

# Optimal Mix of Price and Quantity Regulation under Uncertainty

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## *Abstract*

This paper takes on the issue of ‘Prices vs. Quantities’, see Weitzman (1974), applied to environmental regulations under uncertainty. It is shown that, from an efficiency point of view, it is generally preferable to divide the economy into two parts, one regulated through a tax and the other through cap-and-trade, rather than letting it be subject to either of these regulation mechanisms. This may be so even when the latter alternatives are cost effective while the former is not. Particular interest is devoted to determining the optimal size of each sector. Generally, a steeper marginal abatement cost function relative to the marginal abatement benefit function implies that a larger part of the economy should be taxed.

JEL classification H23, L51, Q28, Q58

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# 1. Introduction

In this paper I deal with the regulation of externalities under uncertainty, concentrating on emissions of harmful substances in air or water. In particular, I show that it is generally preferable from an efficiency point of view to adopt a policy under which some emitters of a particular pollutant are subject to an emissions tax and the remaining emitters are subject to cap-and-trade, even though this policy can be shown not to be cost effective. This is of particular interest, as such policies are used in practice, *e.g.*, in connection with the upcoming European Trading Scheme which includes only 46% of the CO<sub>2</sub> emissions while the rest will have to be confronted by another policy instrument.<sup>1</sup>

It is well known that, when marginal abatement costs (*MAC*) and marginal abatement benefits (*MAB*) are known with certainty, an optimal emissions tax is equivalent to an optimal cap-and-trade system, both of which achieve an efficient policy outcome<sup>2</sup>. However, as first formally analyzed by Weitzman (1974) this conclusion is not generally relevant since in most cases at least the *MAC* function is uncertain at the point in time when the tax or the cap is set. Given linear *MAC* and *MAB* functions, an emissions tax is then preferred to a cap-and-trade system when the *MAC* function is steeper than the *MAB* function. Similarly, when the *MAB* function is steeper than the *MAC* function, cap-and-trade is preferred to an emissions tax<sup>3</sup>. However, in both cases some distortions remain on the market. That is, given that only one of the two systems should be used, the Weitzman analysis provides a second best solution.

Many authors have tried to reduce the remaining distortion on the market by combining a quantity instrument with a price instrument. The seminal paper in this branch of the literature is Roberts and Spence (1976), from which the hybrid regulation mechanism known as a ‘safety valve’ originates. Other studies include Weitzman (1978) who introduces a penalty

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<sup>1</sup> This policy may be justified entirely by other reasons than those highlighted in the present paper, but it nevertheless illustrates a real example of the kind of policies discussed herein.

<sup>2</sup> For this to be true from a ‘double dividend’ perspective the permits under cap-and-trade have to be auctioned, see, *e.g.*, Bohm (2002). For the purpose of this paper this is not crucial.

<sup>3</sup> When the functions are non-linear these conclusions are not always correct, see Malcolmson (1978) and Watson and Ridker (1984)

function that operates if an emitter deviates from a pre-set emissions target and Yohe (1981) who studies the use of so called ‘sliding control’, *i.e.*, a scheme in which the emissions tax varies with the emitters’ emissions level, drawing on work by Ireland (1977) and Laffont (1977), among others. A recent paper by Kaplow and Shavell (2002) argues in favor of the use of non-linear tax schemes. The outcomes under these and similar mechanisms are often preferred to those under a standard emissions tax or pure cap-and-trade. However, it seems that such mechanisms are rarely used in practice, perhaps because they are perceived as being too complex.

The driving force behind the results derived in the present paper is that a market regulated through an emissions tax responds differently to a given realization of the *MAC* than one regulated through cap-and-trade. For argument’s sake, assume for now that only the *MAC* is uncertain. If the *MAC* emerges as higher (lower) than expected, the optimal emissions level under cap-and-trade is below (above) the efficient one, where the *MAB* equates the realized *MAC*. The opposite is true under an optimal tax regime: if the *MAC* is realized as high (low) the market will emit more (less) than the efficient volume. Consequently, when only a subset of all emitters is subject to cap-and-trade while the other emitters are taxed the aggregate emissions volume will be closer to the efficient volume regardless of whether the *MAC* is high or low, assuming that the realization applies to both groups. This is obviously appealing but it has a drawback. When all emitters trade and when all emitters are subject to a tax, marginal costs are equated *ex post* and, hence, both solutions are cost effective. Here, if the *MAC* is realized as higher than the expected *MAC* the price in the trading sector is higher than the tax in the taxed sector and *vice versa*. That is, marginal costs will not be equated and the solution is not cost effective. Thus, the following model deals with two sources of efficiency loss. First, the emissions volume may differ from the efficient level. This will be referred to as the ‘volume error’. Second, abatement efforts may be allocated between agents in an ineffective way. This will be referred to as the ‘allocation error’.

The rest of the paper is organized as follows. Section 2 outlines the formal model. Section 3 contains the main analysis, presented both in a setting in which all emitters are identical and in a setting in which they differ in respect to the slope of their individual *MAC* functions. Section 4 concludes.

## 2. The model

Throughout the paper I assume that the market is competitive, that transaction costs and income effects are negligible and that all agents are in compliance. For simplicity let the *MAC* and *MAB* functions be linear in emissions. The *MAB* is given by

$$MAB = f + ge_{tot} + \delta \quad (1)$$

where  $f$  and  $g$  are non-negative parameters,  $e_{tot}$  denotes total emissions and  $\delta$  is a continuous stochastic variable that is symmetrically distributed around zero. Let there be  $N$  emitters and let each emitter have a *MAC* given by

$$MAC_i = K - \frac{L}{1 + \left(i - \frac{N+1}{2}\right)\Theta} e_i + \varepsilon \quad (2)$$

where  $K$  and  $L$  are parameters,  $e_i$  is emitter  $i$ 's emissions volume and  $\varepsilon$  is a continuous stochastic variable that is symmetrically distributed around zero. Note that  $\varepsilon$  has an economy-wide realization common to all emitters. Furthermore, I assume that  $\varepsilon$  and  $\delta$  are independent.  $\Theta$  is a parameter that relates each emitter's index ( $i$ ) to the slope of its *MAC* function. If  $\Theta$  equals zero all emitters have identical *MAC* functions with a slope of  $-L$ . If  $\Theta$  is positive, emitters with a low index (low  $i$ ) will have steeper *MAC* functions than those with a high index. In order for all individual *MAC* functions to decrease in emissions we must have that  $-2/N \leq \Theta \leq 2/N$ . Furthermore, let  $K > f$  and let  $g$  and  $L$  be strictly positive.

Aggregating over all emitters yields an aggregated *MAC*

$$MAC_{tot} = K - \frac{L}{N} e_{tot} + \varepsilon \quad (3)$$

Note that  $MAC_{tot}$  is independent of  $\Theta$  by design. This simplifies the analysis since a change in the distribution of agents' individual *MAC* functions does not influence the aggregate *MAC*. The efficient emission volume,  $e_{tot}^*$ , is the one that equates the aggregated *MAC* with the *MAB*

$$e_{tot}^* = \frac{N(K - f + \varepsilon - \delta)}{L + gN} \quad (4)$$

## 2.1 Efficiency loss due to the volume error under a single sector regime

When the entire market is subject to a cap-and-trade regime, the optimal cap,  $Q$ , is such that the expected  $MAB$  equals the expected  $MAC_{tot}$ , which amounts to (4) with  $\varepsilon$  and  $\delta$  equal to zero. Integrating  $MAC_{tot} - MAB$  from  $Q$  to the efficient emissions level and taking expectations yields the expected remaining distortion on the emissions market,  $E\{DWL_{CnT}\}$ , under the optimal cap-and-trade regime<sup>4</sup>

$$E\{DWL_{CnT}\} = \frac{N\sigma^2}{2(L + gN)} + \frac{N\tau^2}{2(L + gN)} \quad (5)$$

where  $\sigma^2$  is the variance of the stochastic variable  $\varepsilon$  and  $\tau^2$  is the variance of  $\delta$ . The optimal tax,  $T$ , is given by the price at which the expected  $MAB$  equals the expected  $MAC_{tot}$

$$T = K - \frac{L(K - f)}{L + gN} \quad (6)$$

An exercise similar to the one in the cap-and-trade case yields the expected remaining distortion on the emissions market when the entire economy is subject to an emissions tax,  $T$ , as

$$E\{DWL_{Tax}\} = \left(\frac{gN}{L}\right)^2 \frac{N\sigma^2}{2(L + gN)} + \frac{N\tau^2}{2(L + gN)} \quad (7)$$

Note that when  $g = L / N$ , *i.e.*, aggregated  $MAC$  has the same slope as the  $MAB$ ,  $E\{DWL_{CnT}\} = E\{DWL_{Tax}\}$  while  $E\{DWL_{CnT}\} > E\{DWL_{Tax}\}$  when  $g < L / N$  and  $E\{DWL_{CnT}\} < E\{DWL_{Tax}\}$  when  $g > L / N$ , which correspond to the results derived in Weitzman (1974).

## 2.2 Efficiency loss due to volume error under a dual sector regime

Rather than treating the entire economy as one sector, the emitters may be divided into two different sectors such that emitters  $i = 1$  to  $n$  belong to sector 1 and emitters  $i = n + 1$  to  $N$  belong to sector 2. Aggregating over sector 1 yields the following  $MAC$  function

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<sup>4</sup> I do not report intermediary steps in these calculations, as they are well known from earlier literature, *e.g.*, Weitzman (1974).

$$MAC_{S1} = K - \frac{2L}{n(2 - (N - n)\Theta)} e_{S1} + \varepsilon \quad (8)$$

where  $e_{S1}$  denotes total emissions from emitters in sector 1. Similarly for sector 2

$$MAC_{S2} = K - \frac{2L}{(N - n)(2 - n\Theta)} e_{S2} + \varepsilon \quad (9)$$

where  $e_{S2}$  denotes total emissions from sector 2.

In this ‘dual sector case’ sector 1 is subject to the emissions tax  $T$  given by (6) and sector 2 is regulated by cap-and-trade, such that the expected total emissions volume amounts to the expected efficient outcome,  $Q$ . The emissions volume from sector 1 as a function of the tax is given by

$$e_{S1} = \frac{n(K - T + \varepsilon)(2 - (N - n)\Theta)}{2L} \quad (10)$$

Substituting (6) into (10) yields

$$e_{S1}^T = \frac{n\left(\frac{L(K - f)}{L + gN} + \varepsilon\right)(2 - (N - n)\Theta)}{2L} \quad (11)$$

For sector 2, the emissions volume as a function of the price,  $p$ , is

$$e_{S2} = \frac{(N - n)(K - p + \varepsilon)(2 + n\Theta)}{2L} \quad (12)$$

From (6) we know that if both sectors are subject to the tax  $T$  the expected total emissions level, *i.e.*,  $E\{e_{S1} + e_{S2}\}$ , amounts to the expected efficient one. Hence, a simple way of calculating the necessary cap for sector 2,  $Q_{S2}$ , is to substitute  $p$  in (12) for  $T$  and set  $\varepsilon$  to zero, which yields

$$Q_{S2} = \frac{(N - n)\left(\frac{L(K - f)}{L + gN}\right)(2 + n\Theta)}{2L} \quad (13)$$

The total emissions volume from both sectors is reached by adding (11) to (13)

$$e_{S1+S2} = e_{S1}^T + Q_{S2} = \frac{2LN(K-f) + n\epsilon(L+gN)(2-(N-n)\Theta)}{2L(L+gN)} \quad (14)$$

In expectation terms, (14) equals (4), *i.e.*, the expected total emissions volume in the dual sector case coincides with the expected efficient volume. The efficiency loss due to the volume error accruing from adopting a dual sector setting may be calculated by the integral

$$\int_{e_{tot}^*}^{e_{S1+S2}} (MAB - MAC_{tot}) de_{tot} \quad (15)$$

Substituting for the relevant expressions — (4), (14), (1) and (3) — solving the integral and taking expectations yields an expression for the expected efficiency loss due to the volume error,  $E\{DWL_{VE}\}$ , as

$$E\{DWL_{VE}\} = \frac{(L(N-n)(2+n\Theta) + gnN((N-n)\Theta - 2))^2 \sigma^2}{8L^2N(L+gN)} + \frac{N\tau^2}{2(L+gN)} \quad (16)$$

As an illustration, let all emitters be identical, *i.e.*, set  $\Theta = 0$ , and let the slope of the *MAB* equal the slope of the aggregated *MAC*, *i.e.*, set  $g = L/N$ , then (16) simplifies to

$$E\{DWL_{VE} \mid \Theta = 0; g = \frac{L}{N}\} = \frac{(N-2n)^2 \sigma^2}{4LN} + \frac{N\tau^2}{4L} \quad (17)$$

which may be compared to the dead weight loss under the corresponding single sector setting, reached by substituting  $g$  in (5) or (7) by  $L/N$

$$E\{DWL_{Tax} \mid g = \frac{L}{N}\} = E\{DWL_{CnT} \mid g = \frac{L}{N}\} = \frac{N\sigma^2}{4L} + \frac{N\tau^2}{4L} \quad (18)$$

We can see that (17) is less than (18) for all  $n$  larger than zero and less than  $N$ ; in other words, the expected efficiency loss due to volume error is, in this particular setting, strictly less under a dual sector regime than when the entire economy is regulated by a single common regulation mechanism. This is in line with the general intuition that if the entire market is subject to an emissions tax, total emissions will exceed (fall short of) the efficient emission level as a response to a realization of  $\epsilon$  larger (less) than zero whereas under a cap-and-trade regime the opposite applies and, consequently, splitting the economy into a taxed sector and a

trading sector will yield an expected total emission volume closer or equal to the efficient one. But, as mentioned, this approach has the disadvantage of not being cost effective, which is the issue we now turn to.

### 2.3 Efficiency loss due to the allocation error under a dual sector regime

In the following I calculate the total costs of reaching the emissions volume under a dual sector regime first in a hypothetical cost minimizing setting<sup>5</sup> and then in the actual dual sector setting. The difference in total cost between these two scenarios constitutes the efficiency loss following from the allocation error. I calculate the total abatement cost by integrating the *MAC* from the actual emission level to the business as usual (*BAU*) emission level. Since we are interested in the difference between two total costs measures, any fixed costs may be safely ignored.

I find the *BAU*-emission level by setting marginal costs to zero and solving for  $e_{tot}$  in (3), which yields

$$e_{tot}^{BAU} = \frac{N(K + \varepsilon)}{L} \quad (19)$$

In the hypothetical cost effective case the total costs of reaching  $e_{S1+S2}$  is given by

$$\int_{e_{S1+S2}}^{e_{tot}^{BAU}} MAC_{tot} de_{tot} \quad (20)$$

Substituting (14), (19) and (3) into (20) and solving the integral yields an expression for the lowest total cost at which the emission volume  $e_{S1+S2}$  is reached, given the realization of  $\varepsilon$

$$TC_{\min} = \frac{(2N(fL + gKN) + (N - n)(L + gN)(2 + n\Theta)\varepsilon)^2}{8LN(L + gN)^2} \quad (21)$$

It must be noted that (21) is only a basis for comparison as it assumes that the tax or cap may be set after the realization of  $\varepsilon$ . This is to be compared with the actual dual sector outcome in which the taxed sector 1 emits  $e_{S1}$  and the trading sector 2 emits  $Q_{S2}$  units. Let us start by

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<sup>5</sup> This might be viewed as using an emissions tax, assuming it is possible to set the tax after the realization of  $\varepsilon$ .

deriving the total costs in sector 1 of reducing emissions from  $e_{S1}^{BAU}$  (sector 1's business as usual emission level) to  $e_{S1}^T$ . Inserting a tax equal to zero in (10) yields  $e_{S1}^{BAU}$  as

$$e_{S1}^{BAU} = \frac{n(K + \varepsilon)(2 - (N - n)\Theta)}{2L} \quad (22)$$

The total cost for sector 1 may be calculated by

$$\int_{e_{S1}^T}^{e_{S1}^{BAU}} MAC_{S1} de_{S1} \quad (23)$$

Making the appropriate substitutions, using (11), (22) and (8), yields

$$TC_{S1} = \frac{n(fL + gKN)^2(2 - (N - n)\Theta)}{4L(L + gN)^2} \quad (24)$$

For sector 2 the business as usual emission level is given by (12) under a price of zero

$$e_{S2}^{BAU} = \frac{(N - n)(K + \varepsilon)(2 + n\Theta)}{2L} \quad (25)$$

The total cost for sector 2 of decreasing emissions from  $e_{S2}^{BAU}$  to the cap,  $Q_{S2}$ , is given by

$$\int_{Q_{S2}}^{e_{S2}^{BAU}} MAC_{S2} de_{S2} \quad (26)$$

Substituting for (13), (25) and (9) and solving the integral yields

$$TC_{S2} = \frac{(N - n)(gN(K + \varepsilon) + L(f + \varepsilon))^2(2 + n\Theta)}{4L(L + gN)^2} \quad (27)$$

Adding  $TC_{S1}$  and  $TC_{S2}$  yields the total cost in the economy following from the dual sector policy. This amounts to

$$TC_{S1+S2} = \frac{(N - n)(gN(K + \varepsilon) + L(f + \varepsilon))^2(2 + n\Theta) + n(fL + gKN)^2(2 - (N - n)\Theta)}{4L(L + gN)^2} \quad (28)$$

Thus, we have the total cost following the benchmark cost minimizing solution, from (21), and the actual total costs following the dual sector policy, from (28). The efficiency loss due

to an allocation error, denoted  $DWL_{AE}$ , accruing from the fact that the dual sector policy is not cost effective is the difference between the actual cost and the benchmark cost, *i.e.*,  $TC_{S1+S2} - TC_{min}$ . In expectation terms this amounts to

$$E\{DWL_{AE}\} = \frac{n(N-n)(2+n\Theta)(2-(N-n)\Theta)\sigma^2}{8LN} \quad (29)$$

which, as the previous discussion suggests, is zero when  $n = 0$  (all emitters are subject to cap-and-trade) and when  $n = N$  (all emitters are subject to an emissions tax).

Thus far we have derived an expression for the expected efficiency loss due to the volume error under a dual sector policy, given by equation (16), and an expression for the expected efficiency loss due to the allocation error, given by (29). Getting the total expected efficiency loss following a dual sector policy, denoted  $DWL_{S1+S2}$ , is simply a matter of summing these two, which yields

$$E\{DWL_{S1+S2}\} = \frac{\sigma^2}{8L^2N} \left( Ln(N-n)(2+n\Theta)(2-(N-n)\Theta) + \frac{(L(n-N)(2+n\Theta) - gNn(2-(N-n)\Theta))^2}{L+gN} \right) + \frac{N\tau^2}{2(L+gN)} \quad (30)$$

In the following analysis (30) will be compared with the expected efficiency loss from letting the entire market be subject to cap-and-trade, given by (5), and an emissions tax, given by (7). Even at this early stage it is clear that any difference in expected efficiency loss between a single and a dual sector regime will follow from uncertainty about the *MAC* not about the *MAB*, since the last term of (30)—containing the variance of  $\delta$ —is identical to the one in (5) and (7), respectively.

### 3. The analysis

In the previous section we derived an expression for the expected total efficiency loss from the dual sector policy. We now turn to analyzing the optimal solution to the problem of choosing  $n$ .

The number of emitters to be included in the taxed sector 1,  $n$ , is a policy variable, which is to be set so that the expected efficiency loss, given by (30), is minimized. Obviously, this optimization is carried out under the constraint that  $n$  must be non-negative and not larger

than  $N$ . For the sake of simplicity, we minimize (30) with respect to  $n$  ignoring the constraint and afterwards check whether or not it is fulfilled.

In the following  $N$  needs to be large enough so that  $n$  may be approximated to a continuous variable. Given this, we may differentiate (30) with respect to  $n$  to get

$$\frac{\partial E\{DWL_{S1+S2}\}}{\partial n} = \frac{((N-2n)\Theta-2)(L+gn((N-n)\Theta-2))\sigma^2}{4L^2} \quad (31)$$

### 3.1 Homogeneous emitters

Let us first concentrate on the case where all emitters are identical, *i.e.*, where the individual  $MAC$  functions all have the same slope. This is modeled by setting  $\Theta$  to zero, in which case (31) simplifies to

$$\frac{\partial E\{DWL_{S1+S2} | \Theta = 0\}}{\partial n} = \frac{(2gn-L)\sigma^2}{2L^2} \quad (32)$$

By setting (32) equal to zero and solving for  $n$ , we find the optimal number of emitters to include in sector 1, *i.e.*, the  $n$  that minimizes (30)<sup>6</sup>

$$n^* = \frac{L}{2g} \quad (33)$$

Equation (33) states that the optimal number of emitters in sector 1 depends on the relative slope of the  $MAC$  function and the  $MAB$  function in a way such that the steeper the  $MAC$ , in relative terms, the higher the  $n^*$ . Here is an analogy to the original Weitzman (1974) results discussed earlier. There, the regulator is indifferent between a (single sector) tax regime and a (single sector) cap-and-trade regime if and only if the aggregated  $MAC$  function has the same slope as the  $MAB$  function, *i.e.*, if  $g = L/N$ . Substituting this into (33) yields the corresponding solution if we allow for a dual sector solution, namely that

$$n^*_{g=\frac{L}{N}} = \frac{N}{2} \quad (34)$$

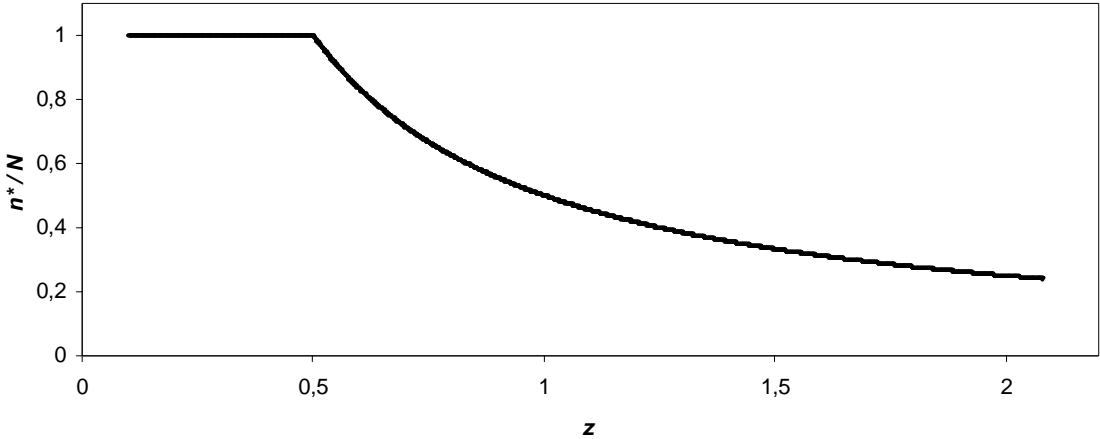
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<sup>6</sup>Since  $\frac{\partial^2 E\{DWL_{S1+S2} | \Theta = 0\}}{\partial n^2} = \frac{g\sigma^2}{L^2} \geq 0$ , the solution constitutes a minimum.

That is, exactly half of the emitters should be in the taxed sector 1 and the other half in the trading sector 2. If  $g$  is slightly higher than  $L / N$ , Weitzman (1974) states that cap-and-trade is preferable to an emissions tax. However, in the dual sector case there should still, in optimum, be some emitters subject to a tax but the number will be less than half of  $N$ . Similarly, if  $g$  is slightly less than  $L / N$  more than half, but generally not all, of the emitters should optimally belong to the taxed sector 1.

We should note that, as we assume  $L$  and  $g$  to be strictly positive,  $n^*$  will never be negative but it may turn out to be larger than  $N$ . In such cases the corner solution of  $n^* = N$  applies, since the number of emitters in the taxed sector cannot exceed the total number of emitters on the market. From (33) we can see that the corner solution is valid for  $g \leq L / (2 N)$ , i.e., for relatively flat  $MAB$  functions.

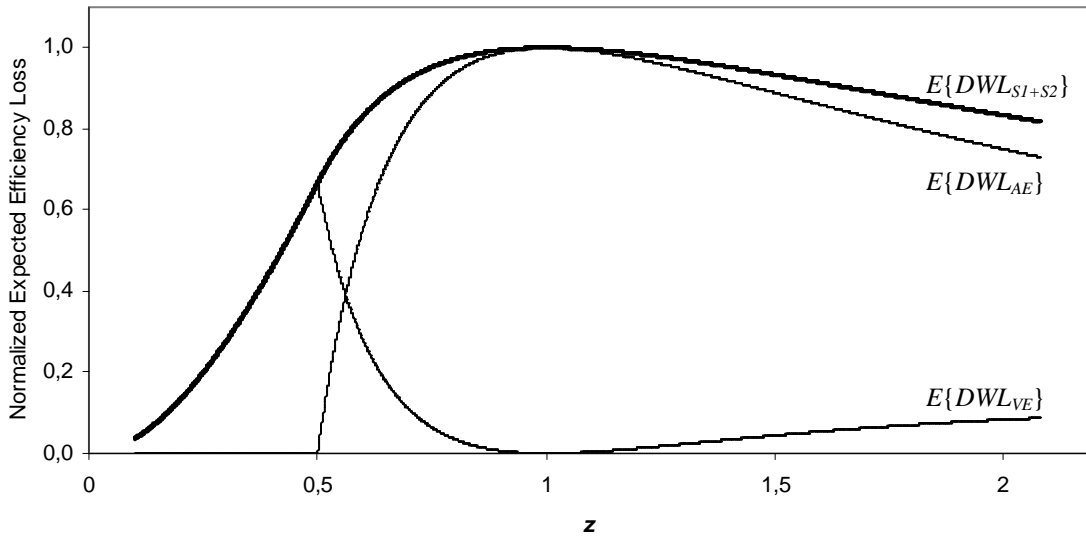
Figure 1 illustrates the optimal proportion of emitters in the taxed sector 1,  $n^* / N$ . On the horizontal axis is  $z$ , which is a measure of the  $MAB$ 's slope defined by  $g = z L / N$ , i.e., a higher value of  $z$  implies a steeper (relative) slope of the  $MAB$ . At  $z$  equal to 1, the  $MAB$  has the same slope as the aggregated  $MAC$ . The horizontal section of the graph captures the formerly mentioned corner solution. Outside this section the optimal proportion of emitters to be included in the taxed sector 1 is convex and decreasing in the relative slope of the  $MAB$  function.



**Figure 1**, Optimal proportion of emitters to be included in sector 1,  $n^* / N$ , as a function of  $z$ , where high values of  $z$  imply a steeper relative slope of the  $MAB$ .

Given an optimal  $n$ , figure 2 decomposes the total expected efficiency loss,  $E\{DWL_{S1+S2}\}$ , following from the optimal dual sector solution into one part due to a volume error,  $E\{DWL_{VE}\}$ , and one part due to an allocation error,  $E\{DWL_{AE}\}$ , assuming  $\tau^2=0$ . Starting at  $z$

close to zero, *i.e.*, a nearly horizontal *MAB* function, it is seen that total expected efficiency loss is small. At  $z = 0$  an emissions tax will result in the efficient outcome and both  $E\{DWL_{VE}\}$  and  $E\{DWL_{AE}\}$  will be zero<sup>7</sup>. As  $z$  increases  $E\{DWL_{VE}\}$  increases, but  $E\{DWL_{AE}\}$  remains at zero due to the aforementioned corner solution. For even higher values of  $z$ ,  $E\{DWL_{AE}\}$  increases and reaches a maximum at  $z = 1$  while  $E\{DWL_{VE}\}$  decreases to reach a minimum at  $z = 1$ . The interaction between the two is such that the total expected efficiency loss,  $E\{DWL_{S1+S2}\}$ , has a maximum at  $z = 1$ . It should be noted that only at the special case of  $z = 1$  will, the efficiency loss due to the volume error be zero. For all other values of  $z$ , larger than 0.5, the optimal solution contains distortions both due to allocation error and volume error.



**Figure 2,**  $E\{DWL_{S1+S1}\}$  as a function of  $z$  decomposed into  $E\{DWL_{VE}\}$  and  $E\{DWL_{AE}\}$ . The graph is normalized so that  $\max(E\{DWL_{S1+S1}\})=1$  and assumes  $\tau^2=0$ .

It is interesting to determine how much better a dual sector regime is compared to the single sector case. Still using  $\Theta = 0$  and looking at the case where  $g = L / N$  the resulting efficiency loss, from (30), amounts to

$$\frac{N\sigma^2}{8L} + \frac{N\tau^2}{4L} \tag{35}$$

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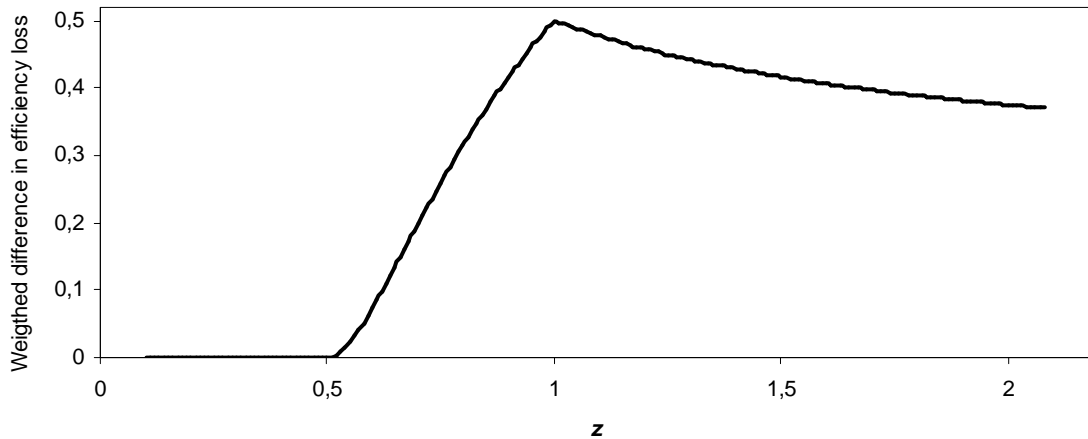
<sup>7</sup> Mathematically, this is not captured by the dual sector model as it would yield a division by zero, but since we know that at  $z = 0$  all emitters will in optimum be taxed, we may use equation (7) instead.

in the dual sector case using the optimal  $n^*$ . The corresponding efficiency loss under a single sector regime, from (5) or (7), amounts to

$$\frac{N\sigma^2}{4L} + \frac{N\tau^2}{4L} \quad (36)$$

The second term in (35) and (36) follows from the uncertainty surrounding the  $MAB$  and is equal for the two cases. The first term follows from the uncertain  $MAC$  and is strictly less in the dual sector case. If there is no uncertainty about the  $MAB$  the expected efficiency loss from the dual sector regime is, in this particular setting, only half that from a single sector solution.

A more general picture is given by figure 3. On the vertical axis is the difference between expected efficiency loss from a single sector case and a dual sector case weighted by the expected efficiency loss from the dual sector case<sup>8</sup>. On the horizontal axis is  $z$ . For  $z < 1$ , a tax is preferred to cap-and-trade under a single sector regime and the opposite for  $z > 1$ . This explains the kink on the curve since at this point the reference regulation mechanism changes from taxes to cap-and-trade.



**Figure 3**, the difference between expected efficiency losses under the single and dual sector cases weighted against the efficiency loss under the single sector case as a function of  $z$ , assuming  $\tau^2=0$ .

As we have seen earlier, for  $z < 0.5$  the optimal solution entails having every emitter in the taxed sector 1. Thus, in this region the optimal policy in the dual sector case coincides with

<sup>8</sup> Algebraically,  $\frac{E\{DWL_{single\ sector}\} - E\{DWL_{S1+S2}\}}{E\{DWL_{single\ sector}\}}$ , where  $E\{DWL_{single\ sector}\} = \min(E\{DWL_{Tax}\}, E\{DWL_{CnT}\})$ .

the single sector tax case and consequently the two will generate identical efficiency losses. Above this region, steeper *MAB* functions will result in increased efficiency gains in switching from the optimal single sector policy to a dual sector policy. The maximum gain possible is at the point where  $z = 1$ , where it amounts to 0.5, which implies that the expected efficiency loss from a single sector regime is twice as high as from a dual sector regime<sup>9</sup>; see (35) and (36). For  $z > 1$ , the efficiency gain from a switch to a dual sector regime decreases in the relative slope of the *MAB*, but it is still positive. As a whole, the dual sector solution performs best, relative to the single sector solution, when the slope of the *MAB* is close to that of the aggregated *MAC*. In cases where these differ greatly the difference in expected efficiency loss between the two systems is smaller.

### 3.2 Heterogeneous emitters

We have shown that generally (subject to the discussion about low values of  $z$ ) a dual sector regime is strictly preferable to a single sector. Furthermore, we did this under the assumption that all emitters are identical, so the result is not a consequence of differences among emitters. Let us now briefly address what happens when this is not the case, *i.e.*, when  $\Theta \neq 0$ . As mentioned, when  $\Theta > 0$ , emitters with a low index have a steeper *MAC* than those with a high index. When  $\Theta < 0$ , the opposite applies. As the *MAC* function is designed we may reverse the order of the emitters—emitter 1 becomes emitter  $N$ , 2 becomes  $N-1$  and so on—without any other changes occurring simply by changing sign of  $\Theta$ . This means it is possible to compare a situation in which those emitters with relatively steep *MAC* functions belong to the taxed sector 1 with a situation in which they belong to sector 2.

Consider the following three policies: 1) let those emitters who have the steepest *MAC* functions belong to the taxed sector 1; 2) let emitters with steep *MAC* functions belong to the trading sector 2; and 3) sort the emitters so that the aggregated *MAC* functions in the two sectors have the same slope. I will now show that, given the use of optimal  $n$ —which differs among the policies—the three policies are equivalent in terms of expected efficiency loss. To

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<sup>9</sup> The figure assumes  $\tau^2 = 0$ . A  $\tau^2 > 0$  adds the term  $\frac{n\tau^2}{2L(1+z)}$  to both  $E\{DWL_{single\ sector}\}$  and  $E\{DWL_{S1+S2}\}$ .

see this, start by deriving an expression for the optimal  $n$ . Setting (31) equal to zero and solving yields the following minimizing root<sup>10</sup>

$$n_{\Theta}^* = \frac{gN\Theta - 2g + \sqrt{4gL\Theta + (gN\Theta - 2g)^2}}{2g\Theta} \quad (37)$$

Note that (37) is not defined for  $\Theta = 0$ , which is why in the previous discussion we had to substitute for this before we solved for the first order condition. Substituting  $g$  for  $zL/N$  and differentiating (37) with respect to  $\Theta$  yields

$$\frac{\partial n_{\Theta}^*}{\partial \Theta} = \frac{N\Theta(z-1) - 2z + \sqrt{z(4N\Theta + z(N\Theta - 2)^2)}}{\Theta^2 \sqrt{z(4N\Theta + z(N\Theta - 2)^2)}} \quad (38)$$

which is positive, *i.e.*,  $n_{\Theta}^*$  increases in  $\Theta$ , as long as  $z > 1/2$ . Under valid values of  $\Theta$ , *i.e.*,  $-2/N \leq \Theta \leq 2/N$ , the derivative is zero at  $z = 1/2$  and negative or lacks a real solution for  $z < 1/2$ . In the previous section we showed that at  $z = 1/2$  and  $\Theta = 0$  all emitters will optimally belong to the taxed sector 1. Since (38) is zero at  $z = 1/2$  this is also the case for all valid values of  $\Theta$ . Furthermore, since

$$\frac{\partial n_{\Theta}^*}{\partial z} = -\frac{N}{z\sqrt{z(4N\Theta + z(N\Theta - 2)^2)}} \quad (39)$$

is negative for all valid  $\Theta$ , a  $z < 1/2$  cannot imply less emitters optimally in sector 1. That is if  $z \leq 1/2$  all emitters are to be taxed regardless of  $\Theta$ , so the interesting situations occur when  $z > 1/2$ . We now restrict our attention to these cases.

Consider an economy with emitters that, if sorted by decreasing *MAC* slope, may be described by a strictly positive  $\Theta$ . We can calculate the optimal number of emitters in the taxed sector under policy 1,  $n_1^*$ , directly using (37). As the order of the emitters is reversed when the sign of  $\Theta$  is changed, we can calculate the optimal number of taxed emitters under policy 2,  $n_2^*$ ,

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<sup>10</sup> To distinguish between the optimal  $n$  under homogenous emitters ( $n^*$ ) we denote the optimal  $n$  under heterogeneous emitters by  $n_{\Theta}^*$ .

using (37) with  $\Theta$  substituted for  $-\Theta$ . Finally, the optimal number of taxed emitters under policy 3,  $n_3^*$ , follows from (33) since this policy corresponds to the case where  $\Theta = 0$ . From (38) it then follows, if  $z > 1/2$ , that  $n_1^* > n_3^* > n_2^*$ . This is in line with the general intuition discussed earlier in the paper. An emitter, subject to an emissions tax, with a relatively flat *MAC* function will respond more to deviations of the *MAC* from its expected realization than one with a relatively steep *MAC* function. It thus seems intuitive that the optimal number of emitters in the taxed sector is larger under policy 1, where the taxed sector contains emitters with relatively steep *MAC* functions, than under policy 2, where it contains emitters with relatively flat *MAC* functions and that policy 3 has an intermediate outcome.

Finally, let us examine the expected efficiency loss. Inserting  $n_\Theta^*$  from (37) into the expression for the total expected efficiency loss, given by (30), and simplifying yields

$$E\{DWL_{S1+S2} | n_\Theta^*\} = \frac{(3gN - L)\sigma^2}{8g(L + gN)} + \frac{N\tau^2}{2(L + gN)} \quad (40)$$

Importantly, (40) does not depend on  $\Theta$ . That is, given the relative slopes and the use of the optimal  $n$ , the choice of policy 1, 2 or 3 has no impact on the expected total efficiency loss. This is perhaps not an intuitive result but there is an explanation, which can be seen by inserting  $n_\Theta^*$  from (37) into the expressions for total emissions from each sector respectively, given by (10) and (13). This yields

$$e_{S1}^{n_\Theta^*} = \frac{L(K - f) + (L + gN)\epsilon}{2g(L + gN)} \quad (41)$$

where  $e_{S1}^{n_\Theta^*}$  denotes sector 1's emissions given tax  $T$  and optimal  $n$ , and

$$Q_{S2}^{n_\Theta^*} = \frac{(K - f)(2gN - L)}{2g(L + gN)} \quad (42)$$

where  $Q_{S2}^{n_\Theta^*}$  denotes the cap for sector 2 given an optimal  $n$ . What is important to note in (41) and (42) is that neither contains  $\Theta$ , *i.e.*, the emission volume from each sector respectively will, under the optimal  $n$ , not differ for different values of  $\Theta$ . Consequently, neither will the total expected efficiency loss.

## **4. Conclusions**

In this paper I have shown that it is, from an efficiency point of view, generally preferable to divide the regulated economy into two sectors, subjecting one sector to cap-and-trade and the other to an emissions tax, rather than adopting the cost effective approach of subjecting the entire economy to either cap-and-trade or an emissions tax. The reason is that the resulting efficiency gains from decreasing distortions on the emissions market outweigh the efficiency loss owing to the dual sector approach not being cost effective. This has been shown to be the case both when the economy consists of identical emitters and when the emitters differ in their slopes of their marginal abatement cost functions.

## References

- Bohm Peter (2002) Comparing Permit Allocation Options: The Main Points, *Working Paper 2002:11*, Department of Economics, Stockholm University
- Ireland N. J. (1977) Ideal Prices vs. Prices vs. Quantities, *Review of Economic Studies*, **44**, pp. 183-186
- Kaplow Louis and Steven Shavell (2002) On the Superiority of Corrective Taxes to Quantity Regulation, *American Law and Economics Review*, **4**:1, pp. 1-17
- Laffont Jean Jacques (1977) More on Prices vs. Quantities, *Review of Economic Studies*, **44**, pp. 177-182
- Malcolmson J. (1978) Prices vs. Quantities: A Critical Note on the Use of Approximations, *Review of Economic Studies*, **45**, pp. 203-207
- Roberts M. and M. Spence (1976) Effluent Charges and Licenses under Uncertainty, *Journal of Public Economics*, **5**, pp. 193-208
- Watson William D. and Ronald G. Ridker (1984) Losses from Effluent Taxes and Quotas under Uncertainty, *Journal of Environmental Economics and Management*, **11**, pp. 310-326
- Weitzman Martin L. (1974) Prices vs. Quantities, *Review of Economic Studies*; **41**, pp. 477-491
- Weitzman Martin L. (1978) Optimal Rewards for Economic Regulation, *American Economic Review*, **68**:4, 683-691
- Yohe Gary W. (1981) Should Sliding Controls be the Next Generation of Pollution Controls? *Journal of Public Economics*, **15**, pp 251-267