# War and peace - cyclical phenomena?*

Adam Jacobsson†

September 20, 2005

**Abstract**

This paper demonstrates how the analysis can differ dramatically between two common modeling approaches to conflict. The first approach uses a one-period setup and associates positive investments in arms with conflict, see, for example, Skaperdas [1992]. The second approach has two periods, where arming decisions are taken in the first period, and the decision on whether to go to war is taken separately in the second, see, for example, Brito and Intriligator [1985]. The second approach is then used to suggest a new possible explanation for the outbreak of war by showing how myopic players may end up in (Edgeworth) cycles of war and peace.

JEL codes: D74, C72
Keywords: Armed conflict, Edgeworth cycles

*I thank Martin Dufwenberg and Sten Nyberg for their advice, Karl Wärneryd and Bengt-Arne Wickström for helpful comments and Lars Johansson for late night computer support. Financial support from the Tore Browaldh Foundation is gratefully acknowledged.

†University of Stockholm, Department of Economics, e-mail: aja@ne.su.se.

# 1 Introduction

> "... history has clearly indicated that periods of stability have continually alternated with periods of instability, turmoil, and conflict."[1]

Most research on the important topic of war and peace has traditionally been done within the domain of political science. However, economic tools are being increasingly used within this field. To date, this literature has produced several explanations for the outbreak of war. As in most fields, the theoretical results differ across various modeling approaches and assumptions.

This paper has two main contributions. The first is to demonstrate in a simple setting how the analysis can dramatically differ between two common modeling approaches to conflict. The benchmark approach uses a one period setup and associates positive investments in arms with conflict, see, for example, Skaperdas [1992], Grossman and Kim [1995], Hirshleifer [1991][2]. In this approach, it is quite straightforward to find a (Nash) equilibrium in pure strategies. The other approach has two periods where arming decisions are taken in the first period, and the decision on whether to go to war is taken separately in the second, see, for example, Brito and Intriligator [1985]. In this case, finding an equilibrium is more complicated. In fact, for a wide range of model parameters, there is no equilibrium in pure strategies. The motivation for analysing this difference in modeling approaches is that the first approach implies that positive investments in appropriative conflict technologies would imply actual conflicts. This is clearly not the case, considering, for example, the fact that most nations have standing armies and are living in peace. Grossman and Kim [1995] note this and try to get around this by distinguishing between investments in purely defensive and offensive conflict technologies. That is, defensive investments cannot be used to attack opponents and vice versa. Such defensive investments could be, for example, fortifications and ground-to-air missiles. However, modern warfare is less and less characterized by fixed fortifications and more by mobile units capable of being used in both defensive as well as offensive roles. Therefore, even though it might seem to be analytically convenient to utilise the first approach when analysing potential and actual conflict, the second approach seems to be, albeit more complicated, much more realistic.

[1] Garfinkel and Skaperdas [1996], p 2.

[2] All papers concern general equilibrium models which capture the trade-off between investing in a consumption- and an appropriation technology. They all have in common the fact that any non-zero investment in a technology that increases the chance of appropriating the opponent's wealth implies the outbreak of a conflict.

The second contribution of my paper is to suggest a new possible explanation for the outbreak of war by showing how myopic players may end up in cycles of war and peace. Even though the setting of the model is specific and simple, I believe it can shed some light on a more general principle that can help explain the phenomenon of instability of peace in history. This phenomenon is also shown to be analogous to Edgeworth cycles in a Bertrand game with capacity constraints.

The economics literature on conflict spans across the behavior of individuals in anarchic conditions, Hirshleifer [1995], Skaperdas [1992], Wärneryd [1993], armed conflict between nations, Hess and Orphanides [1997], civil wars and insurrections, Grossman [1991]. This literature offers several explanations to the outbreak of war. Two classic examples are asymmetric information among adversaries, see, for example, Brito and Intriligator [1985], Bester and Wärneryd [1998], and the inability to credibly commit to a peaceful resolution of conflict. In the first explanation, antagonists may be uncertain about some characteristics of their opponent such as military strength, aversion to casualties etc. This may then urge the uninformed player to wage war as he might believe himself to be very likely to win such a conflict at a sufficiently low cost. Skaperdas and Syropoulos [2000] contend that the long-run effects of armed conflict may make war more likely. This "inverted folk-theorem" states that if going to war will now seriously weaken your opponent for a long time, the costs of war today may be outweighed by future gains. Other explanations concern shifts in model parameters such as changes in civilian productivity, Skaperdas [1992], the discovery of unclaimed valuable resources which provokes a violent contest, Alesina and Spolaore [2001] etc.

I will demonstrate yet another possible mechanism that may lead to war. In contrast to the farsightedness of decision-makers in Skaperdas and Syropoulos [2000], this paper shows how myopia may cause cycles of periods of peace and war. Myopic behavior entails that countries only care about today and fail to grasp the future consequences of today's choices. Neither do they learn from the past. This could be the case if, for example, politicians fail to comprehend future implications of today's actions and/or do not care a great deal about the distant future. If terms in office are short, for example, political decision-makers might not give the future much attention. Also, a sequential model setup with respect to decisions about armament levels appears reasonable, as it can take years to fully implement new defence budgets. Further, countries are not likely to be coordinated in the timing of defence budget decisions.

The paper is structured as follows: Section 2 presents a one-stage simultaneous move benchmark model and its main results. Section 3 uses the same basic set-up as in section 2, but within a two-stage framework and separating between the decisions on arming levels and whether to go to war. Section 4 considers a sequential, multistage version of the two-stage model with myopic players, and shows how war may occur cyclically. This section also explores an analogy to Edgeworth cycles in Bertrand competition with capacity constraints. Section 5 contains the concluding remarks.

# 2 A one-stage benchmark model

Consider two identical countries, 1 and 2. Each country consists of some contestable and some non-contestable land. The contestable land (could also be a territory of water) borders both countries and is initially equally divided between countries 1 and 2, and contains some resource with a value of one. Such a resource could, for example, be an oil field, a diamond mine, water supply, strategic coastline, industrial region or some other valuable piece of land.

Each country has an exogenous budget of size $b$, which can be spent on arms $G_i \in [0, b]$, $i \in \{1, 2\}$ or consumption. Utility equals consumption (assuming linear utility). Country $i$ can increase consumption by fighting over the contestable land with country $j$, where arms investments determine the expected outcome of the conflict. If either country invests any positive amount in arms, a war will occur.[3] This assumption may seem stark, but is common in the literature on conflict where resources are distributed according to relative investments in a conflict technology. The next section will show the consequences of relaxing this assumption.

Following for example Tullock [1980], I assume a simple conflict technology where the probability of winning is determined by the ratio of a country's military strength to the sum of both countries' strength.[4] The probability of winning a conflict for country $i$ is

$$P(G_i, G_j) = \frac{G_i}{G_i + G_j}, \tag{1}$$

if the denominator is non-zero while $P(0, 0) = \frac{1}{2}$. $P(G_i, G_j)$ can be interpreted in different ways. The most literal interpretation is that it reflects the chance of appropriation of the entire "cake". Another interpretation is that it is a weight representing the expected split of the land that would

[3]Some models in the literature allow for negotiated transfers of resources in the shadow of war. As the difference in analysis between the two modeling approaches of this paper is clearer without the transfer mechanism, I choose not to include it.

[4]For an axiomatized description of this functional form, see Skaperdas [1996].

emerge from a war. As I have assumed risk neutral preferences, the choice of interpretation is of no importance; see Skaperdas [1992] for a discussion.

Actual wars are generally destructive. Any analysis of the appropriative motives for war must take this into account. I denote the fraction of resources not destroyed as a result of war by $\delta$, where $\delta \in [0,1]$. Note that the degree of destruction is the same whatever the level of arming, which may seem like a stark simplification. An alternative functional specification could make the degree of destruction a positive function of the level of armaments as in, for example, Brito and Intriligator [1985]. However, doing this will make the analytics much more cumbersome while not adding crucial information. Therefore I choose the above simple functional form as does, for example, Grossman and Kim [1995]. The important feature is that the winner gains less than what the loser loses. Apart from physical destruction, this additional loss can be caused by severed trade links and psychological (stock) market effects which may appear at the event of a declaration of war even though the actual armament intensity of the conflict may be low. The payoff from war to country $i$ is then:[5]

$$U_i^{war} = b - G_i + \delta \frac{G_i}{G_i + G_j}. \tag{2}$$

The corresponding payoff from not investing in the conflict technology and enjoying peace, given that country $j$ does not invest, is:

$$U_i^{peace} = b + \frac{1}{2}. \tag{3}$$

How low can $\delta$ then be for war to be possible? Clearly, should the entire resource be destroyed by war, that is, $\delta = 0$, then nothing can be gained by war and neither country would invest in arms. Comparing the expected payoffs from war and peace:

$$U_i^{war} - U_i^{peace} = \delta \frac{G_i}{G_i + G_j} - G_i - \frac{1}{2}.$$

[5] Due to the assumption of linear utility, we can disregard the utility of holding non-contestable land. An alternative specification which would make the size of the "cake" a negative function of total armaments (capturing the effect that resources spent on arms could have been used to produce some valuable good), could be: $U_i^{war} = \delta \frac{G_i(1-G_i-G_j)}{G_i+G_j} - G_i$. This formulation makes analytical representation much more difficult, but simulations indicate that the qualitative results are the same in my analysis. See Neary [1997] for a discussion on the difference between these two approaches.

War will not be profitable if $U_i^{war} - U_i^{peace} \leqslant 0$. We note that $\frac{G_i}{G_i+G_j}$ has a maximum value of 1, and the cheapest possible way of achieving this for country $i$ is by spending an arbitrarily small amount on arms while country $j$, for some reason, does not arm at all. This is the best possible war payoff, and $\delta$ must not be smaller than $\frac{1}{2}$ for $U_i^{war} - U_i^{peace} \geqslant 0$ to hold. This observation is summarized in lemma 1.

**Lemma 1** *If war is sufficiently destructive, that is, $\delta \leq \frac{1}{2}$, no player can gain from going to war.That is, the unique Nash equilibrium is then $G_1^*= G_2^*= 0$.*

Given that $\delta > \frac{1}{2}$, how much will then each country invest in arms? Let us assume a non-binding budget constraint, that is, $G_i \leq b$, $i = 1, 2$. Taking the first-order condition with respect to $G_i$:

$$\frac{\delta U_i^{war}}{\delta G_i} = \delta \frac{G_j}{\left(G_i + G_j\right)^2} - 1 = 0. \tag{4}$$

Solving for the best response level of armaments for country $i$ holding $G_j$ constant:

$$G_i\left(G_j\right) = -G_j + \sqrt{\delta G_j.} \tag{5}$$

Figure 1 plots both countries' best reply functions for $\delta = 0.8$ where the dashed line denotes $G_1\left(G_2\right)$ and the full line $G_2\left(G_1\right)$:

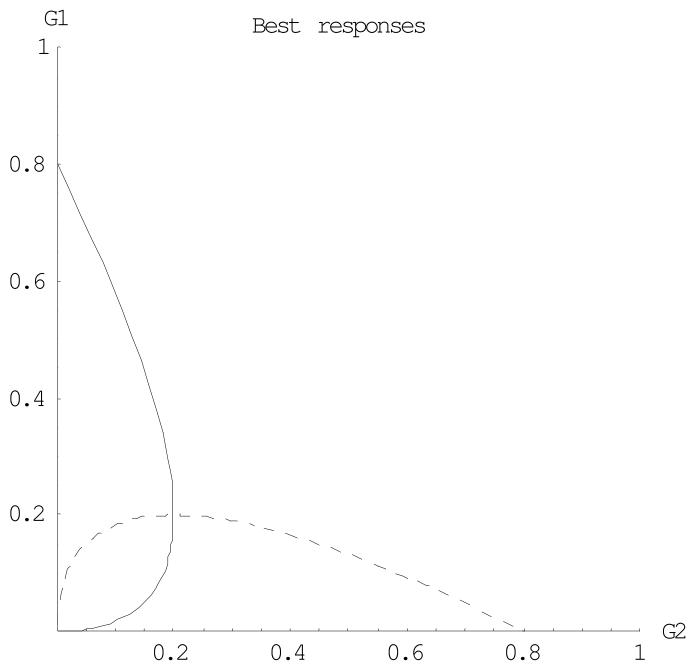


Figure 1. Best response functions for $\delta = 0.8$.

Using equation 5 to solve for the symmetric Nash equilibrium, given that $\delta > \frac{1}{2}$:

$$G_1 = G_2 = G^* = \frac{\delta}{4}, \tag{6}$$

which corresponds to the intersection of the two best response curves in figure 1. Could the strategy profile $G_1 = G_2 = 0$ be a Nash equilibrium? No, since unilateral deviation from this profile, by for example country 1, by setting $G_1$ arbitrarily small to $\varepsilon > 0$, would win the entire resource, less destruction.

This representation is clear cut, as we have a unique symmetric war equilibrium point for the interesting interval of $\delta \in (\frac{1}{2}, 1]$. However, let us proceed to see what the consequences of allowing countries to arm without actually entering into a conflict are. It appears natural to allow for this as arming is quite often intended to deter enemy attacks rather than for offensive warfare. Further, using a two-stage model will better reflect the fact that it may take years to implement armament decisions.

# 3 A two-stage simultaneous move model with rational behavior

3 *A Players, strategies and payoffs*

Consider the same setup as in the previous section, the only difference being that spending a positive amount on arms does not necessarily mean war. I set this up as a two-stage game similar to that of Brito and Intriligator [1985]. In period one, the countries simultaneously choose their level of armaments, $G_i \in [0, \frac{1}{2}]$. We could think of this as parliaments deciding on the size of defense budgets. In period two, the countries observe each other's levels of armaments and decide whether to go to war. Hence, player $i$'s strategy space is $S_i = \big(G_i \in [0, \frac{1}{2}],\ \text{war or peace}\big)$. War will be the outcome if at least one player opts for war as the other player is then called upon to defend himself. As positive armaments are allowed in the peace outcome, the expected payoff from peace is now:

$$U_i^{peace} = b - G_i + \frac{1}{2}. \tag{7}$$

*3 B Best response functions*

Let us proceed by solving for the subgame perfect equilibria. When deciding on whether to go to war in period two, the countries observe realized levels of armaments and compare the payoffs from war and peace. The difference in utility from going to war and opting for peace for country $i$ is:

$$U_i^{war} - U_i^{peace} = \delta \frac{G_i}{G_i + G_j} - \frac{1}{2}. \tag{8}$$

Country $i$'s period two best choice between war and peace is conditioned on the difference in payoffs, given period one armament levels ($G_i$ and $G_j$):

$$\text{Choice period 2} = \begin{cases} \text{War} & \text{if } U_i^{war} - U_i^{peace} > 0 \\ \text{War or peace} & \text{if } U_i^{war} - U_i^{peace} = 0 \\ \text{Peace} & \text{if } U_i^{war} - U_i^{peace} < 0 \end{cases}. \tag{9}$$

To solve for the best response functions, we need to separate between peace and war outcomes. If country $i$ expects peace, we see from equation 7 that its payoff is monotonically decreasing in $G_i$. Consequently, it will set as small a $G_i$ as possible, just making country $j$ indifferent between war and peace. Setting equation 8 equal to zero and solving for the minimal level of $G_i$ that will make country $j$ indifferent, denoted by $G_i^{peace}$, yields:

$$G_i^{peace} = G_j[2\delta - 1]. \tag{10}$$

Hence, if country $i$ sets $G_i$ below $G_i^{peace}$, country $j$ goes to war. If a country expects war, it wants to maximize its payoff function from war, equation 2. The best response function for the war case was derived earlier in equation 5, which is now denoted by $G_i^{war}(G_j)$.

When solving the game, we need to know which best response function is relevant for each combination of $G_i$ and $G_j$. We then need to examine under which conditions opting for war is optimal. Looking at equation 8 and noting that $\frac{G_j}{G_i+G_j} \in [0,1]$, we see, as in the one-stage model, that $\delta$ must not be smaller than $\frac{1}{2}$ for there to be any gains from war, whatever the armament levels.

Now consider the case when $\delta \in (\frac{1}{2}, 1]$. Let us return to the comparison of country $j$'s war and peace payoffs. This time, we let country $j$ play a best response to $G_i$ and then solve for the minimum absolute value of $G_i$ that keeps country $j$ indifferent, denoted by $G_i^{\min}$, which, by symmetry, equals $G_j^{\min}$. To solve for $G_i^{\min}$ I insert $G_j^{war}(G_i)$ and $G_j^{peace}(G_i)$ into equations 2 and 7 and take the difference:

$$U_j^{war} - U_j^{peace} = \delta \frac{G_j^{war}(G_i)}{G_i + G_j^{war}(G_i)} - G_j^{war}(G_i) - (\frac{1}{2} - G_j^{peace}(G_i)).$$

Set this difference to zero and solve for $G_i^{\min}$:

$$G_i^{\min} = G_j^{\min} = G^{\min} = \frac{1}{\delta}\left(\frac{1}{2} - \sqrt{\frac{1-\delta}{2}}\right)^2 \tag{11}$$

which is an increasing function in $\delta$ for $\delta \in [\frac{1}{2}, 1]$. For example, if country $i$ believes that country $j$ is going to set $G_j > G^{\min}$, the best country $i$ can do is to play $G_i^{peace}(G_j)$. On the other hand, if it believes that country $j$ will set $G_j < G^{\min}$, it will play $G_i^{war}(G_j)$. Thus, if any country sets its level of armaments lower than $G^{\min}$, there will be war.

Now, we know in which intervals of $G_i$ and $G_j$ each best response function is relevant. Country $i$'s best reply correspondence can be summarized as:

$$G_i(G_j) = \begin{cases} -G_j + \sqrt{\delta G_j} & \text{if } G_j < G^{\min} \\ -G_j + \sqrt{\delta G_j} \text{ or } G_j[2\delta - 1] & \text{if } G_j = G^{\min} \\ G_j[2\delta - 1] & \text{if } G_j > G^{\min} \end{cases}. \tag{12}$$

*3 C. Destructiveness of war and the scope for peace.*

Clearly, the destructiveness of war determines how much may be gained by initiating a conflict. For example, consider the case where $\delta = 0$, that is, war destroys everything of value. As nothing can be gained from war, no arming will take place, that is, the unique Nash equilibrium armament levels are zero. However, these levels are compatible with two possible Nash equilibria $s^{NE_1}(\delta = 0) = \{(G_i = 0, \text{ peace}), (G_j = 0, \text{peace})\}$, and $s^{NE_2}(\delta = 0) = \{(G_i = 0, \text{ war}), (G_j = 0, \text{ war})\}$. However, the strategy $s_i^{'}(\delta = 0) = (G_i = 0,$ war) is weakly dominated by $s_i(\delta = 0) = (G_i = 0, \text{ peace})$ as the former always yields a payoff of zero, while the latter yields either zero or $\frac{1}{2}$. I henceforth eliminate weakly dominated strategies. It also appears unintuitive to assume that countries would ever go to war in this setting,

How destructive can war be for its possible existence as an equilibrium outcome? By lemma 1, we know that if $\delta \leq \frac{1}{2}$, no country can gain by going to war. That is, if war will destroy at least half of all resources (that is, the entire share of one country), there is no point in trying to appropriate a neighbor's wealth. Proposition 1 summarizes these findings:

**Proposition 1** *If war is sufficiently destructive, that is,* $\delta \leq \frac{1}{2}$*, the unique Nash equilibrium, when eliminating weakly dominated strategies, is* $s^{NE} = \{(G_i = 0,\ \text{peace}), (G_j = 0,\ \text{peace})\}$.

What can we say about the outcome if war is less destructive, that is $\delta > \frac{1}{2}$? First, consider the extreme example when war is not destructive, that is, $\delta = 1$. Let us start by calculating $G^{\min}$ :

$$G^{\min}(\delta = 1) = \left(\frac{1}{2}\right)^2 = \frac{1}{4}, \tag{13}$$

which is what a country needs to arm if it wants peace. We look for the Nash equilibria by first solving the war scenario. We know that the best response for country $i$ in the war case is $G_i(G_j) = -G_j + \sqrt{\delta G_j}$. Due to symmetry, we can rewrite this as: $G = -G + \sqrt{\delta G}$. Solving for the symmetric Nash equilibrium level of armaments yields: $G^{NE}(\delta = 1) = \frac{1}{4}$. From equation 13, we see that the countries will be indifferent between war and peace at $G_i = G_j = \frac{1}{4}$. Hence, for $\delta = 1$ we have four Nash equilibria:

1. $s^{NE_1} = \left\{\left(G_i = \frac{1}{4}, \text{peace}\right), \left(G_j = \frac{1}{4}, \text{peace}\right)\right\}$
2. $s^{NE_2} = \left\{\left(G_i = \frac{1}{4}, \text{war}\right), \left(G_j = \frac{1}{4}, \text{peace}\right)\right\}$
3. $s^{NE_3} = \left\{\left(G_i = \frac{1}{4}, \text{war}\right), \left(G_j = \frac{1}{4}, \text{war}\right)\right\}$
4. $s^{NE_4} = \left\{\left(G_i = \frac{1}{4}, \text{peace}\right), \left(G_j = \frac{1}{4}, \text{war}\right)\right\}$.

Let us now consider the more general case where $\frac{1}{2} < \delta < 1$, that is, war is destructive, but less than half of the total resources are lost. Here, the interaction turns out to be more complex and we need to consider the countries' best responses in more detail. Figure 2 plots the best response correspondences for $\delta = 0,9$, where the dashed lines denote $G_1(G_2)$ and the full lines denote $G_2(G_1)$:

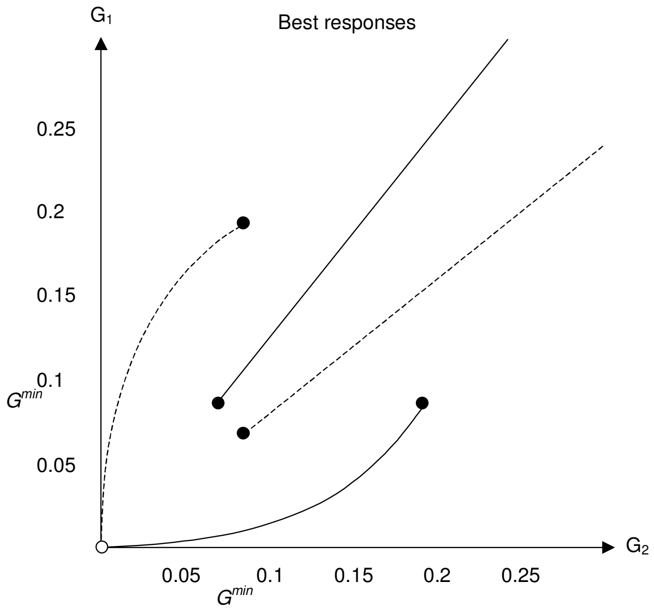


Figure 2

The straight lines correspond to the best response functions when expecting peace, while the concave curves denote the war case. We note that if country 2 sets $G_2 < G^{\min} \approx 0.0849$, country 1 applies its war best response and, for $G_2 > G^{\min}$, its peace best response.

Are there any Nash equilibria in pure strategies? From the best response functions, we see that the point $G_1 = G_2 = 0$ would be a likely candidate, with both countries opting for peace. However, starting in that point, either country would have an incentive to unilaterally deviate from that strategy profile by increasing $G_i$ by an arbitrarily small amount, and going to war. This deviation, given that the opponent does not alter its strategy, would win country $i$ the entire "cake" (less destruction). Hence, $G_1 = G_2 = 0$ and peace cannot be a Nash equilibrium. In fact:

**Proposition 2** *For $\frac{1}{2} < \delta < 1$, there exist no subgame perfect Nash equilibria in pure strategies.*

**Proof** *See Appendix.*

A similar result can be found in Brito and Intriligator [1985]. However, they can prove the existence of mixed equilibria.[6] As the model of this paper has a discontinuous payoff function it is not straightforward to determine

[6] See also Glicksberg [1952].

whether a mixed equilibrium exists.[7] The discontinuity in the payoff function is due to the fact that if country $i$ sets $G_i$ lower than $G_j[2\delta - 1]$ (see equation 10), holding $G_j$ constant, war will occur. At $G_i = G_j[2\delta - 1]$, there is a discrete jump in payoffs as a fraction $(1 - \delta)$ of the contested resource is destroyed if war occurs. However, it would be difficult to perceive a realistic interpretation of a mixed strategy in this setting.

Does the non-existence of pure strategy equilibria mean that the behavior is entirely erratic? In a one-shot rational expectations world - yes. By extending this model with sequential moves and myopic players, we can better understand the behavior associated with the lack of equilibria.

# 4 A sequential move model with myopic behavior

To analyze the lack of equilibria, let us consider a case where countries move sequentially and behave myopically. Apart from myopic behavior and the sequential move structure outlined below, this model is identical to that in section 3.

Specifically, at time $t = 1$, country $i$ chooses a best response to some exogenously given starting value for $G_j$, given peace, and then decides on whether to go to war. At $t = 2$, country $j$ sets its best response and opts for war or peace and so on. A country is thus committed to each armament level for two periods.

Assume also that it is the "moving" country deciding on whether there is war or peace. If war occurs, the contested land is divided according to relative armaments. The new split will then be asymmetric. We can consider this as the border moving in the direction of the losing country. Let us also assume that there will always be some contested land with value one, extending symmetrically in both directions from the border. Hence, after a war, the contested land now extends further into the territory of the losing country as compared to before the war.[8] Another interpretation could be that the spoils of war only last one period. Hence, after one period of occupation, troops are withdrawn and things return to status quo.

[7] A more general existence result for mixed equilibria can be found in, for example, Reny [1999], corollary 5.2, p 1044. However, it is not straightforward whether this result is applicable to my model and I leave the issue aside.

[8] A more complex model could take into consideration that conquests (losses) increase (decrease) the capacity to fight in the future. See, for example, Hirshleifer [2000] for a discussion.

Within this new framework, let us consider an example of the case where $\delta \in \left(\frac{1}{2}, 1\right)$ from the model in section 3.

*4.A An example of cyclical behavior*

The following example is illustrated in table 1 and figure 3 below. Set $\delta = 0.9$ and assume that country 2 has disarmed to the point that $G_2 = 0.0760$. As $G^{\min} \approx 0.0849$, country 1 sees an opportunity to arm for war and plays its war best response according to equation 5 in period $t = 1$. Inserting $G_2 = 0.0760$ into equation 5 yields $G_1(G_2) \approx 0.1855$ (point 1). A war breaks out and land is redistributed. In $t = 2$, country 2 responds by increasing armaments just sufficiently to deter country 1 and opts for peace. In accordance with its peace best response function, equation 10, it sets $G_2 \approx 0.1484$ (point 2). In period $t = 3$, given the peaceful conditions, country 1 sees an opportunity to cash in on the peace dividend and disarms just sufficiently to keep country 2 from attacking (point 3). In $t = 4$, country 2 keeps disarming (point 4). Country 1 disarms further at $t = 5$. At this point, the process of disarmament has made it profitable for country 2 to rearm and go to war as $G_1$ is now less than $G^{\min}$. Yet another war occurs after a period of peace and disarmament and the border is pushed back. Proceeding in this manner, we cycle through points 1 to 10 in table 1 and figure 3 below.

| $t$ | Country | $G$ | Outcome |
|---|---|---|---|
| Start | 2 | 0.0760 | - |
| 1 | 1 | 0.1855 | War |
| 2 | 2 | 0.1484 | Peace |
| 3 | 1 | 0.1187 | Peace |
| 4 | 2 | 0.0950 | Peace |
| 5 | 1 | 0.0760 | Peace |
| 6 | 2 | 0.1855 | War |
| 7 | 1 | 0.1484 | Peace |
| 8 | 2 | 0.1187 | Peace |
| 9 | 1 | 0.0950 | Peace |
| 10 | 2 | 0.0760 | Peace |

Table 1

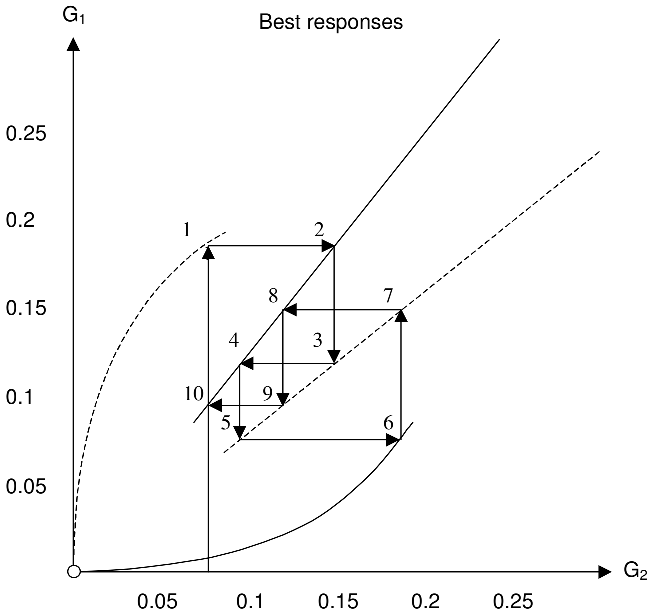


Figure 3.

This cycle captures an interesting intuition. As long as there is peace, countries will attempt to "underbid" each other so as to increase their peace time payoffs. However, as this underbidding progresses, the countries eventually reach a stage where the arming levels are so low that it pays for one country to rearm and go to war. The other country will then respond by rearming enough to deter its opponent. And hence, the cycle continues. Consider, for example, the situation following World War 1 in Europe. Europe's population was tired of war and cashing in on the peace dividend appeared a natural post-war policy. Defense expenditure dropped for a long time in the interwar period; long enough for Adolf Hitler to turn his aspiration of "Lebensraum" into a concrete and violent plan. This is not to say that disarmament is bad, but if the future consequences of disarming today are not well understood or foreseen, it may prove a dangerous path to tread.

*4.B Analogies to Edgeworth cycles*

Edgeworth (1925) found non-existence of equilibria in a static Bertrand game with capacity constraints. Firms compete in prices where the firm with the lowest price captures the market. However, each firm has a limited capacity and cannot serve the entire market at very low prices. In this game, firms underbid each other in a "price war" until one firm reaches its capacity and there is excess demand. At this point, the "price war" has become very costly and one firm "relents" and increases its price as it will be able to sell to the excess demand not served by the other firm. The low price firm now sees an opportunity to increase its price just undercutting the high price firm, thereby increasing its profits. Edgeworth had thus "resolved" the nonexistence problem with a cycle.

This firm behavior appears to be somehow analogous to what is observed in my model in section 3. Instead of prices, my model has armament levels falling sequentially in search of the peace dividend. However, when armament levels become sufficiently low, the opportunity cost of not rearming and attacking becomes too high and what is a "relenting phase" in Edgeworth's model becomes a war phase in this paper.

# 5 Concluding remarks

This paper has demonstrated how the analysis can differ dramatically between two common modeling approaches to conflict. Further, strategic interaction between parties with an option of arming and appropriating each other's wealth may exhibit unstable, cyclical behavior. In my mind, this example offers food for thought with respect to quick, especially unilateral, disarmament in seemingly peaceful environments.

# References


[1] Alesina, A., Spolaore, E., "War, Peace and the Size of Countries", Discussion Paper Number 1937, Harvard Institute of Economic Research, Cambridge, 2001.

[2] Bester, H., Wärneryd, K., "Conflict Resolution Under Asymmetric Information", Working Paper No. 264, Stockholm School of Economics, 1998.

[3] Brito, D., Intriligator, M., "Conflict, War, and Redistribution", *The American Political Science Review*, Vol. 79, 1985.

[4] Brown, M., et al (eds), *Theories of war and peace*, MIT Press, Cambridge, 1998.

[5] Edgeworth, F., *Papers relating to political economy*, Vol 1, Macmillan and Co., London, 1925.

[6] Garfinkel, M., Skaperdas, S., (eds), *The political economy of conflict and appropriation*, Cambridge University Press, Cambridge, 1996.

[7] Glicksberg, I., "A Further Generalization of the Kakutani Fixed Point Theorem", *Proceedings of the American Mathematical Society*, 1952.

[8] Grossman, H., "A General Equilibrium Model of Insurrection", *American Economic Review*, Vol. 81, No. 4, 1991.

[9] Grossman, H., Kim, M., "Swords or Plowshares? A Theory of the Security of Claims to Property", *Journal of Political Economy*, Vol. 103, No 6, pp 1275-1288, 1995.

[10] Hess, G., Orphanides, A., "War and Democracy", MIMEO, University of Cambridge, St. John's College, 1997.

[11] Hirshleifer, J., "The Paradox of Power", *Economics and Politics*, Vol. 3, No 3, 1991.

[12] Hirshleifer, J., "Anarchy and its Breakdown", *Journal of Political Economy*, Vol. 103, No 1, 1995.

[13] Hirshleifer, J., "The Macrotechnology of Conflict", *Journal of conflict resolution*, Vol. 44, No 6, 2000.

[14] Neary, H., "A comparison of rent-seeking models and economic models of conflict", *Public Choice*, 93, pp 373-388, 1997.

[15] Reny, P., "On the existence of pure and mixed strategy Nash equilibria in discontinuous games", *Econometrica*, Vol. 67, No. 5, pp 1029-1056, 1999.

[16] Skaperdas, S., "Cooperation, Conflict, and Power in the Absence of Property Rights", *American Economic Review*, Vol. 82, No. 4, 1992.

[17] Skaperdas, S., "Contest Success Functions", *Economic Theory*, Vol. 7, iss. 2, pp. 283-90, February 1996.

[18] Skaperdas, S., Syropoulos, C., "Can the shadow of the future harm cooperation?", *Journal of Economic Behavior and Organization*, Vol. 29, pp 355-372, 1996.

[19] Tullock, G., "Efficient rent seeking", in Buchanan, J., and Tollison, R., Tullock, G., (eds), *Toward a theory of the rent seeking society*, Texas, 1980.

[20] Wärneryd, K., "Anarchy, Uncertainty, and the Emergence of Property Rights", *Economics and Politics*, Vol. 5, No. 1, 1993.

# 6 Appendix

*A 1 Proof of proposition 2*

We need to show that no subgame perfect Nash equilibria (SPNE) exist for $\frac{1}{2} < \delta < 1$ in the following cases:

*(i) Symmetric equilibria*

- $G^* > G^{\min}$ : This implies that we have peace. A best response in this case is to set $G = G^* [2\delta - 1]$. As $[2\delta - 1] < 1$, this means setting a lower $G$. Hence, $G^*$ is not SPNE.

- $G^* < G^{\min}$ : This implies war. I will show that $G^{war}(G^*) > G^*$. To do this, we need to establish the following three conditions: *i)* $G^{war}(0) = 0$, *ii)* $G^{war}(G)$ is strictly concave in $G$, and *iii)* $G^{war}(G^{\min}) > G^{\min}$:

*i)* $G^{war}(0) = 0 + \sqrt{\delta * 0} = 0$.
*ii)* $\frac{\partial^2 G^{war}(G)}{\partial G^2} = -\frac{G^{-\frac{3}{2}}\sqrt{\delta}}{4}$. Thus, strictly concave.
*iii)* $G^{war}(G^{\min}) > G^{\min}$. Inserting $G^{\min}$ and simplifying yields: $\sqrt{1-\delta}\left(\sqrt{2} - \sqrt{1-\delta}\right) > 0$. As $\delta \in \left(\frac{1}{2}, 1\right)$, the left-hand side is indeed strictly positive.

Hence, $G^* < G^{\min}$ cannot be a SPNE.

- $G^* = G^{\min}$ : Both countries are indifferent between setting their war or peace best responses. Whatever they choose, they deviate from $G^*$and we have no SPNE!

*(ii) Asymmetric equilibria, that is $G_i < G_j$ in the following two cases:*

- Peace: From the best response function, we know that player $j$ will want to decrease $G_j$. Hence, this cannot be a SPNE!

- War: From the best response function, we know that player $i$ will want to increase $G_i$. Hence, this cannot be a SPNE!

QED