Trademark dilution, whereby a firm associates its product with that of another firm and takes advantage of the goodwill created by that firm, is illegal in the EU and in the US. We investigate this regulation from a welfare perspective, considering short-term effects on profits and consumers’ surplus, as well as long-run effects on investment. We find the circumstances under which laws against trademark dilution are welfare-enhancing to be limited. Under Bertrand competition, trademark dilution is never an equilibrium outcome since a decrease in the amount of product differentiation is always associated with a decrease in the prices and profits of both firms. Under Cournot competition anti-dilution laws may change equilibrium investment patterns, but only for intermediate levels of investment costs. If legislation does have an impact, the welfare effects are ambiguous.

\textit{JEL codes}: K11, K13, L15  
\textit{Keywords}: trademark dilution, marketing
1. Introduction

Trademark dilution is a legal term typically referring to a situation where a (low-quality) producer without permission relates to another firm’s (high-quality) product in its marketing. The producer of low quality hereby takes advantage of the goodwill created by the high-quality firm and possibly also dilutes its trademark. Trademark dilution essentially means that some marketing activity creates a spill-over of goodwill from one product to another. If, for example, a low-quality manufacturer launched a new brand called *Trabant Silver Shadow*, Rolls Royce could argue that it was subject to trademark dilution.

There are rules against trademark dilution in the EU as well as in the US. The European Parliament (1984) has for example stated that:

Comparative advertising shall, as far as the comparison is concerned, be permitted when the following conditions are met: ..... (g) it does not take unfair advantage of the reputation of a trade mark, trade name or other distinguishing marks of a competitor or of the designation of origin of competing products.  

(p. 4)

and, similarly, according to the 1996 Federal Anti Dilution Act of the US:

The owner of a famous mark shall be entitled, subject to the principles of equity and upon such terms as the court deems reasonable, to an injunction against another person’s commercial use in commerce of a mark or trade name, if such use begins after the mark has become famous and causes dilution of the distinctive quality of the mark, and to obtain such other relief as is provided in this subsection.

According to one estimate around 60 percent of all advertisements are at least indirectly comparative in the sense that it relates to another firm’s product.\(^1\) Consequently, these rules will have an impact on a large share of the world’s advertising investments. It is interesting to

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\(^1\) See Pechmann & Stewart (1990).
note that the informational interests of consumers generally are protected by separate rules. For example, in the EU a clear distinction is made between misleading advertising and comparative advertising.² Basically the same distinction is made in the US.³ This means that owners of famous trademarks may bring an action for dilution even where there is no likelihood of confusion, as for example when firms belong to different markets. Obviously, advertising could have elements of trademark dilution and be misleading at the same time. In this study, however, we will focus on mechanisms that are purely related to trademark dilution and disregard informational aspects, i.e., consumers are assumed to be fully informed.

The overall purpose of the legislation against trademark dilution seems to be the protection of investments in marketing. From a dynamic point of view there are often good reasons to protect private investments with public good characteristics, for example investments in R&D. The reason is of course that in the absence of regulation there is a risk that firms will under-invest. The motivation for protecting investments in marketing is however less clear. For example, it is not evident whether or not marketing contributes to welfare or if it just shifts demand between products. If marketing affects preferences, it is far from clear what are the “true” consumer preferences.⁴ Finally, it is not obvious that firms will in fact have an incentive to dilute other firms’ trademarks. These observations motivate a study of trademark dilution from an economics perspective. Specifically, our study aims to:

1) analyze theoretically firms’ incentives to dilute the trademarks of competing firms and

² See The European Parliament (1984). Here, misleading advertising means any advertising which, in any way, including its presentation, deceives or is likely to deceive the persons to whom it is addressed or whom it reaches and which, by reason of its deceptive nature, is likely to affect their economic behaviour or which, for those reasons, injures or is likely to injure a competitor. On the other hand, comparative advertising means any advertising which explicitly or by implication identifies a competitor or goods or services offered by a competitor.
³ See the 1996 Federal Anti Dilution Act where misleading and confusing advertising is treated separately from the issue of dilution of famous marks.
⁴ The welfare effects of advertising are complicated by the possibility that advertising affects tastes. This issue is discussed by Dixit & Norman (1978) who argue that there is excessive advertising in equilibrium, even when the point of reference is post-advertising tastes. See also Fischer & McGowan (1979) and Shapiro (1980) for a critique.
2) explore the circumstances under which laws against trademark dilution are likely to increase welfare.

We take an agnostic position with respect to the relevant welfare measure. That is, the welfare aspects of legislation are discussed both in terms of post-advertising tastes and in terms of pre-advertising tastes.

In order to discuss these issues we need a framework where investments in marketing determine the perceived quality levels, i.e., the degree of vertical product differentiation. Moreover, consumers must be different in the sense that some (presumably poor) prefer low-quality products while others (presumably rich) prefer heavily advertised high-quality products. Our choice of workhorse model is Shaked & Sutton (1982), which is extended to allow for sequential investments in marketing by two firms. In period one, firm $A$ chooses between investing in high quality or in low quality. In period two, firm $B$ also has the choice between investing in high quality or in low quality. In addition, it can choose a dilution strategy which, at a low cost, increases the consumers’ valuation of firm $B$’s product, possibly at the expense of the perceived quality offered by firm $A$. In the third and final period firms compete in prices or quantities.

Our main findings are the following. The circumstances under which laws against trademark dilution are welfare-enhancing are quite limited. Under Bertrand competition, trademark dilution is never an equilibrium outcome since a decrease in the amount of product differentiation is always associated with a decrease in the prices and profits of both firms. Under Cournot competition, anti-dilution laws may change the amount of product differentiation and also equilibrium investments patterns, but only for intermediate levels of investment costs. If legislation does have an impact the outcome is dependent on investment
decisions, implying that short-term and long-run effects are different. Welfare results are ambiguous.

The paper is organized as follows. First, the model is introduced. Then we examine incentives under Bertrand and Cournot competition. After a discussion of static welfare effects we characterize the sub-game perfect equilibrium of the investment game. Next, we analyze the welfare implications of banning trademark dilution. The final section concludes and discusses the policy implications of the results.

2. The Model

Consider a market with two sellers, firm A and firm B, each selling goods with one of two exogenously determined quality levels, high quality (or service) $S_A$ and low quality $S_B$.

Trademark dilution, if it occurs, has the effect of increasing the (perceived) quality of the low-quality good. At the same time it may reduce (i.e. dilute) the perceived quality of the high-quality good. Although the signs of the quality effects of trademark dilution are thus clear, the relative sizes of the quality increase effect versus the dilution effect may differ depending on e.g. the type of good.

Each consumer buys maximum one unit of each good and consumers derive utility from the quality level of that good and from consumption of other goods on which the rest of the income is spent. Assuming Cobb-Douglas utility the utility function for an individual with income $V$ can be defined as

$$U(V) = \max\{(V - P_A)S_A, (V - P_B)S_B\}$$

where $P_A$ and $P_B$ are the prices charged by firms.
Consumers will only consider buying low quality if its price is lower than that of high quality. For \( P_A > P_B \) we can determine the income level \( V^* \) such that a consumer is indifferent between buying high or low quality,

\[
V^* = \frac{P_A S_A - P_B S_B}{S_A - S_B}.
\]  

(2)

It follows from the utility function that higher income increases the utility of buying high quality more than the utility of buying low quality. Thus consumers with incomes above \( V^* \) will purchase one unit each of high quality. Consumers with incomes below \( V^* \) will either purchase one unit of low quality or make no purchase. The decision whether to remain outside the market depends on whether utility is positive or not for the given \( S_B \) and \( P_B \). Assume that income levels are uniformly distributed over the interval \([0, b]\). The firms then face demand functions

\[
Q_A = b - V^*,
Q_B = V^* - P_B.
\]

(3)

The structure of demand is shown in Figure 1.

The cost structure is simple: the marginal cost of production is zero and the additional cost of supplying high quality is modeled as a fixed cost \( K \). This formulation of the cost structure allows for different interpretations of the concept of quality, or high quality.\(^5\)

Specifically, the perceived high quality may to some extent be due to marketing, in which

\(^5\) A high quality product may be the result of an investment in product development that yields e.g. a new design, which is superior.
case the issue of whether pre-advertising tastes or post-advertising tastes are used when evaluating welfare effects is of interest.

The timing is that firm A acts first by making a once-and-for-all choice of high or low quality. Firm B then chooses the quality level of its product. When dilution is allowed the quality level of firm B’s product is determined as a combination of its choice of quality and its choice of dilution. Dilution is only possible when firm A selects high quality and firm B low quality, i.e. we assume that it is only the first mover in terms of quality choice that can be exposed to dilution. Once quality levels are determined firms will set prices or quantities.

3. Incentives and Profit Effects in Bertrand and Cournot Competition

Trademark dilution can affect the choices of the players at different stages. For example, by reducing the higher quality level, dilution may reduce Firm A’s incentive to invest. On the other hand, for given investments dilution has the potential to increase welfare since it increases the level of the lower quality. A welfare analysis of trademark dilution must take all such effects into account. To do this we will work backwards and start with the last stage.

In describing the competitive structure at the last stage there is the basic choice between two models, price competition as in the Bertrand model or quantity competition as in the Cournot model. It is known from other policy-oriented studies that this choice can have a critical influence on welfare results. This is true e.g. for strategic trade policy (Eaton & Grossman, 1986), information-sharing in oligopoly (Vives, 1990), endogenously determined quality levels (Motta, 1993), and profit effects of horizontal mergers (Deneckere & Davidson, 1985). Thus welfare effects of trademark dilution may depend on whether the strategic variable used by firms is price or quantity. To be able to draw conclusions that are reasonably general, we investigate both cases.
We examine Bertrand competition first. From Expressions (2) and (3) we can derive quantities demanded as functions of prices. We need only consider the case when firm A supplies high quality and firm B low quality since otherwise trademark dilution is not an issue. In this case the profit functions are of the form \( \pi_A = P_A \cdot Q_A(P_A, P_B, S_A, S_B) - K \) and \( \pi_B = P_B \cdot Q_B(P_A, P_B, S_A, S_B) \). To simplify interpretation let \( z \) denote the ratio between the quality levels, \( z = S_B / S_A, \ 0 < z \leq 1 \). First-order conditions for profit maximization with respect to own price, together with a Nash equilibrium condition in prices, then yields an expression for the profit of the firm producing low quality:

\[
\pi_B = \frac{b^2(1 - z)}{(4 - z)^2} \tag{4}
\]

Dilution increases the low quality and reduces the high quality and thereby increases \( z \). It is straightforward to show that \( \pi_B \) is decreasing in \( z \), i.e. trademark dilution reduces profits for the firm that produces low quality. We summarize the result in:

**Observation 1:** In a market with Bertrand competition there is no incentive for the firm producing low quality to engage in trademark dilution.

This result is the combination of two opposing effects of trademark dilution. Firstly, by increasing the quality level of the diluting firm’s product it increases consumers’ willingness to pay for the product at given prices. Secondly, by reducing the distance between the two products in the quality dimension, trademark dilution increases the strength of price competition between the two firms and reduces prices and profits. In Bertrand competition the
second effect is the stronger one. This means that trademark dilution is an unlikely phenomenon in markets with price competition.

We next look at Cournot quantity competition. Equilibrium prices as functions of quantities are derived from Expressions (2) and (3), yielding profit functions of the form

$$\pi_A = Q_A \cdot P_A(Q_A, Q_B, S_A, S_B) - K$$ \quad and \quad $$\pi_B = Q_B \cdot P_B(Q_A, Q_B, S_A, S_B).$$

Using first-order conditions for profit maximization with respect to own quantity for each firm and the Nash equilibrium condition in quantities, we arrive at expressions for the variable profits (i.e., investment costs not deducted) of the firms producing high and low quality.

$$\pi_A = \frac{b^2(2-z)^2}{(4-z)^2} \equiv \gamma(z)$$ \quad (5)

$$\pi_B = \frac{b^2}{(4-z)^2} \equiv \lambda(z)$$ \quad (6)

$$\pi = \pi_A + \pi_B = \frac{b^2(5+z^2-4z)}{(4-z)^2}$$ \quad (7)

As before, profits do not depend on absolute levels of quality but only on the ratio $z$. Here $\pi_B$ is increasing in $z$, implying that the firm producing low quality has an incentive to engage in trademark dilution. We now turn to a closer investigation of the effects of trademark dilution on profits and utility under Cournot competition.

4. Welfare Effects for Given Quality Levels

Trademark dilution increases the level of low quality produced by firm $B$. Let the amount of increase in quality for firm $B$ be denoted by $D$. Assume that the same amount of dilution reduces the level of high quality that firm $A$ produces by $\alpha D$, where $0 \leq \alpha \leq 1$. After dilution there are thus two new quality levels, $S_A = S_H - \alpha D$ for firm $A$ and $S_B = S_L + D$ for firm $B$. 


where \( S_L \) and \( S_H \) denote exogenous pre-dilution quality levels such that \( S_H > S_L \). We assume the extent of dilution to be limited to \( D \) such that \( S_A > S_B \) which, in turn, is equivalent to assuming that \( D < (S_H - S_L)/(1 + \alpha) \equiv \bar{D} \). It is straightforward to show that \( dz / dD > 0 \), i.e., dilution reduces the relative quality difference. Using this and differentiating Expressions (5), (6) and (7) with respect to \( z \) we derive the profit effects of trademark dilution:

\[
\frac{\partial \pi_A}{\partial z} \frac{\partial z}{\partial D} = \frac{-4b^2(2-z)}{(4-z)^3} \frac{\partial z}{\partial D} < 0
\]

(8)

\[
\frac{\partial \pi_B}{\partial z} \frac{\partial z}{\partial D} = \frac{2b^2}{(4-z)^3} \frac{\partial z}{\partial D} > 0
\]

(9)

\[
\frac{\partial \pi}{\partial z} \frac{\partial z}{\partial D} = -\frac{2b^2(3-2z)}{(4-z)^3} \frac{\partial z}{\partial D} < 0.
\]

(10)

**Observation 2:** In a market with Cournot competition, trademark dilution increases the variable profit for the firm producing low quality while reducing the variable profit for the firm producing high quality.

**Observation 3:** The effect of trademark dilution on total variable profits is negative, i.e. the reduction in the profit of the firm producing high quality is greater than the increase in the profit of the firm producing low quality.

We now turn to investigating the effects of trademark dilution on consumer welfare, measured in terms of consumers’ surplus. We look at two cases: (i) when high quality is a real product effect and leads to higher utility, and (ii) when consumer willingness to pay for high quality is
mainly a result of marketing and both goods are of the same and constant quality. In the latter case we assume that both goods are still valued at the original quality level, $S_L$.

First note that when high quality is valued at $S_A \neq S_L$, the inverse demand functions facing firms are given by $P_A = b - Q_A - zQ_B$ and $P_B = b - Q_B - Q_A$ respectively. Hence, consumers’ surplus in the two markets at equilibrium prices and quantities equal:

$$CS_{H}^{post} = \frac{b^2 (2-z)^3}{2(4-z)^2}$$

$$CS_{L}^{post} = \frac{b^2}{2(4-z)^2}$$

where superscript $post$ refers to post-advertising. This, in turn, implies the following aggregate consumers’ surplus

$$CS^{post} \equiv CS_{H}^{post} + CS_{L}^{post} = \frac{b^2 (5+z^2-4z)}{2(4-z)^2}.$$

Differentiating with respect to $D$ we find the effect of trademark dilution:

$$\frac{dCS_{H}^{post}}{dD} = \frac{\partial CS_{H}^{post}}{\partial z} \frac{\partial z}{\partial D} = \frac{-2b^2 (2-z)}{(4-z)^3} \frac{\partial z}{\partial D} < 0$$

$$\frac{dCS_{L}^{post}}{dD} = \frac{\partial CS_{L}^{post}}{\partial z} \frac{\partial z}{\partial D} = \frac{b^2}{(4-z)^3} \frac{\partial z}{\partial D} > 0$$

$$\frac{dCS^{post}}{dD} = \frac{\partial CS^{post}}{\partial z} \frac{\partial z}{\partial D} = \frac{-2b^2 (3-2z)}{(4-z)^3} \frac{\partial z}{\partial D} < 0.$$
Hence, dilution decreases consumers’ surplus in the market for high-quality goods and increases it in the market for low-quality goods. The net effect is however negative since high-income individuals have a higher willingness to pay for product quality.

Next, consider the case that high quality is valued at $S_L$. This means that consumers’ willingness to pay for the high-quality product is now independent of $z$. The willingness to pay for the low-quality product is the same as before. Hence the inverse demand functions are given by $P_A = b - Q_A - Q_B$ and $P_B = b - Q_B - Q_A$ respectively. Consumers’ surplus in the two markets evaluated at equilibrium prices and quantities then equal:

$$\text{CS}_H^{\text{pre}} = \frac{b^2(2-z)}{2(4-z)^2}$$  \hspace{1cm} (16)$$

$$\text{CS}_L^{\text{pre}} = \frac{b^2}{4(4-z)^2}$$  \hspace{1cm} (17)$$

where superscript $\text{pre}$ refers to pre-advertising tastes and Expression (17) is the same as Expression (12). This, in turn, implies the following consumers’ surplus

$$\text{CS}_H^{\text{pre}} + \text{CS}_L^{\text{pre}} = \frac{b^2(3-z)}{2(4-z)^2}.$$  \hspace{1cm} (18)

Differentiating with respect to $D$ we find the effect of trademark dilution under pre-advertising tastes:

$$\frac{d\text{CS}_H^{\text{pre}}}{dD} = \frac{d\text{CS}_L^{\text{pre}}}{D} \frac{d\text{CS}_L^{\text{pre}}}{dD} = \frac{-b^2z}{2(4-z)^2} \frac{dz}{dD} < 0$$
As before, dilution decreases consumers’ surplus in the market for the high-quality good and increases it in the market for the low-quality good. However, when high quality is valued at $S_L$, trademark dilution increases aggregate consumers’ surplus. This is the result of the interaction of several effects. Firstly, dilution increases the price of low quality by increasing its quality level in the eyes of consumers. This in itself reduces consumers’ surplus. Secondly, dilution lowers the price of high quality by reducing it in the eyes of consumers and this has a positive effect on consumers’ surplus. Finally, dilution increases $V^*$ and thereby the number of consumers who buy low quality, and these pay less than when they bought high quality.

Thus we find that trademark dilution unambiguously reduces consumers’ surplus only in the case when high quality is valued at post-advertising preferences. The results are summarized below.

*Observation 4: (i):* When post-advertising preferences are used for welfare evaluation, trademark dilution reduces consumers’ surplus. *(ii):* When pre-advertising preferences are used for welfare evaluation, trademark dilution increases consumers’ surplus.

To find total welfare effects we aggregate consumers’ surplus and profits. Let us define total welfare $W$ as the sum of consumers’ surplus and profits, disregarding investment costs. For post-advertising tastes it is then the case that:
\[ W_{\text{post}} = \frac{3b^2(z^2 - 4z + 5)}{2(4-z)^2} \]  

(21)

and

\[ \frac{dW_{\text{post}}}{dD} = \frac{\partial W_{\text{post}}}{\partial z} \frac{\partial z}{\partial D} = \frac{3b^2(2z-3)}{(4-z)^3} \frac{\partial z}{\partial D} < 0 \]  

(22)

and for pre-advertising tastes we have that:

\[ W_{\text{pre}} = \frac{b^2(2z^2 - 9z + 13)}{2(4-z)^2} \]  

(23)

and

\[ \frac{dW_{\text{pre}}}{dD} = \frac{\partial W_{\text{pre}}}{\partial z} \frac{\partial z}{\partial D} = \frac{b^2(7z-10)}{(4-z)^3} \frac{\partial z}{\partial D} < 0 \].  

(24)

Observation 5: The static effect of trademark dilution is a reduction in total welfare.

The result that aggregate welfare is negatively affected by dilution under post-advertising preferences follows trivially from Observations 3 and 4. Under pre-advertising tastes the effect on profits dominates the effect on consumers’ surplus.

5. Investment Strategies in Quantity Competition

We have investigated the effects of trademark dilution on profits and welfare when the investment strategies of the two firms were given, i.e. under the assumption that firm A would
always produce high quality and firm B always low quality. However, a purpose of legal restrictions on trademark dilution may be to signal that investments in marketing will be protected. This suggests that without legal restrictions, firms may invest less in producing and/or marketing high quality products when considering the risk of dilution. For example, if dilution is ruled out it is possible that firm B would also consider investing in high quality. Thus in order to determine the effects of restrictions against trademark dilution on investment behavior, we need to investigate incentives for investment with and without trademark dilution.

The introduction of an initial investment phase makes it necessary to consider the extensive form version of the game, described in Figure 2. Firm profits (net of investment costs) are denoted in such a way that subscripts refer to firms (A and B), the first letter of the superscript refers to the strategy of firm A (H for high quality or L for low quality) while the last letter refers to the strategy of firm B (H for high quality, L for low quality and D for dilution). Since firms may choose any quality level at the investment stage, we now denote the quality levels produced under dilution by $S_A^D$ and $S_B^D$. Specifically, profits are defined in the following way:

$$\pi_A^{HD} = \gamma(S_B^D / S_A^D) - K$$

$$\pi_B^{HD} = \lambda(S_B^D / S_A^D)$$

$$\pi_A^{HL} = \pi_B^{HL} = \gamma(S_L^D / S_H^D) - K$$

$$\pi_A^{HH} = \pi_B^{HH} = \gamma(1) - K$$

$$\pi_A^{LL} = \pi_B^{LL} = \gamma(1)$$
where $\gamma(z)$ and $\lambda(z)$ are equivalent to Equations (5) and (6).

The firms’ equilibrium strategies are determined by whether trademark dilution is allowed, together with the cost of investing in high quality. To categorize the different cases we define four levels of investment costs, $K_1 < K_2 < K_3 < K_4$ (see the Appendix for detailed derivations). Moreover, we define a threshold level of dilution $D' < \overline{D}$ (also in the Appendix).\(^6\) Since firm A is the first mover it will take firm B’s actions into account when determining its investment. We thus start by examining firm B’s optimal investment strategies, given the different possible actions taken by firm A and whether dilution is allowed or not.

Consider first the case when firm A produces high quality and dilution is allowed. We can then show the following.

**Lemma 1.** If $S_A = S_H$ and dilution is allowed then $S_B^D$ is an optimal strategy for firm B as long as the investment cost is large ($K > K_1$). Otherwise, the optimal strategy is $S_H$.

**Proof:** See the Appendix.

Thus if the cost of investing in quality is sufficiently low, the fact that firm A has invested in high quality will not prevent firm B from doing so too. In this case it does not make any difference if trademark dilution is legal or not, since it will not be used anyway. For investment costs higher than $K_1$ firm B will however profit from engaging in trademark dilution.

Next consider the case when firm A produces high quality but trademark dilution is not allowed. We then have

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\(^6\)To be precise, $K_3$ plays a role only for $D > D'$ and in that case $K_2 < K_3 < K_4$. 
Lemma 2. If \( S_A = S_H \) and dilution is not allowed then \( S_L \) is an optimal strategy for firm B when the investment cost is large (\( K > K_2 \)). Otherwise, the optimal strategy is \( S_H \).

Proof: See the Appendix.

Thus with investment costs above \( K_1 \) but below \( K_2 \), firm B will invest in high quality if it is prevented from engaging in trademark dilution. If firm A instead produces low quality, firm B may invest in high quality without risking exposure to trademark dilution. Here firm B’s incentives are the following

Lemma 3. If \( S_A = S_L \) then \( S_L \) is an optimal strategy for firm B as long as the investment cost is large (\( K > K_4 \)). Otherwise, the optimal strategy is \( S_H \).

Proof: See the Appendix.

Thus when firm A produces low quality, it is relatively more profitable for firm B to invest in high quality, and firm B will do so unless the investment cost is above \( K_4 \).

Having determined firm B’s optimal strategies as dependent on the actions of firm A and whether trademark dilution is allowed or not, we now determine the equilibrium strategies by considering firm A’s optimal strategies in the investment stage. These are summarized in Propositions 1 and 2 below.
**Proposition 1.** When trademark dilution is allowed equilibrium strategies are

\[
\begin{align*}
K > K_4 & \quad \rightarrow \quad \pi_A^{LL} > \pi_A^{HD} \quad \rightarrow \quad S_A = S_L \text{ and } S_B = S_L \\
K_4 > K > K_1 \text{ and } D < D' & \quad \rightarrow \quad \pi_A^{HD} > \pi_A^{LH} \quad \rightarrow \quad S_A = S_A^D \text{ and } S_B = S_B^D \\
K_4 > K > K_1 \text{ and } D > D' & \quad \rightarrow \quad \pi_A^{LH} > \pi_A^{HD} \quad \rightarrow \quad S_A = S_L \text{ and } S_B = S_H \\
K_4 > K_3 > K_1 \text{ and } D > D' & \quad \rightarrow \quad \pi_A^{HD} > \pi_A^{LH} \quad \rightarrow \quad S_A = S_A^D \text{ and } S_B = S_B^D \\
K < K_1 & \quad \rightarrow \quad \pi_A^{HH} > \pi_A^{LH} \quad \rightarrow \quad S_A = S_H \text{ and } S_B = S_H
\end{align*}
\]

**Proof:** This follows from \( K_4 > K_2 > K_1 \) and \( K_4 > K_3 > K_1 \) if \( D < D' \).

To summarize, if the investment cost of high quality is very low (\( K < K_1 \)) both firms will invest in high quality, and if it is very high (\( K > K_4 \)) none of the firms will do so. For intermediate investment costs firm \( A \) will invest in high quality while firm \( B \) will instead engage in trademark dilution, except when the investment cost is quite high (\( K_3 < K < K_4 \)) and dilution above the threshold level \( D' \) is possible. In that case firm \( A \) will abstain from investing in high quality and firm \( B \) will do so instead.

If trademark dilution is not allowed, the optimal strategies of firm \( A \) and thus the equilibrium strategies are somewhat different.

**Proposition 2.** When trademark dilution is illegal, equilibrium strategies are

\[
\begin{align*}
K > K_4 & \quad \rightarrow \quad \pi_A^{LL} > \pi_A^{HL} \quad \rightarrow \quad S_A = S_L \text{ and } S_B = S_L \\
K_4 > K > K_2 & \quad \rightarrow \quad \pi_A^{HL} > \pi_A^{LH} \quad \rightarrow \quad S_A = S_H \text{ and } S_B = S_L \\
K < K_2 & \quad \rightarrow \quad \pi_A^{HH} > \pi_A^{LH} \quad \rightarrow \quad S_A = S_H \text{ and } S_B = S_H
\end{align*}
\]

**Proof:** This follows from \( K_4 > K_2 \).
The equilibrium strategies with and without trademark dilution are summarized in Table 1.

The effect on equilibrium strategies of making trademark dilution illegal is essentially that it extends the conditions under which it is profitable for both firms to invest in high quality: when $K_2 > K > K_1$, both firms invest in high quality if dilution is not allowed, but only firm A if it is. In this sense trademark dilution reduces investments in quality and this may reduce welfare. On the other hand, when trademark dilution is allowed it \textit{de facto} increases the quality of the low-quality product, and depending on the effect of dilution on high quality this may increase welfare.

Table 1

<table>
<thead>
<tr>
<th>EQUILIBRIUM QUALITY LEVELS</th>
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| \begin{tabular}{|c|c|c|c|}
| \hline
| Dilution & $D < D'$ & $D > D'$ & $D > D'$ & $D > D'$ \\
| \hline
| No Dilution & $S_A = S_H$ & $S_A = S_H$ & $S_A = S_H$ & $S_A = S_H$ \\
| & $S_B = S_H$ & $S_B = S_H$ & $S_B = S_H$ & $S_B = S_H$ \\
| & $S_A = S_H$ & $S_A = S_H$ & $S_A = S_H$ & $S_A = S_H$ \\
| & $S_B = S_H$ & $S_B = S_H$ & $S_B = S_H$ & $S_B = S_H$ \\
| \hline
| & $S_A = S_H$ & $S_A = S_H$ & $S_A = S_H$ & $S_A = S_H$ \\
| & $S_B = S_H$ & $S_B = S_H$ & $S_B = S_H$ & $S_B = S_H$ \\
| & $S_A = S_H$ & $S_A = S_H$ & $S_A = S_H$ & $S_A = S_H$ \\
| & $S_B = S_H$ & $S_B = S_H$ & $S_B = S_H$ & $S_B = S_H$ \\
| \hline
| \end{tabular} |

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The investment effects of trademark dilution imply that banning trademark dilution also leads to dynamic welfare effects. We now turn to an investigation of these.

6. Dynamic Welfare Effects of Banning Trademark Dilution

To determine the welfare effects of trademark dilution we consider the impact on profits and consumers’ surplus for those cases where allowing trademark dilution or not will change either the investment decisions or the production and pricing decisions of one or both firms. As in Section 4 above, we will check if it makes a difference whether quality is evaluated using post-advertising or pre-advertising consumer preferences.

It is obvious from Table 1 that policies against trademark dilution affect investment patterns only for intermediate levels of investment costs. One interesting case in this respect is when \( K \in [K_1, K_2] \), when both firms invest in high quality if dilution is banned, while when dilution is allowed firm A invests in high quality and firm B chooses the dilution strategy. Let us denote this case by Case 1. The other interesting case concerns quality levels but not investment. For \( K \in [K_2, K_3] \) when \( D < D' \) (in the upper panel) and for \( K \in [K_2, K_3] \) when \( D > D' \) (in the lower panel), firm A invests in high quality and firm B supplies low quality when dilution is banned. When dilution is allowed, firm A invests in high quality while firm B chooses the dilution strategy. Let us denote this case by Case 2. There is a third case, where anti-dilution policies actually affect investment patterns but not welfare. For \( K \in [K_3, K_4] \) in the lower panel, firm A alone will invest high quality if dilution is banned. If dilution is allowed the reverse relation holds. This case is neutral in terms of welfare and will not be discussed further.

In Case 1 aggregate investment costs are affected by anti-dilution policies and investment costs must be incorporated in the definition of profits. The results are summarized in Table 2. Subscripts \( D \) and \( ND \) refer to the situations where dilution is allowed and not
allowed. Aggregate profit net of investment costs is denoted by $\pi$, and $TS$ refers to total welfare net of investment costs. For details, see the Appendix.

**Table 2**

THE WELFARE EFFECTS OF BANNING DILUTION

*Case 1*: $S_A = S_B = S_H$ when dilution is banned

<table>
<thead>
<tr>
<th>Post-advertising tastes</th>
<th>Pre-advertising tastes</th>
</tr>
</thead>
<tbody>
<tr>
<td>$CS_{ND}^{post} &lt; CS_{D}^{post}$</td>
<td>$CS_{ND}^{pre} &gt; CS_{D}^{pre}$</td>
</tr>
<tr>
<td>$\pi_{ND}^{post} &lt; \pi_{D}^{post}$</td>
<td>$\pi_{ND}^{pre} &lt; \pi_{D}^{pre}$</td>
</tr>
<tr>
<td>$TS_{ND}^{post} &lt; TS_{D}^{post}$</td>
<td>$TS_{ND}^{pre} &lt; TS_{D}^{pre}$</td>
</tr>
</tbody>
</table>

In *Case 1*, banning trademark dilution reduces the degree of product differentiation, since it implies that both firms will invest in high quality. This, in turn, reduces consumers’ surplus in the market for high-quality goods and increases it in the market for low-quality goods. The net effect is negative for post-advertising tastes and positive for pre-advertising tastes. This follows from Expressions (15) and (20). In terms of aggregate profits, dilution is favorable for firms since it reduces total investment costs. For post-advertising tastes the consumers’ surplus effect and the profit effect have the same sign, and for pre-advertising tastes the profit effect dominates. It is interesting to note that in *Case 1* the welfare effect of trademark dilution is completely reversed compared with the static case with exogenously determined quality levels. Intuitively, banning trademark dilution here leads to over-investment in quality.

The welfare results in *Case 2* are summarized in *Table 3*. For details, see the Appendix.
### Table 3

**WELFARE EFFECTS OF BANNING DILUTION**

*Case 2*: \( S_A = S_H \) and \( S_B = S_L \) when dilution is banned

<table>
<thead>
<tr>
<th>Post-advertising tastes</th>
<th>Pre-advertising tastes</th>
</tr>
</thead>
<tbody>
<tr>
<td>( CS_{ND}^{\text{post}} &gt; CS_D^{\text{post}} )</td>
<td>( CS_{ND}^{\text{pre}} &lt; CS_D^{\text{pre}} )</td>
</tr>
<tr>
<td>( \pi_{ND} &gt; \pi_D )</td>
<td>( \pi_{ND} &gt; \pi_D )</td>
</tr>
<tr>
<td>( TS_{ND}^{\text{post}} &gt; TS_D^{\text{post}} )</td>
<td>( TS_{ND}^{\text{pre}} &gt; TS_D^{\text{pre}} )</td>
</tr>
</tbody>
</table>

In *Case 2*, banning trademark dilution increases the degree of product differentiation. This, in turn, increases consumers’ surplus in the market for high-quality good and reduces it in the market for low-quality goods. The net effect is positive for post-advertising tastes and negative for pre-advertising tastes. Again, this follows from Expressions (15) and (20).

Moreover, we know from Expression (10) that aggregate variable profits increase in the degree of product differentiation. The overall welfare effects are obvious in the left column while in the right column the effect on profits dominates.

*Observation 6:* For investment costs \( K \in [K_1, K_2] \), banning trademark dilution increases investment in quality and reduces welfare as well as product differentiation (*Case 1*). For \( K \in [K_2, K_4] \) \( (K \in [K_2, K_3] \) when \( D > D' \), banning trademark dilution leads to increased product differentiation and higher welfare (*Case 2*).

In all results so far only relative quality turns out to matter. For instance, this implies that an increase in \( S_B \) reduces aggregate consumers’ surplus under post-advertising preferences. The intuition is that an increase in the level of quality of low-quality goods reduces the willingness to pay for high quality to such an extent that the overall effect is negative. For alternative
welfare measures, such as utility, absolute quality levels would matter, but at the expense of a loss of comparability between profits and consumer welfare. In addition, in our framework the level of trademark dilution does affect the results indirectly in the sense that it determines the likelihood of different scenarios. Specifically, both $D$ and $\alpha$ affect the threshold levels for investment costs that determine equilibrium investment patterns and, in turn, the likelihood of Case 1 and Case 2. Specifically, using the fact that $D \in [0, \overline{D}]$, it can easily be shown that

\[
\begin{align*}
\frac{\partial K_1}{\partial \alpha} &< 0 & \frac{\partial K_1}{\partial D} &< 0 \\
\frac{\partial K_2}{\partial \alpha} &= 0 & \frac{\partial K_2}{\partial D} &= 0 \\
\frac{\partial K_3}{\partial \alpha} &< 0 & \frac{\partial K_3}{\partial D} &< 0 \\
\frac{\partial K_4}{\partial \alpha} &= 0 & \frac{\partial K_4}{\partial D} &= 0.
\end{align*}
\]

This leads to our last observation

Observation 7: (i): The Case 1 scenario is more likely the more efficient the dilution strategy is, i.e., the higher $D$ and $\alpha$ are. (ii): The Case 2 scenario is less likely the more efficient the dilution strategy is, i.e., the higher $D$ and $\alpha$ are.

The intuition for the first result is simply that the more efficient the dilution strategy is, the more attractive it is for firm B to use it. The second result follows from the fact that it is less attractive for firm A to invest in high quality the more efficient the dilution strategy is for firm B.
7. Conclusions and Policy Implications

An important motivation for this study was to explore the circumstances under which laws against trademark dilution are likely to increase welfare. The overall results for welfare effects are ambiguous to say the least. One general conclusion that can however be drawn is that anti-dilution laws cannot be justified on the grounds that they protect welfare-enhancing investments in marketing.

If legislation is unlikely to affect investment patterns, implying that a static framework of analysis is appropriate, banning trademark dilution does seem to be beneficial in terms of aggregate welfare. Under Bertrand competition such legislation does not affect firm behavior, but under Cournot competition welfare is higher if dilution is not an option for firms. The reason is partly that dilution puts an upward pressure on the price of low-quality products, which has the consequence that the poorest consumers are excluded from the market. In addition, trademark dilution tends to greatly reduce rich consumers’ willingness to pay for high quality.

In a dynamic setting results are less clear-cut. With a total welfare standard, banning trademark dilution is beneficial to society only if investment costs are relatively high (Case 2). In this case total investments in quality are unaffected by the policy chosen, but banning dilution puts a downward pressure on the price of the low-quality product which increases aggregate demand. It also increases the willingness to pay for high quality. For lower levels of investment costs (Case 1) the policy affects investment, and banning trademark dilution means that firms will tend to over-invest in product quality, which reduces welfare.

It can be noted that the results presented above hold independently of whether pre-advertising preferences or post-advertising preferences are used for the welfare evaluation. It is evident that more narrowly defined welfare measures, such as the consumer welfare standard or the producer welfare standard, also yield ambiguous results. Hence, our study
provides no basis for clear-cut policy advice. Instead, the main conclusion is perhaps that economic theory does not provide general support for laws against trademark dilution.
References


Appendix:

Definition of investment costs:

\( K_1, K_2, K_3, K_4 \) are defined by \( \pi^{HD}_B = \pi^{III}_B, \pi^{III}_B = \pi^{HL}_B, \pi^{HD}_A = \pi^{II}_A \) and \( \pi^{II}_A = \pi^{HL}_A \).

\[
K_1 = \frac{b^2 (S_H - S_L - D(\alpha + 1))(7S_H - S_L - D(7\alpha + 1))}{9(4S_H - S_L - D(4\alpha + 1))^2}
\]

\[
K_2 = \frac{b^2 (S_H - S_L)(7S_H - S_L)}{9(4S_H - S_L)^2}
\]

\[
K_3 = \frac{b^2 X_1X_2}{(4S_H - S_L)^2 (4S_H - S_L - D(4\alpha + 1))^2}
\]

\[
K_4 = \frac{4b^2 (S_H - S_L)(5S_H - 2S_L)}{9(4S_H - S_L)^2}
\]

where

\[
X_1 = 2\alpha D(2S_H - S_L) + D(3S_H - S_L) - (S_H - S_L)(4S_H - S_L)
\]

\[
X_2 = 2\alpha D(6S_H - S_L) + D(5S_H - S_L) - (3S_H - S_L)(4S_H - S_L).
\]

For \( 0 \leq z \leq 1 \) and \( D \in [0, \overline{D}] \) we have that \( K_1 < K_2 < K_4 \) and \( K_2 < K_3 \).

A threshold level of dilution:

\[
D' = \frac{(4S_H - S_L)(X_3 + 43S_H^2 - S_L(26S_H - S_L))}{X_3 + 115S_H^2 - 44S_HS_L + S_L^2}
\]

where

\[
X_3 = 2\alpha(14S_H^2 + 2S_HS_L - 7S_L^2) - 6(\alpha S_L + S_H)\sqrt{29S_H^2 - 28S_HS_L + 8S_L^2}
\]

\[
X_4 = 4\alpha^2(28S_H^2 + 40S_HS_L - 23S_L^2) + 4\alpha(86S_H^2 - 16S_HS_L^2 - 7S_L^2).
\]

\( D > D' \) implies \( K_3 < K_4 \). For \( D \leq D' \), \( K_3 \) is not relevant for investment or dilution decisions since when \( K > K_4 \) the highest attainable profit is \( \pi^{II}_A = \pi^{II}_B \).
Proofs of

Lemma 1: Since $K_4 > K_2 > K_1$ it follows that
\[ K > K_2 \quad \rightarrow \quad \pi_B^{HD} > \pi_B^{HL} > \pi_B^{HH} \quad \rightarrow \quad S_B = S_B^D \]
\[ K_2 > K > K_1 \quad \rightarrow \quad \pi_B^{HD} > \pi_B^{HH} > \pi_B^{HL} \quad \rightarrow \quad S_B = S_B^D \]
\[ K_2 > K_1 > K \quad \rightarrow \quad \pi_B^{HH} > \pi_B^{HD} > \pi_B^{HL} \quad \rightarrow \quad S_B = S_H \]

Lemma 2: Since $K_4 > K_2 > K_1$ it follows that
\[ K > K_2 \quad \rightarrow \quad \pi_B^{HL} > \pi_B^{HH} \quad \rightarrow \quad S_B = S_L \]
\[ K_2 > K \quad \rightarrow \quad \pi_B^{HH} > \pi_B^{HL} \quad \rightarrow \quad S_B = S_H \]

Lemma 3: Since $K_4 > K_2 > K_1$ it follows that
\[ K > K_4 \quad \rightarrow \quad \pi_B^{LL} > \pi_B^{LH} \quad \rightarrow \quad S_B = S_L \]
\[ K_4 > K \quad \rightarrow \quad \pi_B^{LH} > \pi_B^{LL} \quad \rightarrow \quad S_B = S_H \]

Tables 2 and 3:
In Table 2 the results for consumers’ surplus follow from Observation 4 since banning trademark dilution implies that $z = 1$, i.e. $z$ increases. When calculating differences in net profits, investment costs have to be included. Define $\psi = \pi_A^{HD} + \pi_B^{HD} - 2\pi_A^{HH}$. By Observation 3, the function $\psi$ is decreasing in $D$. Moreover $\psi(D) = K$. Thus $\psi > 0$ for $0 < K_1 < K_2$. For post-advertising preferences the aggregate welfare result follows directly since effects on consumers’ surplus and profits have the same sign. For pre-advertising preferences we define the function $\chi = TS_{D}^{pre} - TS_{ND}^{pre}$. It can be shown that $\chi$ is decreasing in $D$, and $\chi(D) = K$. Thus for $0 < K_1 < K_2$ we have that $\chi > 0$. The results in Table 3 follow directly from Observations 3, 4 and 5.