Public Ownership and Income Redistribution

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September 11, 2002

Abstract
The large differences among advanced OECD countries in the shares of workers that are employed by the government can probably only to a small part be explained by factors that are in the center of modern organization theory explanations for public vs. private ownership. This paper explores a new hypothesis for explaining the share of government employment. It is based on asymmetric information about individual worker productivity between the taxman, and workers and their employers. Hence, government employment opens up policy options, not available with only private production. The hypothesis is that government employment is an efficient element of redistribution policy. The mechanism is that the government can, through its employment policy, increase the relative scarcity in the private sector of the workers the government wants to redistribute in favor of. That increases their wages and lowers the need for redistribution through the tax- and transfer systems, which mitigates distortions. One can therefore expect large government employment in countries where the tolerance of inequality is low.

JEL Classification: H11, H110, H21 and H23
Keywords: Structure and Scope of Government, Optimal non-linear income taxation, public production, production efficiency

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1. Introduction

The role of government in-house production of goods and services is a longstanding and central economic and political issue. For long the issue was highly relevant in the context of the debate about central planning vs. markets and the related issue, socialism vs. capitalism. Essentially, that debate was about which economic system is better in delivering social welfare (see, von Mises (1974) and Lange (1936)). Now when that debate is no longer active, other issues are in the forefront such as effects of government in-house production on unemployment (see Yann, Cahuc and Zylberberg (2002)), how choices between private and government in-house production affect political processes and outcomes (see Alesina, Danninger and Rostagno (2002)) and the boundary between government in-house production and private production in mixed economies. The theoretical analyses of private vs. public production in mixed economies are welfare oriented and in the context of partial equilibrium models in which ownership plays a role because of asymmetric information and incomplete contracts (see Laffont and Tirole (1993), Tirole (1992) on asymmetric information and, Hart, Vishny and Schleifer (1997) and Schleifer (1998) incomplete contracts).  

This paper deals with the boundary between government in-house and private production from a new and different perspective. Rather than focusing on particular characteristics of the commodities produced or the internal organization of firms, the issue in this paper is effects of government employment on the distribution of labor income. The reasons for taking this perspective is that there are significant differences between advanced OECD countries in the share of the workforce that is employed by the government. Hence, in Denmark, Norway and Sweden governments account for some 30% of total employment while governments in Germany and the US employ some 15% of the total workforce. Those large differences in employment shares among different countries can probably not be fully explained by differences in which information different agents have available, differences in completeness of contracts or by differences in which commodities that are produced in different countries.

Table 1: Government employment as a percentage of total employment and Gini-coefficients for equivalent disposable household income per individual (mid 90s). OECD (1999).

2 There are also empirically oriented studies, see e.g., López-de-Silanes, Shleifer and Vishny (1997) and Vickers and Yarrow (1991).
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<td>Italy</td>
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<td>13.0</td>
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However, an instance where there are significant and important differences among the advanced OECD countries is in their tolerance of income inequality. That is clearly manifested in different degrees of progressivity of tax- and income support systems. Denmark, Norway and Sweden are doubtless among countries in the lower end of the scale while income inequality seems much more acceptable in, e.g., the US. Hence, it appears that countries that have large shares of government employment are also countries that have low tolerance of income inequality.³ Table 1 reports employment shares and Gini-coefficients for a number of OECD countries. A potential explanation for such a pattern is that extensive redistribution through the tax- and transfer systems give rise to large distortions and that government employment and in-house production might have a role to play to mitigate these.⁴

Such a role is possible because a major source of income inequality is wage inequality and a government that is a large player on the labor market can affect the market outcome. A channel for that when the private sector labor market is competitive and the government offers the same working conditions as private producers (the government adheres to horizontal equity) is that the government deviates from cost minimizing factor demand and uses the workers that are favored by the redistribution more intensively. Such a policy increases the relative demand for and the wages of the favored workers and it reduces the need for redistribution through the tax- and transfer systems. However, a large deviation from the cost-minimizing factor mix in a small part of the economy causes large distortions

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³ Germany might seem as a disturbing observation in this context. However, in Germany health-care production is to large extent carried out by non-profit organizations and therefore not recorded as public production. To what extent these production units follow private sector behavior is an open question.
⁴ Another, more ideological explanation is that low tolerance against income inequality is an important element of left-wing political views and people holding such views are usually more favorable to government in-house production than people with right-wing views.
and is in some cases not even feasible. Therefore, extending the range of goods and services that are produced in-house by the government can potentially reduce the efficiency cost of such a policy. That creates a link between the degree of redistribution and the size of government production.

However, according to the micro-oriented analyses based on asymmetric information and incomplete contracts different types of production are differently suitable for government ownership. For some types production such as running foreign policy or setting up and training the army government in-house production is the only realistic alternative. For other types of production, such as fashion clothes or consumer electronics, private ownership seems to be the only realistic alternative. This implies that the interesting border-line cases for government ownership and wage affecting policies are those where first, private ownership is the more efficient alternative but the loss of internal efficiency under government ownership is not large and, second, the production technology allows factor substitution at low cost. Little loss of internal efficiency under government ownership implies that state-of-the-art technology as well as consumer demand should be well known and not shifting too rapidly.

There are of course other wage affecting policy options open to the government. It can for example hold back sectors that use unfavored workers intensively and expand sectors that use favored workers intensively. Government ownership is not generally required for such a policy but may be needed in cases where it is desirable from an income distribution point of view to hold back a sector while its output cannot be priced too highly. Suppose for instance that the government wants to hold back spending on health care because that sector employs high skilled workers intensively. If it, at the same time, wants health care to be available to those who need it, prices might not be the best rationing device. Screening by doctors and physical rationing may perform better than prices alone but such rationing system may not function well under private ownership where high-powered incentives are in action.

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5 Technology may not allow factor substitution.
6 The implication is not that government ownership of low-wage intensive production is particularly interesting.
7 In cases where the government wants to hold back a sector it should be possible to identify consumption for which consumer surplus is low or in cases where consumption is paid for by an insurer, consumption that is worth less than its cost.
The analysis in this paper addresses only two issues namely the size of public production and to what extent the government should deviate from cost-minimizing factor demand in order to achieve constrained Pareto-efficiency. The analysis builds on the same lack of information on the part of governments as that manifested in tax schedules and assumed in theories of optimum income taxation (see, Mirrlees (1971)). Hence, the government, as a tax collector, can adequately observe income at individual worker level but it cannot observe productivity or wage per hour, and number of hours worked at that level. However, each worker and his or her employer know the worker’s productivity. That lack of information on the part of the government is an obstacle to subsidies (or similar measures) to the use of low wage workers since such a policy would have to rely on truthful reporting in cases where false reporting is feasible and beneficial. However, when the government is the employer it is reasonable to assume that it has the very same information about its employees as private employers about its employees. Hence, government employment opens up a policy option that is otherwise not available. The government can for example use low-wage workers more intensively than private producers would for the corresponding production. That would increase the relative demand for low-wage workers, which tends to increase wages of those workers and reduce the need for transfers.

The analysis is related to the discussion about whether production efficiency (i.e., marginal rates of transformation between factors are the same in all production units) is desirable or not (see Diamond and Mirrlees (1971), Dasgupta and Stiglitz (1971, 1972) for early contributions and Naito (2000) for a recent contribution). Naito (2000) is the closest to the analysis here in that the motivation for the deviation from cost efficiency at going market wages for a public producer is distortions caused by optimum non-linear income taxation. The most important difference between the previous analyses and the analysis in this paper is that the boundary between government in-house production and private production is assumed in those analyses while it is endogenous in this analysis. Another difference is that in our analysis the public sector produces the same commodity as the private sector. That feature of the model reflects the fact that certain commodities are in some countries

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8 In this literature the reason for production inefficiency is some (in many cases exogenous) restriction on the set of feasible tax instruments. Our model is similar in this respect but the restriction on tax instruments is determined endogenously since we employ an income tax the design of which is only restricted by standard self-selection constraints.
produced by private firms and in other in-house by the government. Previous studies do not address the issue of boundary between government in-house- and private production.

The model in this paper essentially is the Stiglitz (1982) two types of workers optimum non-linear income taxation model with endogenously determined wages. The two types of workers are the less- and the more productive workers where the former are called the unskilled and the latter the skilled workers. Only one consumption good, which is a private good, is produced with the two types of labor. In addition to Stiglitz (1982) we add the option for the government to produce commodities in-house. The government and private producers have access to essentially the same production technology which offers some limited substitution possibilities between the two types of labor (the elasticity of substitution is strictly larger than zero and not infinity). Due to weaker incentives on the part of the government, compared to private entrepreneurs (who are residual claimants and face hard budget constraints) to quickly reduce cost and to change products when market conditions alter we assume that government in-house production is subjected to some waste. Hence, we assume a factor neutral lower productivity in government in-house production as compared to private production. That assumption limits the size of government in-house production. The analysis is normative in the sense that we explore Pareto-efficient policies. However, it can be argued that such an approach might have predictive power since Pareto-efficient policies can be expected to dominate non-efficient policies as election programs (see Wittman (1989). Our position on this issue is open because although allocations that are Pareto efficient dominate some non-efficient allocations it seems quite possible that the bargaining and coordination processes that are required to reach efficiency can fail. Our view is therefore that the efficiency approach is a first attempt to model the issue.

The paper is organized as follows. In Section 2 the equilibrium model is presented. Section 3 deals with dominant policies. We show three results. The first is for situations where the incentive compatibility constraints do not bind. In those situations government in-house production is not part of Pareto-efficient policies. The second and the third results are for situations where one of the incentive compatibility constraints binds. In those situations (i) government in-house production enhances efficiency and (ii) it is indeed efficient that the government tilts factor intensity towards the favored type of worker as compared to the cost efficient intensity at going market wages. In this model it is never efficient that the

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9 In Mirrlees (1971) the elasticity of substitution between differently productive workers is infinitely high.
government is cost efficient at going market wages. In Section 4 we carry out a series of simulations in order to obtain information on the size of government employment. Hence, we calculate solutions for government employment of the two types of workers for different income distributions under somewhat realistic parameter assumptions. The results of the simulations are that with ambitious redistribution the efficient level of government employment is significant and factor intensities are significantly tilted against low-wage workers. It therefore seems that differences in the degree of tolerance against income inequality can potentially explain the large differences in the sizes of government employment among advanced OECD countries.

2. Model
The model relies as much as possible only on technological factors and competitive behavior of private agents. A critical choice is how to model the technology for redistribution. We have chosen to not impose other constraints on it than those given by the lack of information on the part of the government and horizontal equity. Additional artificial constraints on for example the tax system would lead to efficiency costs, which may be possible to eliminate through democratic decisions. For those reasons we assume that the government imposes non-linear income taxation and employs workers to produce the consumer good in-house so as to achieve (information) constrained Pareto-efficiency.

2.1. Individuals
Consider an economy with two types of workers indexed $i=1,2$ and two types of work. There are $n_i$ individuals of type $i$. Type 2 workers are assumed to be skilled workers and type 1 workers to be the unskilled. The market wage rates (which are endogenously determined) for the two types of work are $w_i$, $i=1,2$. All individuals have identical preferences represented by the concave utility function $U \left( c_i, \frac{y_i}{w_i} \right)$, which is increasing in $c_i$ and decreasing in $\frac{y_i}{w_i}$, where $c_i$ denotes consumption (or equivalently after tax income) and $\frac{y_i}{w_i} = l_i$, where $y_i$ is gross income, $w_i$ is the wage rate and $l_i$ is labor supply. The price of consumption is normalized to 1.

Since the government cannot observe individual labor supply, a type $i$ worker can pretend to be a type $j$ worker and vice versa. That can be done either (i) through a type $i$
worker works in her own type of work but earning the same income as a type \( j \) worker when she works in her own type of work (income replication) or (ii) a type 2 worker takes up a type 1 work and work in the same fashion as an unskilled worker (job replication). We assume that it is technically feasible for skilled workers to take up an unskilled work but it is not feasible for an unskilled worker to take up a skilled work. Workers are given incentives to not mimic the other type’s income if the following holds.

\[
U^i\left(c_i, \frac{y_i}{w_i}\right) \equiv V^i(c_i, y_i) \geq V^j(c_j, y_j) \equiv U^j\left(c_j, \frac{y_j}{w_i}\right),
\]

where \( i, j = 1, 2 \). When a type 2 worker earns a higher wage rate than type 1 workers it follows that a type 2 worker that earns the same income as a type 1 worker obtains a higher utility than a type 1 worker. Therefore, it follows that when income replication does not pay for the skilled workers, then job replication does not pay either.

2.2. Production

We now turn to production of the consumption good. Essentially only one production technology is available. It is given by a strictly concave production function \( F(L^s_i, L^s_j) \) in the private sector and \( \alpha F(L^s_i, L^s_j) \) in the public sector, where \( L^s_i \) indicates number of work hours of type \( i \) in sector \( s \). The production function exhibits constant returns to scale (CRS) and satisfies the Inada-conditions. The parameter \( \alpha \) (between 0 and 1) reflects lower efficiency in the public sector. CRS implies that we without loss of generality may assume that there is only one private firm that earns no pure profit and only one public firm. This is just a normalization since we assume competitive conditions on all markets.

We consider a situation where the government offers the same wages and working hours as are present in the private sector for the same type of work. Also, private producers take wage rates as given and where wages equate demand and supply for the two types of labor. Profit maximization by the private firm implies that

\[
\frac{\partial F(L^s_i, L^s_j)}{L^s_i} = w_i, \ i = 1, 2.
\]

That is, the wage rate of type \( i \) workers equals the market value of their marginal product in the private sector. In factor intensity form we have, \( f'(x^s) = w_i \) and

\[\text{That is, } \lim_{L^p_i \to 0} F_1(L^p_i, L^s_j) = \lim_{L^p_i \to 0} F_2(L^p_i, L^s_j) = \infty \text{ and } \lim_{L^p_i \to \infty} F_1(L^p_i, L^s_j) = \lim_{L^p_i \to \infty} F_2(L^p_i, L^s_j) = 0 \text{ are assumed to hold.}\]
\(f(x^p) = w_1 + w_2 x^p\), where \(f(x^p) = F(1, x^p)\) and \(x^p = \frac{L^p}{L^p_1}\). This implies that private sector production is \(F(L^p_1, L^p_2) = L^p x f(x^p)\). Public sector production is \(\alpha F(L^p_1, L^p_2) = L^p x f(x^p)\), where \(x^s = \frac{L^s_2}{L^s_1}\).

2.3. Equilibrium

Given that only type \(i\) workers earn type \(i\) income we have the following market-clearing conditions on the labor markets, \(L^p_i + L^s_i = n_i l_i\), \(i = 1, 2\), where \(l_i = \frac{y_i}{w_i}\). Market-clearing on the market for the consumer good implies \(n_1 c_1 + n_2 c_2 = L^p f(x^p) + L^s x f(x^s)\).

3. Dominant policies

The policy making problem for the government is to choose income before and after tax for the two types of workers and the number of workers of the two types that work in public production, so as to satisfy the market clearing conditions and to achieve Pareto-efficiency. We choose to write the problem as maximization of type 2 worker utility given (i) that type 1 workers achieve (the feasible) utility level \(\hat{u}\), (ii) it does not pay for type 1 (2) workers to replicate type 2 (1) income, (iii) the consumption good market clears, (iv) there is no pure profit in the private sector, (v) type two worker wage coincides with type two worker marginal productivity in the private sector and (vi) type two workers are more productive than type 1 workers. The government maximizes over \(c_1, c_2, y_1, y_2, n^s_1, n^s_2\). The Lagrange function is

\[
L = V^2(c_2, y_2) + \mu_1 [\hat{u} - V^1(c_1, y_1)] + \mu_2 [V^1(c_1, y_1) - V^1(c_2, y_2)] + \frac{\lambda_1}{w_1} [V^2(c_2, y_2) - V^2(c_1, y_1)] - \gamma [n_1 c_1 + n_2 c_2 - (n_1 - n^s_1) \frac{y_1}{w_1} f(x^p) - n^s_1 \frac{y_1}{w_1} \alpha f(x^s)] + \phi_1 [w_1 + w_2 x^p - f(x^p)] + \phi_2 [w_2 - f'(x^p)] + \delta [w_1 - w_2]
\]

First order conditions to this problem are found in the Appendix. Although we explicitly stipulate that the type 2 work wage rate cannot be smaller than the type 1 work wage rate we assume that it always holds with strict inequality. That implies \(\delta = 0\) in optimal points.
Let us first note that this problem is not well behaved: In order to apply the Kuhn-Tucker theorem we need $V^2$ to be strictly concave and all constraints to be quasi-convex. The latter condition only holds for the utility constraint on type 1, which is strictly convex and for the productivity constraint, $(w_1 - w_2)$, which is linear. However, the self-selection constraints are differences between two concave functions. As a consequence we cannot be sure that the Lagrange multipliers $\lambda_1$ and $\lambda_2$ are non-negative. However, in the following we assume that these multipliers are non-negative. That is also the standard assumption in the literature.

The same holds for the multiplier $\varphi_2$ which is associated with the wage determining condition; it may be either negative, zero or positive. The interpretation of that Lagrange multiplier is that it gives the direction of change in the value function when the type 2 wage rate increases above its marginal product. Therefore its sign will be essential in determining how the policy maker should distinguish herself from the producers in the private sector.

The solution to the maximization problem can be of different kinds. It can either be a first-best Pareto efficient solution (where none of the self-selection constraints are binding) or second-best Pareto-efficient where one of the self selection constraints are binding. The natural starting point for analysis is the situation where redistribution is limited in the sense that the allocation does not deviate much from a laissez-faire equilibrium. Therefore none of the self-selection constraints bind.

**Proposition 1.** In a first-best Pareto efficient solution, $\lambda_1 = \lambda_2 = 0$, there is no public production. That is $n^1 = n^2 = 0$.

All proofs of propositions are also found in the Appendix. This result is intuitive. Redistribution is limited and not driven to far enough to make one of the self-selection constraints binding. Then differentiated non-distortionary taxation can be used to carry out the redistribution. This is the same type of result as that in the Second Welfare Theorem.

We now turn our attention to the cases with public production. We can immediately dismiss the case with public production such that $x^e = x^p$. Such factor intensities can never

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11 Our main result that there will be public production in a second-best optimum is unaffected by this assumption. However, in which direction public sector factor intensity will deviate from that in the private sector depends crucially on this assumption.
be a part of a Pareto-efficient solution because of the waste factor $\alpha$. Reduction of public employment will always increase the value of the objective function.

**Proposition 2.** Public production ($n_i^x > 0, i = 1,2$) with $x^s = x^p$ is never (constrained) Pareto-efficient.

For public production to make sense it must do something different compared to the private sector. Since the same private commodity is produced using the same production technology, except for the waste factor, the only thing the government as a producer can do differently is to choose a different factor intensity compared to that used in the private sector. Since the private sector factor intensity is cost minimizing at going wage rates, public production will always generate a loss and require additional tax revenue. If the public sector uses the same factor mix as the private sector it would, on the margin, not affect the relative demand for the different types of labor. Hence, public production would be costly but generate no gain.

**Proposition 3.** In (constrained) Pareto-efficient solutions in which (i) type 1 is favored by redistribution ($\lambda_1 = 0$ and $\lambda_2 > 0$), public production is such that $x^s < x^p$, and (ii) type 2 is favored by redistribution ($\lambda_2 = 0$ and $\lambda_1 > 0$), public production is such that $x^s > x^p$.

Hence, public production is always (constrained) Pareto-efficient in a second-best solution. Furthermore, the factor intensity will be such that relatively more is used of the factor that is favored by redistribution via the tax system. Hence, it will be optimal for the public sector to employ type 1 (2) workers beyond the point where their marginal product equals type 1 (2) wage rate, if redistribution goes in the direction of type 1 (2) workers. Such an employment policy increases the wage of type 1 (2) workers and lowers the need for distortionary redistribution via the tax system.

Finally, let us say something about the results for the optimal non-linear income tax. Basically, it can be shown that the present model arrives at the same results as in Stiglitz (1982): E.g., for the case of redistribution in favor of type 1 (i.e., ($\lambda_1 = 0$ and $\lambda_2 > 0$)) the optimal marginal tax rate for type 2 is negative and for type 1 positive.

4. **Simulations**

In order to explore the potential significance of the hypothesis put forward in this paper we have carried out a number of simulations under what we regard as reasonably realistic parameter assumptions and for different degrees of redistribution. We have parameterized
the model with CES utility and production functions. The utility function parameters have been chosen to, in the laissez faire equilibrium, generate somewhat less than 40 work hours per week for unskilled workers and somewhat more than 40 work hours per week for skilled workers. The elasticity of substitution has been chosen to generate a small, close to 0.10, uncompensated elasticity of labor supply. The latter seems consistent with empirical estimates (see Hansson-Brusewitz and Blomquist (1990)). Production function parameters have been chosen to generate a wage rate ratio of 1.4 in the laissez-faire equilibrium. That is close to the observed difference in the US (see Gottschalk (1997)). For the elasticity of substitution between skilled and unskilled workers (worker with or without college degree) there are some recent estimates on a macro level (see Krusell, Ohanian, Rios-Rull and Violante, (2000)). A value for the elasticity of substitution of 1.5 seems quite in line with their estimates. A parameter, for which there is not much information to obtain in the literature, is the relative efficiency of government in-house production. We have chosen 0.95. A higher value would lead to a larger share of public production while a lower value would lead to a smaller share of public production. It is of course possible that there are large differences among countries in the relative efficiency of government in-house production of particular commodities. But it is not obvious that Denmark, Germany, Norway, Sweden and the US are very different in this respect.

Hence, the simulations have been carried out with the following utility and production functions.

\[
U(c_i,l_i) = \left[ \beta c_i^\rho + (1 - \beta)(168 - l_i)^\rho \right]^{\frac{1}{\rho}},
\]

\[
F(L_1^p, L_2^p) = \left[ \phi(n_1 - n_1^p)l_1^p + (1 - \phi)(n_2 - n_2^p)l_2^p \right]^{\frac{1}{\nu}},
\]

and

\[
\alpha F(L_1^p, L_2^p) = \alpha \left[ \phi n_1^p l_1^p + (1 - \phi)n_2^p l_2^p \right]^{\frac{1}{\nu}}.
\]

The following parameter values have been used; \( \beta = 0.29, \rho = 0.12, \phi = 0.4, \nu = 1/3 \) and \( \alpha = 0.95 \). The number 168 refers to the number of hours in a week.

The simulation solutions have been carried out in Mathematica: The program used is included in the Appendix. To get around the problem of non-uniqueness of solutions to the first-order conditions, we made use of the fact that the laissez-faire equilibrium is unique. Hence, we first calculated the laissez-faire equilibrium. Then we moved away in small steps
from this equilibrium with starting values close to the earlier equilibrium. Given that the optimum solution does not make dramatic jumps that method should guarantee that we obtain the global optimum.

The results of the simulations are presented in Table 2 and Table 3. Table 2 is for the case where public employment is not an available instrument. Table 3 is when public employment is an available instrument. From Table 2 we obtain that the skill-premium, $\frac{w_2}{w_1} - 1$, decreases from 2.26 to 0 as utilities are equalized. Relative after tax income decreases from 64 to 1.7 and relative gross income increases from 1 to 1.8. This pattern comes back in Table 3, which also shows Pareto-efficient levels of public employment. Hence starting in a situation where redistribution favors skilled workers, the public sector tilts skill intensity in favor of skilled workers. Moreover, moving to solutions (down in the table) where skilled workers are less favored public employment shrinks to disappear in the segment where no incentive compatibility constraint is binding. When the self-selection constraint binds for type 2 workers the redistribution goes in favor of unskilled workers. In that segment public employment is again interesting but now the skill intensity is tilted in favor of unskilled workers. It is interesting to note that increased equality can easily result in public employment, which is up to about 9% of total employment. Hence, it seems that the hypothesis put forward in this paper has potential to explain rather much of between country differences in public employment.

Tables 2 and 3 about here.

There are also a few other things that are interesting to note about the simulations. Wage rate inequality is at its largest value when redistribution in favor of skilled workers is driven very far. Wages are equalized when utilities are equalized. Gross incomes are equalized when redistribution in favor of skilled workers is driven very far. Gross income inequality is at its largest when redistribution is in favor of the unskilled is at its largest but it is still modest (i.e., less than 50 per cent). When redistribution is driven further in that direction gross income inequality is reduced. The result that gross income inequality is larger when there is large redistribution in favor of the unskilled than in the laissez faire equilibrium seems not to accord with reality. The technical explanation is that low skilled work time decreases significantly. In reality not all low skilled workers are employed and therefore a possible interpretation of that result is that the income represents an average among low skilled workers.
5. **Concluding Comments**

The aim of this paper is to put forward a new hypothesis on public employment, namely that public employment may serve the purpose to mitigate labor market distortions created by taxation. The analysis is in a context of a model with two types of workers and where the public sector produces the same commodities as the private sector but with a somewhat lower efficiency. The only constraint on taxes and transfers is that the government cannot observe individual worker productivity. The result is that whenever incentive compatibility constraints affect tax schedules, distortions can be reduced by means of public employment. The public sector should distinguish itself from the private sector by employing a larger share of workers of the types favored by redistribution policy than would the private sector. Such an employment policy increases wages and thereby welfare of the favored types of workers, which relieves the tax and transfer policies from some burden.

In a more realistic context where there are several types of workers, which are rather close to each other in terms of productivity, incentive compatibility constraints will almost always affect tax schedules. The implication is that whenever a government wants to redistribute income there will almost always also be an argument for production inefficiency and public employment.

The simulation results show, under somewhat realistic parameter constellations, that public employment is not negligible. This implies that political programs that amounts to achieving redistribution may for efficiency reasons also include significant elements of government production and employment. Political programs in which large scale redistribution is not a prominent element could therefore be expected to not politicize the issue of public vs. private ownership of certain production facilities. That is potentially an important explanation for the large differences in the shares of government employment among the advanced OECD countries.

6. **References**


Appendix

A.1. First order conditions

Throughout we use $\hat{V}_i^j \equiv V_i^j(c_i, y_j)$, $i, j = 1, 2$ and $i \neq j$. Using the definition of factor intensities $x^p = \frac{n_2 - n_2^x}{n_1 - n_1^x} \frac{w_1 y_2}{w_2 y_1}$ and $x^s = \frac{n_2^s}{w_2 y_1}$, the first order conditions needed to derive the results regarding public production can be written as follows:

$$\frac{\partial L}{\partial y_1} = (\lambda_1 - \mu)V_1^1 \lambda_2 V_1^2 + \gamma \left[ (n_1 - n_1^s) + \alpha n_1^s \frac{f(x^s) - x^s f'(x^s)}{w_1} \right] + \phi_2 f'''(x^p) \frac{x^p}{y_1} = 0, \quad (1)$$

$$\frac{\partial L}{\partial y_2} = (1 + \lambda_2) V_2^1 \lambda_1 V_2^2 + \gamma \left[ (n_2 - n_2^s) + \alpha n_2^s \frac{f'(x^s)}{w_2} \right] - \phi_2 f'''(x^p) \frac{x^p}{y_2} = 0,$$  \quad (2)

$$\frac{\partial L}{\partial w_1} = \mu V_1^1 y_1 \frac{w_1}{w_2} - \lambda_1 y_1 \left( V_1^1 - V_1^{1\#} \right) - \gamma y_1 \frac{w_1}{w_2} \left[ (n_1 - n_1^s) + n_1^s \alpha \frac{f(x^s) - x^s f'(x^s)}{w_1} \right] + \phi_1 - \phi_2 f''(x^p) \frac{x^p}{w_2} + \delta = 0,$$  \quad (3)

$$\frac{\partial L}{\partial w_2} = -V_2^2 y_2 \frac{w_2}{w_2} - \lambda_2 y_2 \left( V_2^2 - V_2^{2\#} \right) - \gamma y_2 \frac{w_2}{w_2} \left[ (n_2 - n_2^s) + n_2^s \alpha \frac{f'(x^s)}{w_2} \right] + \phi_1 \frac{x^p}{w_2} + \phi_2 \left[ 1 + f''(x^p) \frac{x^p}{w_2} \right] - \delta = 0,$$  \quad (4)

$$\frac{\partial L}{\partial n_1^s} = \gamma_1 \left[ \alpha \frac{f(x^s) - x^s f'(x^s)}{w_1} - 1 \right] - \phi_2 f'''(x^p) \frac{x^p}{n_1 - n_1^s} \leq 0, n_1^s \geq 0 \quad \text{and} \quad (5)$$

$$\frac{\partial L}{\partial n_2^s} = \gamma_2 \left[ \alpha \frac{f'(x^s)}{w_2} - 1 \right] + \phi_2 f'''(x^p) \frac{x^p}{n_2 - n_2^s} \leq 0, n_2^s \geq 0.$$  \quad (6)

A.2. Proofs

Note first that one can show that $\gamma > 0$ (i.e., it is never optimal to have some resources unused and more resources would increase the value of the objective function) and $\mu < 0$ (i.e., type 1 will be on its exogenously given utility level, the reduction of which would increase the value of the objective). Throughout we consider solutions such that $w_2 > w_1$ and therefore $\delta = 0$.  

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Proof of Proposition 1: Suppose no self-selection constraint is binding so that \( \lambda_1 = \lambda_2 = 0 \). Multiply equation (3) with \( \frac{w_i}{y_1} \) and subtract the result from equation (1). The result implies \( \varphi_1 = 0 \). Multiply equation (4) \( -\frac{w_i}{y_2} \) and subtract the result from equation (2). The result implies \( \varphi_2 = 0 \). Suppose also there is some public production so that \( n_i^g > 0 \quad i = 1,2 \). Using \( \lambda_1 = \lambda_2 = \varphi_1 = \varphi_2 = 0 \) and the first order conditions for cost minimization in the private sector, i.e., \( f'(x^p) = w_2 \) and \( f(x^p) = w_1 + w_2 x^p \), equations (5) and (6) implies

\[
\frac{f(x^g) - x^g f'(x^g)}{f'(x^g)} = \frac{w_1}{w_2} = \frac{f(x^p) - x^p f'(x^p)}{f'(x^p)}.
\]

Hence, the factor intensity in the public sector is the same as that in the private sector. However, since the public producer replicates cost minimization it follows from equation (5) and (6) that \( \frac{\partial L}{\partial n_i^g} = -\gamma y_i(1-\alpha) < 0 \) contradicting an interior solution for the public demand of both types of labour. Hence, there is no interior solution with \( \lambda_1 = \lambda_2 = 0 \), which proves Proposition 1.

Proof of Proposition 2: Consider a solution with public production so that \( n_i^g > 0 \quad i = 1,2 \). Suppose also that \( x^g = x^p \). From equations (5) and (6) then immediately follows that \( \varphi_2 \) is both strictly positive and strictly negative in the optimal point. A contradiction and therefore \( x^g \neq x^p \) if optimality implies public production. Hence, Proposition 2 follows.

Proof of Proposition 3: Since the production function satisfies the Inada-conditions, c.f., footnote 9, it follows directly from the first order conditions (5) and (6) that any interior solution to \( n_1^g \) or \( n_2^g \) is ruled out given an appropriate choice of \( x^g \) such that \( x^g \neq x^p \); i.e., the value of the objective function can always be increased if the public sector uses both types of labor at a different factor intensity than the private sector.

Consider then the case when redistribution favors type 1, i.e., \( \lambda_1 = 0 \) and \( \lambda_2 > 0 \). From above we know that \( n_1^g > 0 \) and \( n_2^g > 0 \). Now, multiply equation (3) with \( \frac{w_i}{y_1} \) and subtract the result from equation (1). The result implies \( \varphi_1 < 0 \). Multiply equation (4) with

16
and subtract the result from equation (2). The result implies \( \varphi_2 > 0 \). In an interior solution the first order conditions with respect to \( n_1^\epsilon \) and \( n_2^\epsilon \), equations (5) and (6), can be arranged to yield

\[
\frac{f(x^\epsilon) - x^\epsilon f'(x^\epsilon)}{f''(x^\epsilon)} = \frac{w_1 + \frac{w_1}{\gamma_1} \varphi_2 f''(x^p) - x^p}{n_1 - n_1^\epsilon} < \frac{w_1}{w_2}.
\]

The implication then is that \( x^\epsilon < x^p \) when \( \lambda_1 = 0 \) and \( \lambda_2 > 0 \). The proof for the case when \( \lambda_1 > 0 \) and \( \lambda_2 = 0 \) (i.e., when redistribution favors type 2) is completely analogous and therefore omitted, but then instead \( x^\epsilon > x^p \). That proves Proposition 3.

A.3 Mathematica program

The following Mathematica notebook simulates our model for CES utility and production functions given the parameter values given in the program. For our simulation procedure, see the main text:

```mathematica
Needs[ "Miscellaneous'RealOnly' " ]

K1 := \left( d + c2 \gamma + (1 - d) \ast \left( t - \left( \frac{y2}{w2} \right) \right) \right) \frac{1}{\gamma}

K2 := \left( d + c2 \gamma + (1 - d) \ast \left( t - \left( \frac{y2}{w1} \right) \right) \right) \frac{1}{\gamma} - \left( d + c1 \gamma + (1 - d) \ast \left( t - \left( \frac{y1}{w1} \right) \right) \right) \frac{1}{\gamma}

K3 := \left( d \ast \left( n1 - n1 g \right) \ast \left( \frac{y1}{w1} \right) \right)^\frac{\gamma}{\rho} + \left( 1 - d \right) \ast \left( n2 - n2 g \ast \left( \frac{y2}{w2} \right) \right)^\frac{\gamma}{\rho} \ast \left( \frac{1}{\rho} \right) + a \ast \left( d \ast \left( n1 g \ast \left( \frac{y1}{w1} \right) \right)^\frac{\gamma}{\rho} + (1 - d) \ast \left( n2 g \ast \left( \frac{y2}{w2} \right) \right)^\frac{1}{\rho} - (n1 \ast c1 + n2 \ast c2) \right)
```

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$K4 := v^2 - \left[ \delta \times \left( (n2 - n1g) \times \left( \frac{v1}{w1} \right) \right)^{\rho} + (1 - \delta) \times \left( (n2 - n2g) \times \left( \frac{v2}{w2} \right) \right)^{\rho - 1} \right]$

$K5 := v^2 \times \left( (n2 - n2g) \times \left( \frac{v2}{w2} \right) \right)^{\rho} + (1 - \delta) \times \left( (n1 - n1g) \times \left( \frac{v1}{w1} \right) \right)^{\rho} - (c2 \times \left( (n2 - n2g) \times \left( \frac{v2}{w2} \right) \right)^{\rho} + (1 - \delta) \times \left( (n1 - n1g) \times \left( \frac{v1}{w1} \right) \right)^{\rho} + \left( d + c1 \right) + \left( 1 - d \right) \times \left( t - \frac{v1}{w1} \right)^{\rho} \right]^{\frac{1}{\rho}}$

$L := K1 - \lambda1 \times K7 + \lambda2 \times K3 + \mu1 \times K4 + \mu2 \times K5 + \mu3 \times K6$

$E1 := D[L, c1]$

$E2 := D[L, c2]$

$E3 := D[L, y1]$

$E4 := D[L, y2]$

$E5 := D[L, w1]$

$E6 := D[L, w2]$

$E7 := D[L, \lambda1]$

$E8 := D[L, \lambda2]$

$E9 := D[L, \mu1]$

$E10 := D[L, \mu2]$

$E11 := D[L, n1g]$

$E12 := D[L, n2g]$

$E13 := D[L, \mu3]$

$E14 := v2 \times \left( d \times c2^\rho + (1 - d) \times \left( t - \left( \frac{v2}{w2} \right) \right)^{\rho} \right) \times \frac{1}{\rho}$
\[ R := \\
0 \ t := \\
1/3.5 \nu := \\
0.12 \ n1 := \\
1 \ n2 := \\
1 \ d := 1.2/3 \rho := \\
1/3 \ t := 16 \theta \alpha := 0.95 \ U1 := 3 \\
\]

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Table 2: Simulations without public production

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Table 3: Simulations with public production

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