments show that the results are substantially unaffected for \( \kappa_1 \leq 0.1 \).
7.1. Cyclical monetary policy and tax instruments

Table II reports the coefficients of the policy rules with the associated welfare level and costs. Panel [a] concentrates on the results with fully-optimized (unconstrained) rules that maximize the measure of conditional welfare $W_0$. It shows that optimal monetary policy follows the Taylor principle, namely the prescription that the coefficients on the inflation objective should be larger than one. When lump-sum taxes are the fiscal-policy instrument, all the values of $\psi_1$ that generate unique rational-expectations equilibria share the same welfare level. This is due to the fact that the real allocation is unaffected by fiscal policy when taxes are lump-sum and follow a passive rule.

The key result from panel [a] of Table II is that the optimal degree of cyclicality in monetary policy depends on the policy mix. Except for the case of time-varying taxes on labor, optimal monetary policy does not respond to output. Instead, with a policy rule for labor-income taxes, the optimized coefficient on the gap between current and long-run output\(^3\) takes the largest value within the admissible range.

Some insight on the role of cyclical monetary policy can be gained by looking at the pattern of macroeconomic adjustment generated by the optimizing policy mix. Figures 1(a)-1(d) report the impulse responses from monetary-policy rules that maximize conditional welfare with and without response to the output gap. Three aspects are worth noting. First, for a given degree of responsiveness of monetary policy to output, the initial sign of the responses of the variables is the same under all the tax instruments, with the exception of the rule for labor-income taxes. The difference lies only in the quantitative magnitude of the fluctuations. Second, like for lump-sum taxes, with a policy rule for taxes on either consumption or capital income, the nominal rate of interest rises modestly when monetary policy is acyclical, and falls when the central bank pursues a cyclical stance after a positive productivity shock. Finally, in the case of labor-income taxes (see figure 1(d)), acyclical monetary policy generates multiple equilibria, and the corresponding impulse responses cannot be computed without further assumptions on the mechanism of equilibrium selection.

The fall of policy rates when the ‘central bank leans against the wind’ appears counterintuitive. In particular, one would expect the presence of a cyclical component to strengthen the response of monetary policy to a positive supply shock. This indicates that there should be an inverse relation between inflation variability and the output coefficient of the Taylor rule. Figure 2 plots the variance of $\pi_t$ as a function of $\alpha_y$, where all the policy parameters are chosen to maximize conditional welfare. Except for the case of time-varying taxes on labor income, figure 2 indicates that inflation variability is a

\(^3\)Coherently with the model-solution method, I use the expression ‘output gap’ to denote the deviation of current output from the distorted deterministic steady state. This differs from the notion usually employed in the literature on monetary-policy rules, which refers to the output deviation from frictionless potential output.
monotonic increasing function of the responsiveness to output. As a result, the fall of inflation after a productivity shock is larger the higher the coefficient on the output gap.

Schmitt-Grohé and Uribe (2006) suggest that inflation changes following positive supply shocks have important welfare implications. Given the structure of the Calvo setting, the fraction of firms that are allowed to change prices makes the relative price $\bar{P}_t/P_t$ drop. Since the markups of these firms will keep close to the steady state, their real marginal costs fall. As a result, the markups of the firms that cannot change prices are bound to increase, thus raising the economy-wide markup. The inefficient dispersion of markups explains why cyclical monetary policy is welfare-reducing.

With time-varying taxes on labor income, the optimal response to a positive productivity shock induces a fall in the nominal rate of interest. Hence, the optimizing policy mix with time-varying taxes on labor does not eliminate the inefficiency from markup dispersion. In order to provide intuition, figure 3 reports the impulse responses from a positive productivity shock for the policy rules that maximize conditional welfare as a function of $\alpha_y$. While varying the coefficient on the output gap, the other policy parameters are set so that conditional welfare is maximized. There are two forces at work. On one hand, the globally-optimizing coefficient on the output gap objective induces the smallest initial fall in the interest rate. This is consistent with the smallest initial fall in prices. On the other, the magnitude of the initial response of the real return of capital drops as a function of $\alpha_y$ because capital accumulation becomes more sluggish. At the same time, the fraction of output absorbed by household bond holdings rises.

Another key ingredient in the macroeconomic adjustment with time-varying taxes for labor income is the fact that hours worked fall after a positive productivity shock (see figure 1(d)). With Calvo pricing, the fraction of firms that cannot change prices reduces the amount of labor services employed as a response to transitory technological improvements. This happens independently from the operative instrument for fiscal policy. However, when labor taxes are the fiscal-policy instrument, the overall quantitative impact of the monopolistic distortion on the aggregate demand for labor is negative.

The decline in hours worked is due also to the sluggish adjustment in consumption and capital. The response of capital to the positive productivity shock is small because of the inertial reaction of investment, which causes the rental rate of capital to fall on impact. Since both consumption and investment respond by little, the time share devoted to leisure must increase in order to contain the rise in output.

These considerations shed light on patterns different from those discussed in the literature. In particular, Schmitt-Grohé and Uribe (2006) argue that responding to the output gap is suboptimal for the class of Taylor rules considered here. Using the standard New Keynesian model, they find that the welfare costs of policy mistakes are increasing in the responsiveness of monetary policy to output. The results discussed here indicate that
this consideration need not hold across tax instruments. Figure 4 plots the percentage changes of welfare costs from the minimized costs as a function of $\alpha_y$, holding the other parameters of the policy rules at the optimal levels. Two lessons emerge. The first one is that, with the exception of time-varying taxes on consumption, policies that are too responsive to output with respect to the optimal setup do not cause large welfare losses. The second is that sound monetary policy need not stick to responding to inflation alone. When labor-income taxes follow a simple policy rule, underestimating the optimal response to output raises the welfare costs by the same magnitude of the policy mistakes under time-varying taxes for capital income.

### 7.2. The value of tax instruments

The negative response of labor supply to a positive productivity shock is at the root of an additional aspect, that is the fact that the minimized welfare costs fall as monetary policy becomes more responsive to output. Figure 5 plots the percentage change of the welfare costs minimized at each admissible value of $\alpha_y$ with respect to the welfare costs of optimal policy mix. All the policy parameters are chosen so that conditional welfare is maximized for each value of the output-gap coefficient of the Taylor rule. The introduction of time-varying taxes for labor income makes the welfare-cost curve downward-sloping in the responsiveness of monetary policy to output. As noticed earlier, this is explained by the fact that the drop in hours worked is an increasing function of the welfare-maximizing values of $\alpha_y$ (see figure 3). The larger the fall of hours worked, the lower the initial drop of the inflation rate, and the lower the dynamic distortions due to markup changes.

Two points are worth noting from figure 2. When taxes on labor income are the instrument for fiscal policy, the variance of inflation is negatively related to $\alpha_y$. This is coherent with the pattern documented above. The important aspect is that the optimal policy mix minimizes the variability of inflation. With the exception of time-varying taxes for labor income, the policy rules achieve perfect inflation stabilization around the distorted steady state. Kollmann (2006) obtains this result by choosing a welfare-maximizing equilibrium with a high inflation coefficient in the Taylor rule. The results presented here, instead, indicate that the size of the feedback parameter on inflation need not matter as long as the Taylor principle holds.

An additional measure of the value of alternative tax instruments is provided in figure 6. This figure plots the percentage change in welfare costs from the minimized cost as a function of the tax responsiveness to government liabilities, holding the other parameters at the welfare-maximizing values. Figure 6 shows that a lack of response to total liabilities exacerbates the welfare loss in the case of time-varying taxes for capital income. Fiscal-policy mistakes have no significant impact with time-varying taxes on consumption and labor income.
7.3. The role of interest-rate inertia

The optimized policy mix with lump-sum taxes incorporates a large degree of interest-rate inertia. However, this feature is not constant across tax instruments. In the model with time-varying taxes on labor income, monetary policy exhibits no smoothing. Panel [b] of Table II restricts the optimized Taylor rules to having no interest-rate inertia. With the exception of the policy rule for lump-sum taxes, there are no large falls in welfare from not responding to the past policy stance. Figure 7 reports the percentage increase in welfare costs due to policy mistakes in the choice of $\alpha_R$. The results indicate that these policy mistakes have only a limited impact on welfare. Hence, although the optimal degree of policy inertia is different across fiscal-policy instruments, deviations from the optimal values can be tolerated. This confirms that autocorrelation in the policy rates plays a minor role for the implementable policy mix.

7.4. Ad-hoc policy rules

A natural benchmark for comparing the performance of the fully-optimized policy mix is the parametrization proposed by Taylor (1993) for monetary policy. To that end, I search for the constrained rules with $\alpha_\pi = 1.5$, $\alpha_y = 0.5$ and $\alpha_R = 0$ that maximize conditional welfare over $\psi_1$. Panel [c] of Table II shows that the model with lump-sum taxes displays the largest fall in the level of conditional welfare. Among the distortionary tax instruments, the rule with time-varying taxes on capital income generates a sizeable increase in welfare costs.

Standard assumptions in dynamic general-equilibrium models are that taxes are lump sum, and that the public budget is balanced in every period. This means that the real value of public debt is constant. As a result, the government flow budget constraint drops out of the model structure. Panel [d] of Table II reports the optimized rules for monetary policy with balanced budget rules. Since there are only few grid points with determinate equilibria, I omit the results for capital-income taxes. The pattern of cyclicality across fiscal-policy instruments is the same as with the fully-optimized policy mix. Balanced budget rules achieve the same welfare levels of simple feedback rules. The only difference concerns the case of labor-income taxes, for which the transition towards the stochastic steady states is more costly than for fully-optimal policy rules.

7.5. Inflation targeting

The optimal policy rules of panel [a] achieve perfect inflation stabilization around the distorted steady state. Hence, it is interesting to consider the optimal policy mix where monetary policy contemplates constant inflation. Panel [e] of Table II reports the welfare-maximizing coefficients for a simple rule for monetary policy with $\hat{\pi}_t = 0$. The optimized
parameters involve a responsiveness of taxes to government liabilities that is not substantially different from those of panel [a]. However, the conditional costs in variance are very small. With time-varying taxes on capital income, the achieved welfare level is slightly higher than that under the rule from panel [a].

8. AN ECONOMY WITH FLEXIBLE PRICES

In a model with price stickiness and nominal debt, the distortions generated by markup changes are the key for interpreting the welfare implications of the mix between fiscal and monetary policy. In this section, I consider a setting with flexible prices where inflation does not lead to inefficient price dispersion. As a result, unanticipated inflation changes that realign the real value of public debt to that prescribed by the intertemporal solvency condition are no longer welfare reducing.

Kollmann (2006) shows that the adoption of an active stance for fiscal policy induces large inflation volatility. In the context of simple linear rules for monetary policy, this can be a source of excessive variability in the nominal interest rate. The local validity of the approximate solution might not hold any longer. Hence, along with the exogenous bound 30, I impose a condition that rules out excessive aggressiveness in the conduct of monetary policy:

\[ E[R_t] > 2\sigma_{R_t}. \]  
(31)

Large deviations of the nominal interest rate from the steady state are also likely to prescribe violations of the zero bound at some point in time (see Schmitt-Grohé and Uribe, 2005b). Enforcing the constraint 31 is consistent with imposing a zero lower bound on policy rates.

Table III reports the policy coefficients that maximize conditional welfare. Three main results emerge. First, except for the case of time-varying taxes on consumption, the optimal policy mix entails active fiscal policy \((\psi_1 \geq 2)\) and passive monetary policy \((\alpha_\pi < 1)\). Second, like in the model with price rigidity, optimal monetary policy can respond to output depending on the tax instrument adopted. Third, optimal monetary policy is characterized by no interest-rate inertia \((\alpha_R = 0)\).

The optimality of active fiscal policy follows from the lack of welfare costs due to price changes. The analysis of Leeper (1991) suggests that, in this case, the response coefficient of taxes to government liabilities should be large. Thus, it is interesting to investigate the reason for which this prescription does not hold in the case of a policy rule for consumption taxes.

Figure 8 plots the impulse responses to a positive productivity shock when the optimized tax rule is restricted to the active and passive regions, as suggested in Leeper
When fiscal policy is constrained to the standard region of activeness — i.e. \( \psi_1 \in [2, 4] \) —, the optimizing rule for monetary policy is \( \hat{R}_t = 1.1\hat{\pi}_t + 0.3\hat{y}_t \), and is very similar to one from panel [a] of Table III. Figure 8 shows that the drop of inflation on impact for \( \psi_1 \in [0, 1.9] \) is larger than the drop for \( \psi_1 \in [2, 4] \). This is due to the fact that, in order to generate a response of consumption of equal magnitude irrespective of \( \psi_1 \), the required increase of the outstanding level of debt is larger under \( \psi_1 \in [0, 1.9] \) than under \( \psi_1 \in [2, 4] \). Figure 9(b) plots the percentage change with respect to the minimized welfare cost as a function of \( \psi_1 \), while holding the other parameters at the welfare-maximizing level. With time-varying taxes on consumption, the closer the tax rule gets to the region with \( \psi_1 \in [2, 4] \), the larger the welfare costs become.

Summing up, with flexible prices and time-varying consumption taxes, a policy mix with a low feedback coefficient on total liabilities supports a Pareto-improving allocation. In particular, the distinction between ‘active’ and ‘passive’ policy based on the quantitative reaction of taxes to government liabilities can lead to misleading conclusions when taxes are distortionary. The reason is that changes to distortionary taxes affect both the inflation rate in the initial period through their expected impact on the government flow budget constraint, and the real allocations at each point in time.

The subsequent point of interest concerns the optimal responsiveness of monetary policy to output with flexible prices. Figure 10 compares the impulse responses to a productivity shock for welfare-optimizing policy rules with \( \alpha_y = 0 \) and \( \alpha_y > 0 \). We can see that, independently from the instrument for fiscal policy, the optimizing degree of responsiveness to output generates the largest reaction of inflation on impact. This confirms the results discussed earlier. Table IV reports the variability of some selected macroeconomic variables under alternative policy rules. The third and fourth columns confirm that the optimal policy mix maximizes the variance of inflation. Table IV shows also that, when inflation variability is large, also the variance of the nominal interest rate is sizeable. Despite this, all the optimal rules underlying Table IV comply with the bound 31 that rules out excessive aggressiveness in monetary policy, and that preserves the stationarity of the solution. Figure 11 suggests that, except in the case of time-varying lump-sum taxes, deviations from the optimal coefficient on the output gap lead to a substantial drop in the optimized welfare level.

A final note concerns the optimal degree of policy inertia. Mistakes in the choice of this parameter lead to non-existence of valid equilibria with time-varying taxes on capital income and lump-sum. Figure 12 shows that, with a policy rule for consumption taxes, the increase in welfare costs can be substantial as it goes up to 15% as \( \alpha_y \) approaches 1.
9. Conclusion

This paper studies the role of tax composition for the optimal design of simple rules for monetary and fiscal policy. I formulate a New Keynesian model with Calvo pricing and three types of distortionary taxes — taxes on consumption, capital and labor income. I assume that one of the tax rates varies at a time as a function of the deviation of government liabilities from the deterministic steady state. The second-order approximation method of Schmitt-Grohé and Uribe (2004) is used to obtain a nonlinear solution, and to compute welfare-maximizing coefficients for the policy rules.

Three main results emerge. In a model with price stickiness, inflation stabilization is optimal independently from the tax instrument considered, and optimal fiscal policy is passive in the sense of Leeper (1991). Differently from Schmitt-Grohé and Uribe (2006), I find that the optimal degree of responsiveness of monetary policy to output can depend on the instrument for tax policy. In particular, when labor-income taxes follow a simple policy rule, underestimating the optimal response to output can lead to a substantial welfare loss. This is due to the fact that acyclical monetary policy is unable to pin down unique macroeconomic equilibria, and the policy mix produces different outcomes that are equally achievable by the policy planner. Finally, with flexible prices, ‘active’ rules for fiscal policy that prescribe unexpected variations in the price level are optimal independently from the tax instrument. This confirms the results obtained by Kollmann (2006).

Several interesting extensions can be envisaged. Ongoing work considers optimal implementable rules when a combination of different tax instruments is used to deliver government solvency. Another interesting topic of ongoing investigation deals with the stabilizing power of tax instruments in a model with both price and wage rigidity. Erceg, Henderson, and Levin (2000) show that optimal monetary policy tolerates a significant amount of deviation from perfect inflation stabilization when also nominal wages are rigid. Hence, the question arises about whether the distortionary tensions from wage stickiness can be softened through an appropriate design of tax policy.
APPENDIX 1: FIRST-ORDER CONDITIONS

The first-order conditions from the firm’s allocation problem are

\[ w_t = mc_{it}(1 - \alpha)z_t k_{it}^{\alpha} (\ell_{it})^{-\alpha}, \tag{A1.1} \]

\[ r_t = mc_{it} \alpha z_t \left[ \frac{\ell_{it}}{k_{it}} \right]^{1-\alpha}. \tag{A1.2} \]

with real marginal costs per unit of output \( mc_{it} \).

Optimal price decisions for firms that can adjust prices at \( t \) follow from the optimality condition

\[ \mathbb{E}_t \sum_{n=0}^{\infty} \Xi_t + n \theta P_{it} \left[ \frac{\pi^n P_{it}}{P_{it+n}} + mc_{t+n} \right] \left( \frac{\pi^n P_{it}}{P_{it+n}} \right)^{-\theta-1} y_{t+n} = 0. \tag{A1.3} \]

After deflating the household’s budget constraint by the price level, I obtain the following from the consumer’s optimization problem

\[ u_c = (1 + \tau c_t) \varsigma_t, \]

\[ u_{it} = -(1 - \tau_{it}) w_t \varsigma_t, \]

\[ \eta_t = \beta \mathbb{E}_t \frac{\varsigma_{t+1}}{s_t \bar{\pi}_{t+1}}, \]

\[ \mathbb{E}_t \frac{\varsigma_{t+1}}{s_t} \left[ (1 - \tau_{k_{t+1}}) r_{t+1} + \tau_{k_{t+1}} \delta_r \right] + (1 - \delta) \mathbb{E}_t \frac{\varsigma_{t+1}}{s_t} = \frac{1}{\beta}, \]

where \( \varsigma_t \) is the Lagrange multiplier on the budget constraint.

APPENDIX 2: EQUILIBRIUM PRICE DISPERSION

\[ s_t := \int_0^1 \left[ \frac{P_{it}}{P_i} \right]^{-\theta} dt \]

\[ = (1 - \phi_p) \left[ \frac{\pi P_{it}}{P_i} \right]^{-\theta} + (1 - \phi_p) \phi_p \left[ \frac{\pi P_{it+1}}{P_i} \right]^{-\theta} + (1 - \phi_p) \phi_p^2 \left[ \frac{\pi^2 P_{it+2}}{P_i} \right]^{-\theta} + \ldots \]

\[ = (1 - \phi_p) \sum_{j=0}^{+\infty} \phi_p^j \left[ \frac{\pi^j P_{it+j}}{P_i} \right]^{-\theta} \]

\[ = (1 - \phi_p) \bar{\pi}_t^{-\theta} + \phi_p \left[ \bar{\pi} \right]^{-\theta} s_{t-1}. \]

PROOF: It follows from the assumptions that only the histories of no price re-optimization matter for price-setting decisions, and that price re-negotiations set the same new prices.

\[ \text{PROPOSITION 2: Equation (A1.3) can be re-written in a recursive fashion:} \]

\[ x_1^{t} = \frac{\theta - 1}{\theta} x_2^{t}, \]
with the following terms:

\[
x_1^t := E_t \sum_{n=0}^{\infty} \Xi_{t+n|t} \phi_p^n mc_{t+n} \left( \frac{\pi^n \tilde{P}_t}{P_{t+n}} \right)^{-\theta-1} y_{t+n}
\]

\[
= mc_t \tilde{p}_t [\theta - y_t + \phi_p E_t \Xi_{t+1|t} \left[ \frac{\pi \tilde{p}_t}{\pi_{t+1} \tilde{p}_{t+1}} \right]^{-\theta-1} x_{t+1}^1]
\]

\[
x_2^t := E_t \sum_{n=0}^{\infty} \Xi_{t+n|t} \phi_p^n \left( \frac{\pi^n \tilde{P}_t}{P_{t+n}} \right)^{-\theta} y_{t+n}
\]

\[
= [\tilde{p}_t ]^{-\theta} y_t + \phi_p E_t \Xi_{t+1|t} \left[ \frac{\pi \tilde{p}_t}{\pi_{t+1} \tilde{p}_{t+1}} \right]^{-\theta} x_{t+1}^2
\]

\[
\tilde{p}_t := \frac{\tilde{P}_t}{P_t}.
\]

**APPENDIX 3: MODEL EQUATIONS**

The equations coded into the solution algorithm are the following:

1. \( \frac{1}{c_t} = \varsigma (1 + \tau c_t) \)
2. \( \gamma = (1 - \tau^k) w_t \ell_t \)
3. \( E_t \frac{R_t}{\pi_{t+1}} = \frac{1}{\beta} \)
4. \( E_t \frac{R_t}{\pi_{t+1}} \left[ (1 - \tau^k r_{t+1} + \tau^k \delta) + (1 - \delta) \frac{R_t}{\pi_{t+1}} \right] = \frac{1}{\beta} \)
5. \( x_1^t = mc_t \tilde{p}_t^{-\theta-1} y_t + \phi_p E_t \beta \frac{\Xi_{t+1|t}}{\pi_{t+1} \tilde{p}_{t+1}} \left[ \frac{\pi \tilde{p}_t}{\pi_{t+1} \tilde{p}_{t+1}} \right]^{-\theta-1} x_{t+1}^1 \)
6. \( x_1^t = \frac{\theta - 1}{\theta} x_2^t \)
7. \( 1 = \phi_p \left( \frac{\pi}{\pi_t} \right)^{1-\theta} + (1 - \phi_p) (\tilde{p}_t)^{1-\theta} \)
8. \( (1 - \alpha) mc_t \frac{y_t}{\ell_t} = w_t \)
9. \( \frac{y_t}{k_t} - mc_t = r_t \)
10. \( y_t = \frac{z_t}{s_t} \ell_t \alpha \ell_t^{1-\alpha} \)
11. \( k_{t+1} = i_t + (1 - \delta) k_t \)
12. \( y_t = c_t + i_t + g_t \)
13. \( s_t = (1 - \phi_p) \tilde{p}_t^{-\theta} + \phi_p \left[ \frac{\pi_t}{\pi} \right]^\theta s_{t-1} \)
14. \( d_t + \tau c_t l_t + \tau^k (r_t - \delta) k_t + \tau^k w_t \ell_t = R_{t-1} d_{t-1}/\pi_t + g_t \)
15. \( \ln[z_{t+1}] = \rho z \ln[z_t] + \sigma z_{t+1} \)
16. \( \ln[g_{t+1}] = \rho g \ln[g_t] + (1 - \rho g) \ln[g_t] + \sigma g_{t+1} \)
The system is closed with the rules for fiscal and monetary policy.

**APPENDIX 4: THE STATE-SPACE REPRESENTATION OF THE MODEL**

The first-order conditions of the model economy can be arranged in the following way:

\[
E_t \mathcal{H} (e_{t+1}, e_t, x_{t+1}, x_t | \sigma) = 0,
\]

where \(y\) is a vector of co-state variables. The state variables are collected in \(x\):

\[
x_t := \begin{bmatrix} x_{1,t} \\ x_{2,t} \end{bmatrix},
\]

with vectors of endogenous state variables \(x_{1,t}\), and exogenous state variables \(x_{2,t}\):

\[
x_{2,t+1} = \Lambda_1 x_{2,t} + \Lambda_2 \sigma \epsilon_{t+1},
\]

with matrices \(\Lambda_1\) and \(\Lambda_2\). The scalar \(\sigma \geq 0\) is known.

With steady-state indexation, and consumption taxes as the fiscal-policy instrument, I define the following vectors and matrices (analogous vectors are defined in the other cases):

\[
x_{1,t} = \begin{bmatrix} k_t & s_t & d_{t-1} & R_{t-1} \end{bmatrix}',
\]

\[
x_{2,t} = \begin{bmatrix} z_t & g_t \end{bmatrix}',
\]

\[
e_t = \begin{bmatrix} y_t & R_t & d_t & m_t & c_t & \pi_t & \ell_t & w_t & \sigma_t \end{bmatrix}',
\]

\[
\Lambda_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},
\]

\[
\Lambda_3 = \begin{bmatrix} 0 & 0 & 0 & \sigma_z & 0 \\ 0 & 0 & 0 & \sigma_g \end{bmatrix}'.
\]

**APPENDIX 5: COMPUTATION OF CONDITIONAL SECOND MOMENTS**

The computation of conditional moments requires analytical formulas for multistep-ahead forecasts. Kim, Kim, Schaumburg, and Sims (2003) suggest that using the expressions of the full second-order approximation for the recursive calculation of these forecasts introduces spurious higher-order terms. This problem can be avoided by exploiting the linear (first-order) part of the solution.

Let \(\hat{\epsilon}_t^{(2)}\) denote the full second-order solution, and \(\hat{\epsilon}_t^{(1)}\) denote the linear part. We can write the system of solutions

\[
\begin{bmatrix} \hat{\epsilon}_t^{(2)} \\ \hat{\epsilon}_t^{(1)} \end{bmatrix} = M_1 \begin{bmatrix} \hat{x}_t^{(2)} \\ \hat{x}_t^{(1)} \end{bmatrix} + K_1,
\]

\[
\begin{bmatrix} \hat{x}_{t+1}^{(2)} \\ \hat{x}_{t+1}^{(1)} \end{bmatrix} = M_2 \begin{bmatrix} \hat{x}_t^{(2)} \\ \hat{x}_t^{(1)} \end{bmatrix} + K_2 + u_{t+1}.
\]

Define

\[
X_t = \begin{bmatrix} \hat{x}_t^{(2)} \\ \hat{x}_t^{(1)} \end{bmatrix},
\]

(A5.3)
Equations A5.2 and A5.1 can be re-written by repeated substitution as

\[ X_{t+k} = M_2^k X_t + \sum_{i=0}^{k-1} M_2^i (K_2 + u_{t+k-i}), \]

(A5.5)

\[ Y_{t+k} = M_1 X_{t+k} + K_1 = K_1 + M_1 M_2^k X_t + \sum_{i=0}^{k-1} M_1 M_2^i (K_2 + u_{t+k-i}). \]

(A5.6)

The expectation conditional on an initial state vector takes the form

\[ E (Y_{t+k} | X_t) = K_1 + M_1 M_2^k X_t + \sum_{i=0}^{k-1} M_1 M_2^i K_2. \]

(A5.7)

The conditional variance can be computed from

\[ Y_{t+k} - E (Y_{t+k} | X_t) = \sum_{i=0}^{k-1} M_1 M_2^i u_{t+k-i}, \]

(A5.8)

\[ \text{Cov}(Y_{t+k} | X_t) = E \left\{ (Y_{t+k} - E (Y_{t+k} | X_t)) (Y_{t+k} - E (Y_{t+k} | X_t))' | X_t \right\}, \]

(A5.9)

\[ = \sum_{i=0}^{k-1} M_1 M_2^i \Sigma_u (M_1 M_2^i)' , \]

(A5.10)

where \( \Sigma_u := E(u_t u_t') \). From Paustian (2003), we know that \( u_t \) takes the form

\[ u_t = \begin{pmatrix} \sigma N_{\epsilon_t} \\
\sigma^2 (N \otimes N) (\text{vec}(I) - \epsilon_t \otimes \epsilon_t) \end{pmatrix}. \]

(A5.11)

Marzo, Strid, and Zagaglia (2006) show that the variance matrix of \( u_t \) is

\[ \text{E} u_t u_t' = \begin{pmatrix} \sigma^2 N N' & 0 \\
0 & 2\sigma^4 (N \otimes N) \text{vec}(I) \text{vec}(I)' (N \otimes N)' \end{pmatrix}. \]

(A5.12)
REFERENCES


Figure 1—: Impulse responses to a positive productivity shock (%), model with price rigidity
Figure 2—: Variance of inflation (%) as a function of $\alpha_y$, model with price rigidity

Legend: For each value of $\alpha_y$, I search for the remaining parameters of the policy rules that maximize $W_0$. This figure reports the percentage variance of inflation at each (constrained) welfare-maximizing combination of parameters.
Figure 3—: Impulse responses to a positive productivity shock (%) as a function of $\alpha_y$ in labor-income taxes, model with price rigidity

Legend: The values of the other policy parameters are chosen to maximize $W_0$ for each value of $\alpha_y$. 
Figure 4—: Percentage change in welfare costs from policy mistakes on $\alpha_y$, model with price rigidity

Legend: Denote by a star the parameter values that maximize $W_0(\cdot)$ over the full grid $\{\alpha_\pi, \alpha_y, \alpha_R, \psi_1\}$. The corresponding conditional welfare cost is $\Phi_c^i(\alpha^*_\pi, \alpha^*_y, \alpha^*_R, \psi^*_1)$. For a given $\alpha_y$, this figure plots

$$100 \times \left[ \Phi_c^i(\alpha^*_\pi, \alpha_y, \alpha^*_R, \psi^*_1) - \Phi_c^i(\alpha^*_\pi, \alpha^*_y, \alpha^*_R, \psi^*_1) \right] / \Phi_c^i(\alpha^*_\pi, \alpha^*_y, \alpha^*_R, \psi^*_1).$$
LEGEND: Denote by a star the unconstrained parameter values that maximize \( W_0 ( \cdot ) \) over the full grid \( \{ \alpha_\pi, \alpha_y, \alpha_R, \psi_1 \} \). The corresponding conditional welfare cost is \( \Phi^c ( \alpha_\pi^*, \alpha_y^*, \alpha_R^*, \psi_1^* ) \). For a given \( \alpha_y \), a new set of welfare-maximizing parameters \( \{ \bar{\alpha}_\pi, \alpha_y, \bar{\alpha}_R, \bar{\psi}_1 \} \) is computed with \( \Phi^c ( \bar{\alpha}_\pi, \alpha_y, \bar{\alpha}_R, \bar{\psi}_1 ) \). This figure plots
\[
100 \times \left[ \Phi^c ( \bar{\alpha}_\pi, \alpha_y, \bar{\alpha}_R, \bar{\psi}_1 ) - \Phi^c ( \alpha_\pi^*, \alpha_y^*, \alpha_R^*, \psi_1^* ) \right] / \Phi^c ( \alpha_\pi^*, \alpha_y^*, \alpha_R^*, \psi_1^* ).
\]
Figure 6—: Percentage change in welfare costs from policy mistakes on $\psi_1$, model with price rigidity

Legend: Denote by a star the parameter values that maximize $W_0(\cdot)$ over the full grid $\{\alpha_{\pi}, \alpha_y, \alpha_R, \psi_1\}$. The corresponding conditional welfare cost is $\Phi_c^t(\alpha^*_\pi, \alpha^*_y, \alpha^*_R, \psi^*_1)$. For a given $\psi_1$, this figure plots

$$100 \times \left[ \Phi_c^t(\alpha^*_\pi, \alpha^*_y, \alpha^*_R, \psi_1) - \Phi_c^t(\alpha^*_\pi, \alpha^*_y, \alpha^*_R, \psi^*_1) \right] / \Phi_c^t(\alpha^*_\pi, \alpha^*_y, \alpha^*_R, \psi^*_1).$$
Legend: Denote by a star the parameter values that maximize $W_0(\cdot)$ over the full grid $\{\alpha_\pi, \alpha_y, \alpha_R, \psi_1\}$. The corresponding conditional welfare cost is $\Phi_\ell (\alpha_\pi^*, \alpha_y^*, \alpha_R^*, \psi_1^*)$. For a given $\alpha_R$, this figure plots

$$100 \times \left[ \Phi_\ell (\alpha_\pi^*, \alpha_y^*, \alpha_R^*, \psi_1^*) - \Phi_\ell (\alpha_\pi^*, \alpha_y^*, \alpha_R^*, \psi_1^*) \right] / \Phi_\ell (\alpha_\pi^*, \alpha_y^*, \alpha_R^*, \psi_1^*) .$$
Figure 8—: Impulse responses to a positive productivity shock (%) with rule for consumption taxes, model with flexible prices

Legend: The parameter values of the underlying policy rules are chosen to maximize conditional welfare. Passive fiscal policy restricts $\psi_1 \in [0, 1.9]$, whereas for active fiscal policy $\psi_1 \in [2, 4]$. The optimal rules with active fiscal policy are: $\hat{R}_t = 1.1\hat{\pi}_t + 0.3\hat{y}_t$ and $\tau^c_t = 0.571 + 2.0(t_{t-1} - t)$. 
Figure 9: Percentage change in welfare costs from policy mistakes on $\psi_1$, model with flexible prices

Legend: Denote by a star the parameter values that maximize $\mathcal{W}_0(\cdot)$ over the full grid $\{\alpha_\pi, \alpha_y, \alpha_R, \psi_1\}$. The corresponding conditional welfare cost is $\Phi_\epsilon^c(\alpha_\pi^*, \alpha_y^*, \alpha_R^*, \psi_1^*)$. For a given $\psi_1$, this figure plots

$$100 \times \left[ \Phi_\epsilon^c(\alpha_\pi, \alpha_y, \alpha_R, \psi_1) - \Phi_\epsilon^c(\alpha_\pi^*, \alpha_y^*, \alpha_R^*, \psi_1^*) \right] / \Phi_\epsilon^c(\alpha_\pi^*, \alpha_y^*, \alpha_R^*, \psi_1^*)$$.
Figure 10—: Impulse responses to a positive productivity shock (%)
Legend: Denote by a star the parameter values that maximize $\mathcal{W}_0(\cdot)$ over the full grid $\{\alpha_x, \alpha_y, \alpha_R, \psi_1\}$. The corresponding conditional welfare cost is $\Phi^c_\epsilon (\alpha^*_x, \alpha^*_y, \alpha^*_R, \psi^*_1)$. For a given $\alpha_y$, this figure plots

$$100 \times \left[ \Phi^c_\epsilon (\alpha^*_x, \alpha_y, \alpha^*_R, \psi^*_1) - \Phi^c_\epsilon (\alpha^*_x, \alpha_y, \alpha^*_R, \psi^*_1) \right] / \Phi^c_\epsilon (\alpha^*_x, \alpha_y, \alpha^*_R, \psi^*_1).$$

Figure 11—: Percentage change in welfare costs from policy mistakes on $\alpha_y$, model with flexible prices.
Figure 12—: Percentage change in welfare costs from policy mistakes on $\alpha_R$, model with flexible prices

Legend: Denote by a star the parameter values that maximize $W_0(\cdot)$ over the full grid $\{\alpha_x, \alpha_y, \alpha_R, \psi_1\}$. The corresponding conditional welfare cost is $\Phi_c^{\alpha}(\alpha_x^*, \alpha_y^*, \alpha_R^*, \psi_1^*)$. For a given $\alpha_R$, this figure plots

$$100 \times \left[ \Phi_c^{\alpha}(\alpha_x^*, \alpha_y^*, \alpha_R, \psi_1^*) - \Phi_c^{\alpha}(\alpha_x^*, \alpha_y^*, \alpha_R^*, \psi_1^*) \right] / \Phi_c^{\alpha}(\alpha_x^*, \alpha_y^*, \alpha_R^*, \psi_1^*)$$.
<table>
<thead>
<tr>
<th>Description</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Subjective discount factor</td>
<td>$\beta$</td>
<td>0.9948</td>
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<tr>
<td>Weight on disutility from work</td>
<td>$\gamma$</td>
<td>1</td>
</tr>
<tr>
<td>Elasticity of substitution of intermediate goods</td>
<td>$\theta$</td>
<td>5</td>
</tr>
<tr>
<td>Rate of capital depreciation</td>
<td>$\delta$</td>
<td>$1.1^{(1/4)} - 1$</td>
</tr>
<tr>
<td>Capital elasticity of intermediate output</td>
<td>$\alpha$</td>
<td>0.4</td>
</tr>
<tr>
<td>Fraction of firms not setting prices optimally</td>
<td>$\phi_p$</td>
<td>2/3</td>
</tr>
<tr>
<td>Steady-state inflation</td>
<td>$\bar{\pi}$</td>
<td>$1.04^{(1/4)}$</td>
</tr>
<tr>
<td>Persistence of productivity shock</td>
<td>$\rho_z$</td>
<td>0.92</td>
</tr>
<tr>
<td>Standard dev. of productivity shock</td>
<td>$\sigma_z$</td>
<td>0.01</td>
</tr>
<tr>
<td>Steady-state average consumption taxes</td>
<td>$\bar{\tau}^c$</td>
<td>0.12</td>
</tr>
<tr>
<td>Steady-state average capital-income taxes</td>
<td>$\bar{\tau}^k$</td>
<td>0.36</td>
</tr>
<tr>
<td>Steady-state average labor-income taxes</td>
<td>$\bar{\tau}^\ell$</td>
<td>0.3168</td>
</tr>
<tr>
<td>Steady state ratio of gov. transfers to output</td>
<td>$\bar{tr}/\bar{y}$</td>
<td>0.1005</td>
</tr>
<tr>
<td>Steady state ratio of gov. consumption to output</td>
<td>$\bar{g}^e/\bar{y}$</td>
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<tr>
<td>Persistence of government spending shock</td>
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<tr>
<td>Standard dev. of government spending shock</td>
<td>$\sigma_g$</td>
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### Table II: Optimized policy rules with price rigidity

<table>
<thead>
<tr>
<th>[a] Fully optimized monetary-policy rules</th>
<th>α_π</th>
<th>α_y</th>
<th>α_R</th>
<th>ψ_1</th>
<th>W_0^0</th>
<th>%Φ^t_c</th>
<th>%Δ^t_E</th>
<th>%Δ^t_V</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lump-sum taxes*</td>
<td>1.1</td>
<td>0.0</td>
<td>0.9</td>
<td>0.6</td>
<td>65.3633</td>
<td>6.14</td>
<td>-9.05</td>
<td>13.930</td>
</tr>
<tr>
<td>Consumption taxes</td>
<td>1.1</td>
<td>0.0</td>
<td>0.5</td>
<td>0.2</td>
<td>-54.4477</td>
<td>-0.002</td>
<td>-16.89</td>
<td>13.931</td>
</tr>
<tr>
<td>Capital-income taxes</td>
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<td>0.0</td>
<td>1.0</td>
<td>1.9</td>
<td>-54.4607</td>
<td>0.004</td>
<td>-16.18</td>
<td>13.931</td>
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<tr>
<td>Labor-income taxes</td>
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<td>1.0</td>
<td>0.0</td>
<td>0.2</td>
<td>-54.4627</td>
<td>0.005</td>
<td>-16.17</td>
<td>13.929</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>[b] No interest-rate smoothing (α_R = 0)</th>
<th>α_π</th>
<th>α_y</th>
<th>α_R</th>
<th>ψ_1</th>
<th>W_0^0</th>
<th>%Φ^t_c</th>
<th>%Δ^t_E</th>
<th>%Δ^t_V</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lump-sum taxes*</td>
<td>2.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.5</td>
<td>65.3254</td>
<td>6.16</td>
<td>-9.05</td>
<td>13.930</td>
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<td>-16.18</td>
<td>13.931</td>
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<td>0.0</td>
<td>0.2</td>
<td>-54.4627</td>
<td>0.010</td>
<td>-16.17</td>
<td>13.929</td>
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</table>

<table>
<thead>
<tr>
<th>[c] Taylor rules (α_π = 1.5, α_y = 0.5, α_R = 0)</th>
<th>α_π</th>
<th>α_y</th>
<th>α_R</th>
<th>ψ_1</th>
<th>W_0^0</th>
<th>%Φ^t_c</th>
<th>%Δ^t_E</th>
<th>%Δ^t_V</th>
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</thead>
<tbody>
<tr>
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<table>
<thead>
<tr>
<th>[d] Balanced-budget targets (ψ_2 = 1)</th>
<th>α_π</th>
<th>α_y</th>
<th>α_R</th>
<th>ψ_1</th>
<th>W_0^0</th>
<th>%Φ^t_c</th>
<th>%Δ^t_E</th>
<th>%Δ^t_V</th>
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<tbody>
<tr>
<td>Lump-sum taxes</td>
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<td>0.0</td>
<td>0.9</td>
<td>0.6</td>
<td>65.3633</td>
<td>6.14</td>
<td>-9.05</td>
<td>13.930</td>
</tr>
<tr>
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<td>-16.19</td>
<td>13.930</td>
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<td>Capital-income taxes</td>
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<td>-</td>
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<tr>
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<td>0.4</td>
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<td>13.929</td>
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<table>
<thead>
<tr>
<th>[e] Inflation targeting (π_t = 0)</th>
<th>α_π</th>
<th>α_y</th>
<th>α_R</th>
<th>ψ_1</th>
<th>W_0^0</th>
<th>%Φ^t_c</th>
<th>%Δ^t_E</th>
<th>%Δ^t_V</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lump-sum taxes</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
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<tr>
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<td>-0.004</td>
<td>0.002</td>
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<td>-</td>
<td>0.0003</td>
<td>-0.001</td>
<td>0.002</td>
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<td>-</td>
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Legend: *Any combination of passive fiscal policy for lump-sum taxes with ψ_1 ∈ [0.1, 1.9] yields the same welfare level.
### TABLE III:

**Optimized policy rules in a model with flexible prices**

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<thead>
<tr>
<th></th>
<th>( \alpha_x )</th>
<th>( \alpha_y )</th>
<th>( \alpha_R )</th>
<th>( \psi_1 )</th>
<th>( W_0^e )</th>
<th>%( \Phi_c^t )</th>
<th>%( \Delta^t_E )</th>
<th>%( \Delta^t_V )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Lump-sum taxes</strong></td>
<td>0.9</td>
<td>0.2</td>
<td>0.0</td>
<td>3.7</td>
<td>65.3317</td>
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<td>6.11</td>
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<table>
<thead>
<tr>
<th></th>
<th>( \alpha_x )</th>
<th>( \alpha_y )</th>
<th>( \alpha_R )</th>
<th>( \psi_1 )</th>
<th>( W_0^e )</th>
<th>%( \Phi_c^t )</th>
<th>%( \Delta^t_E )</th>
<th>%( \Delta^t_V )</th>
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<tr>
<td><strong>Lump-sum taxes</strong></td>
<td>0.9</td>
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<td>6.159</td>
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<th>( \alpha_x )</th>
<th>( \alpha_y )</th>
<th>( \alpha_R )</th>
<th>( \psi_1 )</th>
<th>( W_0^e )</th>
<th>%( \Phi_c^t )</th>
<th>%( \Delta^t_E )</th>
<th>%( \Delta^t_V )</th>
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<table>
<thead>
<tr>
<th></th>
<th>( \alpha_x )</th>
<th>( \alpha_y )</th>
<th>( \alpha_R )</th>
<th>( \psi_1 )</th>
<th>( W_0^e )</th>
<th>%( \Phi_c^t )</th>
<th>%( \Delta^t_E )</th>
<th>%( \Delta^t_V )</th>
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</thead>
<tbody>
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</tbody>
</table>

Legend: *The welfare function is flat over the values of \( \psi_1 \) that generate unique valid equilibria.*
TABLE IV: 
STANDARD DEVIATIONS FOR THE FULLY-OPTIMIZED RULES

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<th>Price rigidity</th>
<th>Price flexibility</th>
<th></th>
</tr>
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<tr>
<td></td>
<td>$\alpha_y = 0$</td>
<td>$\alpha_y \neq 0$</td>
<td></td>
</tr>
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<td>[a] Lump-sum taxes</td>
<td></td>
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<tr>
<td>Output</td>
<td>0.71</td>
<td>0.69</td>
<td>0.69</td>
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<td>[b] Consumption taxes</td>
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<td>[c] Capital-income taxes</td>
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Legend: This table reports unconditional standard deviations of selected variables generated by the optimized rules of panel [a] in Table II and Table III.