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Price Competition, Advertising and Media Market Concentration*

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Abstract

In media markets, the value of advertisement exposure depends on circulation, and media consumers' valuation is affected by advertising. This paper analyzes media market competition in a duopoly framework. There exist symmetric and asymmetric equilibria in terms of firm size. There is less scope for asymmetry when products are more differentiated or of higher intrinsic quality. Some media exhibit public good features. This increases the scope for asymmetry when consumers value advertising positively. If their valuation is negative only symmetric equilibria exist. Regulations limiting price competition increase the scope for natural monopoly. Finally, monopoly is a less likely outcome in subscription-based media markets.

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"A magazine is simply a device to induce people to read advertising."

- James Collins (1907), ad executive.[1]

1. Introduction

Media markets tend to share two common features. First, they are often highly concentrated. Second, media firms, such as newspapers, magazines and commercial television channels, operate simultaneously in two sub-markets. Not only do they sell their products to readers, viewers or listeners, they also sell advertising space to firms. Moreover, these markets are generally interrelated on the demand side. For example, the value of placing an ad in a local newspaper depends on the paper's circulation, and the subscribers' valuation of the newspaper is, at least to some extent, affected by the type and amount of advertising.[2]

It is sometimes argued that demand linkages of this type give rise to positive spirals that partly explain the strong tendency towards concentration in media markets. For instance, a newspaper with a growing circulation may enjoy a strong demand for advertisements. This allows it to charge more for its ads and still attract a large advertising volume which, in turn, makes it more appealing to subscribers. Needless to say, the fact that the production of media content often involves high fixed costs further increases the benefits conferred by size.

From a policy point of view, the economic efficiency aspects of media concentration, e.g. the price-cost margins on newspaper copies, are likely to be outweighed by concerns that increased media concentration may have adverse effects on the democratic process.[3] This issue

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[1]Quoted in Lears (1994, p. 201).

[2]Media market concentration is discussed in e.g. Chaudhri (1998) and Rosse & Dertouzos (1979). The interdependency between sub-markets has been examined in e.g. Rosse (1970), Masson, Mudambi & Reynolds (1990) Blair & Romano (1993) and Chaudhri (1998).

[3]For example, owners of media monopolies could have private incentives to misrepresent news and to introduce a political bias in reporting. Therefore, a diversified free press is often considered to be a merit good; see e.g. Strömberg (1998).

is reflected in special provisions in, or amendments to, competition laws, government subsidies and other policies designed to counter concentration tendencies or mitigate the effects of increasing concentration. For instance, the British Fair Trading Act contains special rules concerning newspaper mergers. In the US, the Newspaper Preservation Act allows joint operating agreements between papers in order to preserve independent editorial offices in markets where otherwise only one paper would remain. In Norway and Sweden, small newspapers receive subsidies with the objective of allowing for political diversity in media markets. Several countries also have various restrictions on ownership of media firms, including limits to foreign ownership and rules concerning cross-ownership.

Despite the policy interest in media concentration there have been relatively few attempts to study the economic mechanisms that may explain media market concentration. In this paper, we analyze the interplay among circulation, advertising and market structure by means of a dual market Bertrand model. The focus is positive rather than normative.[4] Since diversity is at the core of the policy discussion we allow for product differentiation in our model. However, the added complexity of product differentiation confines our analysis to a duopoly framework. Market structure, which is endogenous in the model, is therefore either monopolistic or duopolistic. In terms of the newspaper industry, this may not be such a restrictive assumption as a large fraction of local newspaper markets are highly concentrated in many countries.[5]

Our objective is to examine the market based mechanisms behind tendencies toward concentration in media markets. In so doing, we abstract from other potentially important

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[4] The welfare effects of advertising are complicated by the possibility that advertising affects tastes. This issue is discussed by Dixit & Norman (1978) who argue that there is excessive advertising in equilibrium, even when the point of reference is post advertising tastes. See also Fischer & McGowan (1979) and Shapiro (1980) for a critique.

[5] For example, Dertouzos & Trautman (1990) claim that fewer than one percent of the newspapers in the US face direct competition from other newspapers published in the same city.

determinants of market concentration, like the industry cost structure. Our discussion is cast mostly in terms of newspaper markets, although the results also have bearing on other markets with a similar structure, such as markets for commercial television and radio broadcasting. In the latter case, the media product often has public good characteristics. This raises the question as to whether or not the public good property affects the tendency towards increased media market concentration.

Since both consumers of media and buyers of advertisements base their decisions on the behavior of the other group, expectations play an important role in the analysis. When someone buys a copy of the *New York Times*, he *expects* to find a certain mix of ads and articles. Similarly, an advertiser bases his decision on beliefs about the *New York Times*' readership. There are no guarantees, however, that these beliefs are fulfilled *ex post*. One interpretation of the monopolization argument is that expectations become self-fulfilling prophecies that constrain small newspapers while large ones benefit, and that it is difficult for firms to change these expectations.

The economics literature on media markets is mainly empirical and has examined issues such as the relation between market concentration and advertising rates. For example, Landon (1971) finds that concentration and advertising rates are positively correlated, while a more recent study by Reimer (1992) reaches the opposite conclusion. Another branch of the literature deals with the relationship between ownership structure and advertising rates. It is argued that crosswise ownership will tend to dampen price competition and increase advertising rates. The empirical support for this hypothesis is weak, however; see Ferguson (1983) and Parkman (1984). There is also a technology oriented strand of the empirical literature. For example, Dertouzos & Trautman (1990) find evidence of significant scale economies in the newspaper industry.

To our knowledge, theoretical contributions in the field are quite rare. An exception is an interesting study by Masson, Mudambi & Reynolds (1990) that examines the relationship between concentration and advertising rates in a symmetric framework where advertising affects consumer utility *negatively* and the media product is a *public good*. The model is tailored to fit markets for commercial broadcasting. It is found that advertising rates are higher, the stronger the negative externality on viewer utility. Furthermore, monopoly advertising rates may be lower than oligopoly rates. A media market oligopolist that lowers advertising rates (thereby increasing the volume of commercials) risks losing viewers to competitors. This, in turn, reduces advertisers' willingness to pay for commercials. A monopolist does not face this restriction and may therefore choose a lower advertising rate. Chaudhri (1998) discusses pricing in industries where consumers have a *positive* valuation of advertising, so-called circulation industries. His study examines the polar cases of *perfect competition* and *monopoly*. It is shown that positive consumption externalities may push the monopoly consumer price, e.g. the price of a single copy of a newspaper, below marginal cost. Similar results were reached by Blair and Romano (1993) in a study that focused entirely on the monopoly case.

Our analysis is complementary to that of Masson, Mudambi & Reynolds (1990) in that we are *mainly* concerned with *positive* advertising externalities in a framework where the media product is a *private good*. We only briefly discuss the public good case.[6] Whereas Chaudhri (1998) examines welfare effects under given market structures, we treat *market structure* as an *endogenous* variable that is determined by the *strategic interaction* between firms.

In our model, positive demand externalities between the markets may result in asymmetric equilibria and in some cases natural monopolies, i.e., equilibria where monopolistic behavior on

[6] It may to some extent be argued that the nature of the advertising externality is related to whether the media product is a public or a private good. For instance, while television viewers can avoid commercial messages by flipping between channels, they cannot avoid the interruption. Consumers of printed media, on the other hand, choose whether to read an ad or not. With practically *free disposal,* consumers are much more likely to put a positive value on advertising.

the part of one firm makes it rational for the other firm not to produce. Specifically, consumers' valuation of advertisements interacts with the degree of product differentiation to determine the existence of asymmetric equilibria. The scope for natural monopoly equilibria is reduced when products are more differentiated and the intrinsic quality of the products is higher. The results concerning media markets where the product has public good characteristics, such as markets for TV and radio broadcasting, are mixed. If the advertising externality is positive, the public good property will tend to increase the scope for natural monopoly equilibria. If the externality is negative, only symmetric equilibria are feasible. Regulations that curtail price competition in the media market will have the same effect as a reduction in the intrinsic quality level, i.e., they will increase the scope for natural monopoly equilibria.

The results concerning market structure are determined implicitly. In order to examine firm profits and discuss incentives for product differentiation, however, we need explicit solutions and thus resort to an analytically tractable example of the case where consumers value advertising positively. We find that firm profits may be decreasing in the degree of market power. Competition is intensified as products become closer substitutes. This, in turn, makes higher production and advertising volumes credible. When consumers' valuation of advertisements is positive, this results in an increase in consumer willingness to pay, which can be exploited by firms. For the same reason, monopoly profits are sometimes lower than duopoly profits.

In subscription based media markets, it is relatively easy for advertisers to predict circulation levels. Hence, it could be argued that timing is essentially sequential. First, consumers decide whether or not to subscribe. This decision is based on *expected* levels of advertising. Second, advertisers base their decisions on *observed* subscription stocks. We find that the tendency towards monopoly is weaker in subscription based markets.

The paper is organized as follows. We introduce the basic model and characterize the feasible media market equilibria in section 2. Next, in section 3, we compare how the equilibria depend on whether the consumer product is a private or a public good. In section 4 we examine firm profitability and the incentives for product differentiation in the context of an example. The model is then extended to allow for subscription based media in section 5. Some concluding remarks are offered in section 6.

2. A Media Market Duopoly

There are two firms competing *à la* Bertrand, both in the consumer market (M) and in the market for advertisements (A). For simplicity, suppose that firms face zero production costs. The total profit of firm i is then simply the sum of its revenues in the consumer market and in the advertising market,

$$\pi_i = \pi_{Mi} + \pi_{Ai} = p_{Mi}q_{Mi} + p_{Ai}q_{Ai}$$

where (p_{Mi}, p_{Ai}) are the prices charged in the consumer and the advertising market, respectively, and (q_{Mi}, q_{Ai}) denote the corresponding demand.

Consumer preferences (of readers, viewers or listeners) are represented by a simple utility function such that utility is quadratic in media consumption and linear in the consumption of other goods, I. The representative consumer is assumed to have a positive demand for *both* media goods. The utility function can be interpreted as an aggregate representation of preferences, i.e., individual consumers may limit their consumption to one of the products:[7]

$$U(\boldsymbol{q_M}, \boldsymbol{q_A}, I) = q_{Mi}(a + bq_{Ai}^r) + q_{Mj}(a + bq_{Aj}^r) - \frac{1}{2}(q_{Mi}^2 + q_{Mj}^2 + 2cq_{Mi}q_{Mj}) + I.$$

[7] This representation is an extension of the model developed by Dixit (1979). Our specification is identical to the original one for $b = 0$.

Boldface characters indicate vectors of corresponding values for each firm, e.g. $\boldsymbol{q_M} = \{q_{Mi}, q_{Mj}\}$ and $\boldsymbol{q_A} = \{q_{Ai}, q_{Aj}\}$. The parameter $c \in [0, 1]$ measures the substitutability between the media content provided by the two firms (i.e., the degree of product differentiation). If $c = 0$, each firm has monopolistic market power, while if $c = 1$, the products are perfect substitutes. Media consumers are assumed to have a positive but decreasing marginal valuation of the advertising volume. This is reflected by the parameter $r \in [0, 1]$. Finally, a measures the intrinsic quality of the media product and b reflects the impact of advertisements on utility.[8]

Consumers maximize utility conditional on the expected advertising volumes $\tilde{\boldsymbol{q}}_{\boldsymbol{A}}$, where a tilde denotes expectation, subject to the budget constraint $p_{Mi}q_{Mi} + p_{Mj}q_{Mj} + I \leq m$, where m is the consumer's income.[9] The consumption of other goods, I, is assumed to consist of two components, advertised goods, S, and non-advertised goods, Z. All else equal, an increase in the advertising intensity will shift demand from non-advertised to advertised products.[10] The price of the composite good is normalized to one. The demand for product i is then given by:

$$q_{Mi}(\boldsymbol{p_M},\ \tilde{\boldsymbol{q}}_{\boldsymbol{A}}) = \max\left\{\frac{a + b\tilde{q}_{Ai}^{\,r} - p_{Mi} - c(a + b\tilde{q}_{Aj}^{\,r} - p_{Mj})}{1 - c^2},\ 0\right\}. \qquad \textbf{(1)}$$

Firms set consumer prices to maximize profits, taking the other firm's price as given. Firm i's reaction function is then: $p_{Mi}(p_{Mj},\ \tilde{\boldsymbol{q}}_{\boldsymbol{A}}) = \max\{[a + b\tilde{q}_{Ai}^{\,r} - c(a + b\tilde{q}_{Aj}^{\,r} - p_{Mj})]/2, 0\}$ which ensures existence of a price equilibrium given by

[8]As regards the impact of negative externalities ($b < 0$), discussed in section 3, it is natural to believe that utility decreases in advertising at an increasing rate so that $r > 1$.

[9]The second-order conditions hold for all interior solutions.

[10]Consumers might, for example, have a weak preference for advertised products. An alternative interpretation would be that consumers are uninformed about the existence of some products and that advertising is informative.

$$p^*_{Mi}(\tilde{\boldsymbol{q}}_{\boldsymbol{A}}) = \max\left\{\frac{(2-c^2)(a+b\tilde{q}^r_{Ai})-c(a+b\tilde{q}^r_{Aj})}{4-c^2},\ 0\right\}. \quad \textbf{(2)}$$

The corresponding equilibrium demand is given by

$$q^*_{Mi}(\tilde{\boldsymbol{q}}_{\boldsymbol{A}}) = \max\left\{\frac{(2-c^2)(a+b\tilde{q}^r_{Ai})-c(a+b\tilde{q}^r_{Aj})}{(4-c^2)(1-c^2)},\ 0\right\}. \quad \textbf{(3)}$$

Let us now turn to the advertising market. On the demand side, there is a representative profit-maximizing advertiser who acts as a price taker in all markets. The purpose of advertising is to generate sales. For simplicity, it is assumed that the consumption of advertised goods, S, can be represented as an additively separable function, $S(\boldsymbol{q_A}, \boldsymbol{q_M}) = q_{Ai}{}^{\alpha}q_{Mi}{}^{\beta} + q_{Aj}{}^{\alpha}q_{Mj}{}^{\beta}$ where $\boldsymbol{q_A}$ is the number of ads per year and $\boldsymbol{q_M}$ daily circulation. Then $q_{Ai}q_{Mi}$ can be interpreted as the total consumer exposure generated by advertising in media i. The sales technology is likely to exhibit decreasing returns, both in terms of advertising and circulation, i.e., $\alpha, \beta \in [0, 1]$. Consumers are either exposed or unexposed to advertising. An increase in the number of ads (for a given circulation) would lead to fewer unexposed consumers and increase the exposure intensity among those exposed. The marginal return on increased exposure intensity is likely to be decreasing. Similarly, an increase in circulation (keeping advertising fixed) has two types of effects. First, there may be an influx of *new* potential customers, i.e., individuals previously unexposed to advertising because they had not consumed the media good. Second, the *existing* stock of media consumers may start purchasing the media good more frequently, which means that they would be exposed to ads more often. The latter effect in particular is likely to yield a diminishing marginal return on increased circulation.

Remember that the price of other goods is normalized to one. Assuming zero cost of production, the advertiser then maximizes profits, $S(\boldsymbol{q_A},\boldsymbol{q_M}) - p_{Ai}q_{Ai} - p_{Aj}q_{Aj}$, with respect to $\boldsymbol{q_A}$ conditional on the *expected* circulation in the media market, $\bar{\boldsymbol{q}}_{\boldsymbol{M}}$. The first-order conditions are independent, which means that there are no cross-price effects in the demand for advertising. Specifically, firm *i* faces the following demand for ads

$$q_{Ai}(p_{Ai},\ \tilde{q}_{Mi}) = \varphi(p_{Ai})\, \tilde{q}_{Mi}^{\frac{\beta}{1-\alpha}} ,$$

where $\varphi(p_{Ai}) = (\alpha/p_{Ai})^{1/(1-\alpha)}$. We assume a common and constant marginal cost of producing advertisements, MC_A. The multiplicative demand structure then implies that the profit-maximizing advertising rates are independent of the expected level of circulation, $\bar{q}_{Mi}$. Moreover, the equilibrium advertising rates will be equal across firms, $p^*_{Ai} = p^*_{Aj} = p^*_A$. Hence, the equilibrium levels of advertising are proportional to the expected circulations in the media market. The equilibrium value of φ is thus determined entirely by the demand conditions in the advertising market and can be treated as a constant. For convenience, we normalize it to one so that the equilibrium demand for advertising equals:

$$q^*_{Ai}(\tilde{q}_{Mi}) = \tilde{q}_{Mi}^{\rho}$$

where $\rho \equiv \beta/(1-\alpha)$. The parameter $\rho\in[0, 1]$ can be interpreted as the average elasticity of demand with respect to circulation and will depend on advertiser characteristics.[11] For simplicity, we

[11]For an individual who advertises a single item in the classifieds section, the return to increasing circulation is likely to be rapidly decreasing. By contrast, for a department store, the profitability of attracting an additional customer is likely to decrease at a much slower rate.

normalize ρ to one.[12] Such a normalization is approximately consistent with empirical findings and the loss in terms of generality is modest.[13] The equilibrium advertising demand facing firm *i* then simply equals its expected circulation

$$q^*_{Ai}(\tilde{q}_{Mi}) \;=\; \tilde{q}_{Mi}\,. \qquad \textbf{(4)}$$

An increase in circulation is assumed to have a positive impact on the advertisers' willingness to pay. If such a demand shift occurs, the firm in question would generally experience an increase in the equilibrium demand for advertisements as well as an increase in the advertising rate. Our specification has the convenient property that *all* adjustments take place in terms of quantity. Firms that enjoy a stronger demand for their media product simply sell more advertisements. Another important feature is that the demand for advertising facing firm *i* is unaffected by changes in firm *j*'s circulation. In other words, there is no strategic interaction in terms of circulation.

In a rational expectations equilibrium, beliefs must be consistent with realized equilibrium quantities. This consistency requirement can be expressed in terms of either circulation or advertising volumes. We follow the latter path and let expected circulation ($\tilde{\boldsymbol{q}}_{\boldsymbol{M}}$ in expression (4)) equal realized circulation ($\boldsymbol{q}^*_{\boldsymbol{M}}$ in expression (3)). This yields two equations relating the equilibrium advertising for each firm to the expected advertising of both firms. Requiring the equilibrium volumes to equal the expected volumes gives us the following condition for the equilibrium beliefs:

[12] The parameters α and β can be interpreted as the elasticities of sales with respect to advertising and circulation, respectively. Assuming ρ = 1 implies β = 1 - α which, in turn, means that the sales technology is characterized by constant returns to scale, i.e. a simultaneous increase in advertising intensity and circulation will lead to a proportionate increase in sales.

[13] In Dertouzos & Trautman (1990) the estimated value of ρ for the US is 0.82.With ρ ≠ 1 the existence of equilibria depends on *r*ρ instead of *r*. Therefore, all of our results except *Corollary 2* (where the sequential structure complicates matters) can easily be established for ρ < 1.

$$\tilde{q}_{Ai} = \max\left\{\frac{(2-c^2)(a+b\tilde{q}_{Ai}^{\,r})-c(a+b\tilde{q}_{Aj}^{\,r})}{(4-c^2)(1-c^2)},\ 0\right\} \quad \forall\ i,j\in\{1,2\}, i\neq j. \qquad \textbf{(5)}$$

In equilibrium this condition must be satisfied for both firms simultaneously. For positive expected circulation, the right-hand side of (5) is increasing in $\bar{q}_{Ai}$. Moreover, it is strictly concave (for $r < 1$) and the marginal increase tends to zero as $\bar{q}_{Ai}$ approaches infinity. Thus for a sufficiently high $\bar{q}_{Ai} = \bar{q}_{Ai}^{max}$, expression (5) is satisfied for $\tilde{q}_{Aj} = 0$. For higher $\tilde{q}_{Ai}$ there is no positive $\tilde{q}_{Aj}$ that satisfies the expectations condition. Three examples of the above condition are illustrated graphically in *Figure 1*. Equilibrium beliefs correspond to intersections.

Figure 1 about here

It is evident from *Figure 1* that there may be multiple equilibria, symmetric and asymmetric. For interior equilibria, it is easy to solve condition (5) for $\tilde{q}_{Aj}$ which is then a continuous function of $\tilde{q}_{Ai}$. Similarly, $\tilde{q}_{Ai}$ can be expressed as a continuous function of $\tilde{q}_{Aj}$. For analytical convenience we use the transformed variables $(\tilde{q}_{Aj})^r = x_j$ and $(\tilde{q}_{Ai})^r = x_i$. Then x_j is a continuous function g of x_i, i.e., $x_j = g(x_i)$, that is concave on $x_i \in [0, x_i^{max}]$ where $x_i^{max} = (\bar{q}_{Ai}^{max})^r$. Since the expectations conditions are symmetric, we also have that $x_i = g(x_j)$. The equilibrium condition can now be expressed in terms of the choice variable of one of the firms, e.g. $x_i = g(g(x_i))$. The composite function $g(g(\cdot))$ may either be single peaked or have two peaks of equal height. *Figure 2* illustrates a single and a double peaked $g(g(\cdot))$, respectively. Graphically, the equilibrium points are given by the intersections between the 45° line and $g(g(\cdot))$.

Figure 2 about here

Equilibrium points where at least one firm has a strictly positive circulation always exist since the function is continuous, starts above the 45° line and eventually decreases to zero. For the special case $a = 0$, there also exists a symmetric zero quantity equilibrium. If the product has no intrinsic value and consumers expect no ads to appear, then they will not buy the product; consequently it does not pay to advertise and the beliefs are fulfilled. We abstract from this equilibrium in the subsequent discussion.

Proposition 1: When the media products have a strictly positive intrinsic quality, $a > 0$, there exists one symmetric equilibrium $\{x^*, x^*\}$. In addition, there may exist at most two asymmetric equilibria; (i) $\{\underline{x}, \hat{x}\}$ where $0 < \underline{x} \leq x^* \leq \hat{x}$, and (ii) $\{0, x^{NM}\}$ where x^{NM} is the natural monopoly quantity.

Proof: See Appendix.

Equilibria with large asymmetries between firms could be interpreted as market outcomes that are close to natural monopolies. To better understand the conditions that may drive the market towards high concentration we characterize the set of equilibria and relate existence to the parameters of the model.

There are three types of equilibria: symmetric equilibria, asymmetric equilibria with strictly positive market shares for both firms and equilibria where one firm is a natural monopolist. There always exists a symmetric equilibrium. The existence of asymmetric equilibria depends on the level of product quality, a, consumers' valuation of advertisements, b, the degree of product differentiation, c, and the extent to which the advertising externality is characterized by decreasing returns, r.

In *Proposition 2* we relate the existence of asymmetric (and symmetric) equilibria to the degree of product differentiation. The most important result is that there exists a unique threshold for the degree of product differentiation, c^*, above which natural monopolies and asymmetric equilibria are feasible. Below c^* only symmetric equilibria exist. In a sense, the lower this threshold, the greater the scope for the emergence of a monopolistic market structure.

Proposition 2: For a, b, $r > 0$ there exist two threshold levels of product differentiation c^* and c^{**} (where $0 \leq c^* \leq c^{**} \leq 1$) such that (*i*) a symmetric equilibrium exists for $c \in [0, 1]$, (*ii*) an asymmetric equilibrium exists for $c \in [c^*, c^{**}]$, while (*iii*) a natural monopoly equilibrium exists for $c \in [c^*, 1]$. (*iv*) For $b < 0$, only symmetric equilibria exist. (*v*) Claims (*i*)-(*iv*) hold regardless of whether the media good is private or public.

Proof: See Appendix.

Although closed-form solutions for the equilibrium quantities cannot be derived, the equilibria can be characterized in qualitative terms. Moreover, we can plot the threshold functions $c^*(r)$ and $c^{**}(r)$ for different values of product quality, a, and consumer valuation of advertising, b; see *Figure 3*.

Figure 3 about here

First, consider the case where media products have no intrinsic quality (*a* = 0) so that demand would be zero in the absence of advertising. Then, a natural monopoly can always be sustained in equilibrium. If consumers expect the advertising volume to be zero, they will not buy the product. Hence, zero volume expectations are always fulfilled in equilibrium. In other words, the low threshold, c^*, is equal to zero. If the media products have a strictly positive intrinsic quality ($a > 0$) and products are so differentiated that they are effectively on separate markets (i.e., if $c = 0$) then obviously a natural monopoly cannot be sustained. Positive product quality in combination with perfect monopoly power guarantees positive circulation and this, in turn, guarantees a positive advertising volume. Hence, for $a > 0$ it must be the case that $c^*(r) > 0$. It is straightforward to verify that the low threshold c^* increases monotonically in the intrinsic quality level, *a*.

Corollary 1: The low threshold c^* increases in *a*, i.e., the scope for natural monopoly equilibria is smaller in high-quality markets.

Thus, for media markets characterized by relatively high quality, say with extensive and in-depth editorial material, there is at least one force that may make them less concentrated. Intuitively, higher quality makes symmetric equilibria more likely since the relative importance of the positive externality between the market diminishes - consumers do not buy the newspaper only for its advertisements. On the other hand, high quality content is often associated with high fixed costs, which we have abstracted from in our analysis. This can be expected to increase the concentration tendencies in media markets.

3. The Role of Price Competition

So far it has been assumed that firms compete in prices in the media market. Sometimes, market characteristics may place constraints on pricing and firms may try to strategically limit price competition. For example, in some media markets firms offer products that are public goods, while in other markets firms may be constrained by regulations or collusive agreements. An interesting question then is whether or not the scope for natural monopoly equilibria is stronger under these circumstances.

3.1 Media Markets with Public Goods

Radio stations and most TV channels do not charge consumers for their broadcasts. Broadcasting technology has strong public good features, even though cable networks and satellite broadcasting technologies have led to an increasing share of pay-per-view services.

In markets with public good characteristics, $p_{Mi} = p_{Mj} = 0$ by assumption. In *Figure 4* we illustrate how the threshold values of product differentiation in the private good case compare with those in the public good case. The left panel of *Figure 4* is identical to *Figure 3* while the right panel illustrates the case where the media product is a public good. In the figure, the threshold values c^* and c^{**} are larger in the former case. This relation can be shown to hold generally.

Proposition 3: All else equal, the critical values c^* and c^{**} are higher when the media product is a private good than when it is a public good.

Proof: See Appendix.

Figure 4 about here

In the presence of positive demand externalities, products offered by small firms are less attractive in the eyes of the consumer as compared to products offered by large firms. Hence, when the media good is *private*, large firms are able to charge higher prices in equilibrium. In the *public* good case, small firms and large firms are bounded to charge the same price, namely $p_{Mi} = p_{Mj} = 0$. Hence, there is no price differential to dampen the asymmetry between firms in terms of size. Consequently, monopoly equilibria are feasible for a wider range of *c*'s. Of course, competition between TV channels and newspapers differs in many other respects, so that the empirical implication of *Proposition 3* is not clear-cut. The public good feature of broadcasting does, however, unambiguously tend to strengthen the scope for monopolization.

Advertising in broadcasting media is often thought to have a negative impact on consumer utility. For example, this is the underlying assumption in Masson, Mudambi & Reynolds (1990). In our framework this would correspond to a negative *b*. In *Proposition 2* it was shown that asymmetric equilibria cease to exist once *b* turns negative.

We may conclude that the results concerning markets where the media good is a public good are mixed and depend on whether consumers perceive advertising as a good or a bad. If advertising affects utility positively, the public good property will tend to increase the scope for monopolistic equilibria. If, on the other hand, consumers place a negative value on advertising then natural monopoly is not an equilibrium outcome. As mentioned earlier, the latter case may be more likely to occur in media markets with public good characteristics, where advertising often means unwelcome interruptions.

3.2 Price Regulations and Collusive Agreements

An alternative interpretation of *Proposition 3* is that a regulation or collusive agreement which discourages price competition may strengthen the tendency towards monopoly. It is in fact the case that any constraint which prevents competition in the media market will increase the scope for monopolization.

Proposition 4: A regulation or collusive agreement that ties the media market prices to some common level p_M (where $p_M > 0$) reduces the critical values c^* and c^{**} as compared to the public good case.

Proof: The media market demand (for arbitrary advertising expectations) is given by expression (1). By letting $p_{Mi} = p_{Mj} = p_M$ we may rewrite this expression in the following form:

$$q_{Mi}(p_M,\ \tilde{\boldsymbol{q}}_{\boldsymbol{A}}) = \max\left\{ \frac{(1-c)(a-p_M)+b(\tilde{q}^{\,r}_{Ai}-c\tilde{q}^{\,r}_{Aj})}{1-c^2},\ 0 \right\}.$$

The public good case is equivalent to $p_M = 0$, while $p_M > 0$ is formally equivalent to a reduction in a. From the proof of *Proposition 2* it is clear that c^* and c^{**} are increasing in a. □

In the light of *Proposition 4*, some government policies aimed at promoting diversity in newspaper markets may have detrimental effects. A good example is the Swedish policy that allows the second largest firm in local newspaper markets to apply for government subsidies. An important prerequisite, however, is that the subscription fee of the applicant firm may not be lower than the average fee within the same category of newspapers.[14] As a consequence, price competition is effectively curtailed. Another example is the Newspaper Preservation Act which has been argued to reduce competitive pressure among newspapers in the US (Barnett (1993)).

4. An Example

While the model allows us to examine how the demand linkages between media and advertising markets affect market structure, it is difficult to consider profitability without explicit analytical solutions for the equilibrium quantities. This also means that we cannot, in general, study issues such as firms' incentives for product differentiation. For these reasons we now examine an analytically tractable example where the media good is private, namely the case where $r = 1/2$, $a = 0$, and $b = 1$. To determine the profit in the advertising market, we also need to assume a value for p_A. Since it is determined by the demand conditions in that market and is independent of everything else, it has thus far been treated as an arbitrary constant. For simplicity we let $p_A = 1$. The example is then used as a point of departure for discussing product differentiation.

Figure 5 shows equilibrium prices and quantities in the media market as a function of the degree of product differentiation, c, for symmetric as well as asymmetric equilibria. The left panel shows the equilibrium consumer prices, p_M, and the right panel shows the corresponding demand, q_M.

Figure 5 about here

[14]The Swedish Code of Statutes (SFS) 1990:524.

Figure 6 illustrates how the firms' profits depend on the degree of product differentiation. The first panel shows the advertising market profit, π_A, the second panel the consumer market profit, π_M, and the third panel the total profit, $\pi = \pi_A + \pi_M$.

Figure 6 about here

Note that the profit in the advertising market must be proportional to the output in the media market since the equilibrium quantities are proportional and the price in the advertising market is a constant determined by demand conditions. Here the advertising profit coincides with the output in the media market since $q_A = q_M$ and $p_A = 1$.

In our example, about eighty percent of the revenues comes from advertising when media products are fully differentiated. In a symmetric equilibrium, this share increases as products become closer substitutes and competition in the media market becomes more intense. This seems intuitive enough, but why is the equilibrium output in the symmetric case U-shaped in c? We would generally expect equilibrium quantities to increase as products become less differentiated. When media products are highly differentiated, firms have a considerable degree of market power. As c increases, and media products become closer substitutes, the representative consumer reallocates demand from media to other consumption goods. One interpretation of this aggregate response is that families or workplaces which previously subscribed to two newspapers (to cater to diverse tastes) may settle for only one as the papers become more similar and spend their money on something else instead. As seen in the left panel of *Figure 5*, price decreases at an approximately constant rate in c which means that the price cuts in percentage terms increase sharply. For low *c*, firms face little competition from each other and lower prices moderately in response to declining demand. For high *c*, competition is more intense and further increases in *c* are met by more aggressive pricing, which in turn results in increased output.

Let us now turn to *asymmetric* equilibria with interior solutions. In this case, firms are actually hurt by having too much market power. In a sense, firms are captives of expectations. Strong product differentiation is consistent with low levels of circulation and advertising. When consumers attach a positive value to advertising, this implies a relatively low willingness to pay for the media product. Less product differentiation can therefore raise the profits of *both* firms in the sense that larger circulations and advertising volumes become credible. Consumers' willingness to pay is then reinforced by positive consumption externalities which, to some extent, can be exploited by the firms.

As products become close substitutes, interior asymmetric equilibria cease to exist. A low degree of differentiation means that the value added of the second largest newspaper is small in the eyes of the consumer and it becomes more difficult to fight a bigger competitor. Hence, natural monopoly is the only feasible asymmetric equilibrium. Similarly, for products that are strongly differentiated, so that the media products are practically on separate markets, asymmetric equilibria, as well as natural monopoly equilibria, are inconsistent with rational expectations.[15]

In *Figure 6* the symmetric duopoly profit (and the larger firm's profit in the asymmetric equilibrium) converges to the monopoly profit as products become maximally differentiated. Clearly, *both* firms would prefer the asymmetric duopoly equilibrium to the natural monopoly equilibrium if products are strongly differentiated, i.e., if *c* is small. In other words, when products are highly differentiated it is valuable to have a competitor. Again, the explanation is that

[15] In the example, $a = c^* = 0$ which means that asymmetric equilibria are indeed consistent with an extreme amount of product differentiation. In the general case (i.e., for $a, c^* > 0$), however, asymmetric equilibria cease to exist when differentiation is strong enough.

competition makes larger volumes credible, which increases profits in the presence of bilateral externalities.

Considering that the low threshold, c^*, is generally greater than zero, the incentives to differentiate products are complex. Firms that are stuck in the symmetric equilibrium would gain by increasing differentiation maximally (since $\pi(c = 0) > \pi(c = 1)$). Firms in an asymmetric equilibrium have conflicting interests. The large firm would prefer an intermediate degree of differentiation, while the small firm would be better off in the symmetric equilibrium. Both a reduction and an increase in the degree of differentiation could trigger such a switch in equilibria. Finally, if there is initially a natural monopoly equilibrium, the only viable strategy for an entrant firm is to produce a remote substitute in relation to the incumbent's product.

It should also be noted that advertising rates per reader (p_A / q_{Mi}) are concave in the degree of product differentiation in the symmetric equilibrium of our example. This reflects that circulation, q_{Mi}, is convex in the same argument. Hence, there exist intervals in which increases in market power will reduce advertising rates as they are measured in Reimer (1992). The importance of this similarity between the implications of our model and Reimer's (1992) empirical findings should not be exaggerated, however, as his concept of market power is based on market concentration rather than on product differentiation.

5. Subscription Based Media Markets

In subscription based media markets, it is relatively easy for advertisers to predict consumer demand. In some newspaper markets, notably in the UK, consumers contract with news agents for deliveries rather than subscribe directly from newspapers. The central feature here, however, is that advertisers are reasonably well informed about the level of circulation when they make their decisions. This makes timing essentially sequential. First, firms set prices for the media good, e.g.

newspaper subscriptions, knowing that the advertising market outcome is determined entirely by the media market circulation. Consumers also recognize this, but since an individual consumer's decisions only have a negligible impact on circulation, they base their subscription decisions on the subscription price and the *expected* advertising volume. Second, advertisers base their advertising decisions on *observed* subscription stocks. Except for the change in the timing of decisions, the model specification remains unchanged. As before, equilibrium advertising rates, p_A, are independent of the level of circulation. Thus, here the equilibrium advertising volume is equal to actual circulation, rather than expected circulation (cf. expression (4)). Consequently, firms in subscription based media markets maximize the following profit function with respect to the subscription fee, p_{Mi}:

$$\pi_i = \pi_{Mi} + \pi_{Ai} = (p_{Mi} + p_A - MC_A)q_{Mi}$$

where q_{Mi} is given by expression (1). Equilibrium demand in the media market (for arbitrary expectations concerning $\boldsymbol{q_A}$) then equals

$$q^*_{Mi}(\tilde{\boldsymbol{q}}_{\boldsymbol{A}}) = \max\left\{\frac{(2-c^2)(a+p_A-MC_A+b\tilde{q}^r_{Ai})-c(a+p_A-MC_A+b\tilde{q}^r_{Aj})}{(4-c^2)(1-c^2)},\ 0\right\}$$

which is identical to expression (3) for p_A - MC_A = 0. Thus we can see that the effect of taking the sequential structure of subscription based media markets into account is formally equivalent to an increase in the intrinsic quality level, a. Since the advertising volume is determined entirely by circulation, the advertising market effectively becomes part of the media market, which is equivalent to an upward shift in demand, i.e. the same effect as a higher a. This, in turn, implies that we can draw on *Corollary 1* and reinterpret it in the following way;

Corollary 2: The scope for natural monopoly equilibria is smaller in subscription based media markets than in markets where advertising and media market volumes are determined simultaneously, i.e., the low threshold c^* is higher in the former case.

Note that whereas there is reason to believe that a higher a is associated with higher fixed costs, this is not the case for markets that are subscription based. Thus, *Corollary 2* has a more clear-cut interpretation than *Corollary 1*.

When the game is sequential, it is possible to compensate for a small *expected* advertising volume by choosing a low price in the media market. This, in turn, will lead to a higher *realized* advertising volume *ex post*. Roughly speaking, in a sequential game, small advertising volumes are not as likely to be part of a rational expectations equilibrium.

Real-world media markets are seldom *pure* subscription markets or *pure* non-subscription markets. We may nevertheless conclude that our results hold for the extreme cases, where circulation is either fully observed or determined entirely by expectations.

6. Concluding Remarks

By explicitly taking into account the demand linkages between media markets and markets for advertisements, we found that the market outcome is likely to differ substantially from what is predicted by standard oligopoly theory. When products are strongly differentiated, equilibrium beliefs are symmetric. This yields a symmetric duopoly outcome. When products are closer substitutes, asymmetric equilibria and natural monopolies are equally plausible outcomes. Hence, there is some theoretical justification for concerns about excessive concentration in media markets, especially in markets where media goods are of low intrinsic quality and where consumers are not bound by subscription contracts. From a policy point of view, we may conclude that regulations which reduce price competition among media firms may increase the scope for monopoly.

An interesting extension would be to allow for different levels of intrinsic product quality. In such a framework, it might be possible to study the incentives to invest in quality enhancing technologies. This could potentially have a strong impact on the process that determines market structure. From an empirical perspective, it would be interesting to examine the relation between product quality, for instance measured in terms of the amount of editorial material, and market structure. A further aspect would be to test whether media markets where subscription contracts are common tend to be less concentrated.

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Appendix

Proof of Proposition 1: Strict concavity, $g(0) > 0$ and $g(x^{max}) = 0$ implies that there exists exactly one point $x^{*} \in [0, x^{max})$ such that $g(x^{*}) = x^{*}$ and consequently $g(g(x^{*})) = x^{*}$.

Suppose $\underline{x}$ is an asymmetric equilibrium point. Then $g(\underline{x}) = \bar{x}$ where $\bar{x}$ is a point such that $g(\bar{x}) = \underline{x}$. Thus $\bar{x}$ is also an equilibrium point since, by assumption, $g(g(\bar{x})) = g(\underline{x}) = \bar{x}$. Moreover, note that $g(x) > x$ for $x < x^{*}$. Thus if $x < x^{*}$ and $g(x) < x^{*}$ then $x < g(x) < g(g(x))$. Hence, such an x cannot be an equilibrium. Consequently, if $\underline{x} < x^{*}$ and $g(g(\underline{x})) = \underline{x}$ then $\bar{x} = g(\underline{x}) > x^{*}$.

To simplify notation let $\mu = (2-c^2)/c$ and $\lambda = (4-c^2)(1-c^2)/(bc)$ where $\lambda \geq 0$ and $\mu \geq 1$, since $c \in [0,1]$. Then $g(x) = (\mu-1)a/b + x(\mu - \lambda x^{1/r-1})$. If $g(g(x))$ can intersect the 45° line from below only once, there exist at most three equilibria. Since $g(g(0)) > 0$, such intersections must lie on the increasing segment between two peaks (the interval [argmax $g(x)$, max$\{x|\ g(x) =$ argmax $g(x)\}$]) where $g'(g(x)) = \mu-(\lambda/r)g(x)^{1/r-1} < 0$ and $g'(x) = \mu-(\lambda/r)x^{1/r-1} < 0$. To prove that there is at most one intersection from below, it suffices to show that the slope of the equilibrium condition $g(g(x)) - x = 0$, which we denote $F(x) = g'(g(x))\, g'(x)-1$, is strictly quasi-concave on the interval.

$$F'(x) = -\lambda\frac{1-r}{r^2}\left[g(x)^{1/r-2}\left(\mu-\frac{\lambda}{r}x^{1/r-1}\right)^2 + x^{1/r-2}\left(\mu-\frac{\lambda}{r}g(x)^{1/r-1}\right)\right] \qquad \textbf{(A1)}$$

$$\begin{aligned} F''(x) = \lambda\frac{1-r}{r^2}\Bigg[&-\frac{1-2r}{r}g(x)^{1/r-3}\left(\mu-\frac{\lambda}{r}x^{1/r-1}\right)^3 \\ &+3\lambda\frac{1-r}{r^2}(g(x)x)^{1/r-2}\left(\mu-\frac{\lambda}{r}x^{1/r-1}\right) - \frac{1-2r}{r}\left(\mu-\frac{\lambda}{r}g(x)^{1/r-1}\right)x^{1/r-3}\Bigg]. \end{aligned} \qquad \textbf{(A2)}$$

Thus, $F''(x)$ is negative for $r \geq 1/2$ since $g'(g(x))$ and $g'(x) < 0$. To show quasi-concavity also for $r < 1/2$, we evaluate $F''(x)$ at points where $F'(x) = 0$. Using $F'(x) = 0$ for substitution in the first and third terms and simplifying, (A2) can be rewritten as follows:

$$\frac{\lambda(1-r)}{r^4 g(x)x}\left(\mu-\frac{\lambda}{r}x^{1/r-1}\right)\left[r(1-2r)\left(x^{1/r-1}+g(x)^{1/r-1}\right)+(1+r)\lambda g(x)^{1/r-1}x^{1/r-1}\right].$$

This expression is strictly negative for $r < 1/2$ and consequently F is strictly quasi concave on the interval.

Finally, there exists at most one natural monopoly equilibrium $\{g(x^{NM}) = 0\ ,\ g(0) = x^{NM}\}$ since there is only one x^{NM} such that $g(x^{NM}) = 0$. (For existence see *Proposition* 2). □

Proof of Proposition 2: The proof proceeds in three steps. First, we characterize c^* and c^{**} and show that if $b > 0$ then (*i*) a symmetric equilibrium exists for $c\in[0, 1]$. Second, we prove that: (*ii*) an asymmetric equilibrium exists for $c\in[c^*, c^{**}]$, and (*iii*) a natural monopoly equilibrium exists for $c\in[c^*, 1]$. Third, it is shown that only a symmetric equilibrium exists if $b < 0$. The same reasoning applies when the media good is public, but then the equilibrium is conditional on $p_{Mi} = p_{Mj} = 0$. The expressions corresponding to the public good case are indicated by a *PG* index.

Step I: Let $c^{**}(r)$ be the set of $\{c, r\}$ for which *1)* the equilibrium is symmetric and *2)* $g(g(x))$ is tangent to the 45° line. In a symmetric equilibrium, expressions (5) and (1) must hold for $x = x_i = x_j$, which yields;

$$x^{1/r} = \frac{a+bx}{(2-c)(c+1)} \quad \textit{and} \quad x_{PG}^{1/r} = \frac{a+bx_{PG}}{c+1}\ . \qquad \textbf{(A3)}$$

These expressions have unique interior solutions for $c\in[0, 1]$ which establishes (*i*). The expressions in (A3) combined with $\partial g(g(x))/\partial x = 1$, implies that $c^{**}(r)$ is implicitly defined by:

$$[ra]^r = \frac{a(2-c^2-c)}{b[r(2-c^2+c)+c^2+c-2]^{1-r}} \quad \textit{and in the PG case} \quad [ra]^r = \frac{a(1-c)}{b[r(c+1)-(1-c)]^{1-r}}\cdot \qquad \textbf{(A4)}$$

Similarly, let $c^*(r)$ be the set of $\{c, r\}$ such that the smallest equilibrium quantity is equal to zero. Thus, $c^*(r)$ must satisfy $0 = g(g(0))$ which, in turn, means that $c^*(r)$ is implicitly defined by:

$$[ra]^r = (rc)^r\left[\frac{a(2-c^2-c)}{bc}\right] \quad \textit{and in the PG case} \quad [ra]^r = (rc)^r\left[\frac{a(1-c)}{bc}\right]. \qquad \textbf{(A5)}$$

The right-hand sides of (A4) and (A5) strictly decrease in c. Hence, $c^*(r)$ and $c^{**}(r)$ are unique.

We now show that $0 \le c^*(r) \le c^{**}(r) \le 1$. First, note that $c^*(1) = c^{**}(1) = 1/2[\sqrt{9-4b}-1]$ and $c^*_{PG}(1) = c^{**}_{PG}(1) = 1-b$ and that $c^{**}(0) = c^{**}_{PG}(0) = 1$. Furthermore,

$$c^*(0) = \frac{\sqrt{9a^2+2ab+b^2}-a-b}{2a} < c^{**}(0) \quad \textit{and} \quad c^*_{PG}(0) = \frac{a}{a+b} < c^{**}_{PG}(0). \qquad \textbf{(A6)}$$

For $c^*(r) = c^{**}(r)$ in the private goods case the right-hand sides of (A4) and (A5) must be equal. This implies that $c/[r(2-c^2+c)+c^2+c-2]^{1-r} = (rc)^r$ which is satisfied only by $r = 1$. The corresponding PG condition, $c/[r(c+1)-(1-c)]^{1-r} = (rc)^r$, is also satisfied only by $r = 1$. Hence, $c^* \le c^{**}$ for public, as well as private, goods. Finally, note that c^* and c^{**} are monotonously decreasing in b. In the limit as $b \to 0$, $c^* = c^{**} = 1$ and as $b \to +\infty$, $c^* = 0$. Hence, $0 \le c^* \le c^{**} \le 1$.

Step II: Asymmetric equilibria cannot exist unless $g(g(x))$ intersects the 45° line more than once. The limiting case occurs for $c^{**}(r)$ when $g(g(x))$ is tangent to the 45° line at x, where x lies on the upward sloping segment between two peaks of $g(g(x))$ (see proof of *Proposition 1*). In this interval $g(g(x))$ is first strictly convex and then strictly concave. Moreover, for $c = c^{**}(r)$, $\partial^2 g(g(x))/\partial x \partial c < 0$ and $\partial^2 g(g(x))/\partial x^2 = 0$. Hence, an increase in c will cause $g(g(x))$ to cut the 45° line from above (one fixed point), while a reduction will cause it to cut from below (three fixed points). Since $c^{**}(r)$ is unique, this result is independent of the magnitude of the change in c. Hence, there are three fixed points for $c < c^{**}$ and one for $c > c^{**}$.

By construction, $c^*(r)$ partitions the c r- space $[0,1]^2$ so that the small firm has a strictly positive market share if and only if $c > c^*$. Thus, an asymmetric equilibrium with strictly positive market shares for both firms exists for $c \in [c^*, c^{**}]$, which establishes claim (*ii*).

We proceed to claim (*iii*). Let firm 2 be the small firm and assume that $\bar{q}_{M2} = \bar{q}_{A2} = 0$. We will show that these beliefs are fulfilled in equilibrium for $c \geq c^*(r)$.

We begin by examining the *public good* case. Suppose $\bar{q}_{M2} = \bar{q}_{A2} = 0$. The demand facing firm 1, $x_{PG}{}^m$, is then the solution to $x_{PG}^{1/r} = a + bx_{PG}$. This follows from expression (1) for $c = q_{A2} = 0$. The threshold advertising level that makes $q_{M2} = 0$ rational for consumers is $\hat{x}_{PG} = a(1-c)/(bc)$. This follows from expression (1) for $q_{M2} = q_{A2} = 0$. It can be shown that $x_{PG}{}^m > \hat{x}_{PG}$ if and only if $c > c_{PG}{}^*$. Hence, if $c < c_{PG}{}^*$ there exists no equilibrium with zero quantity expectations. We can also conclude that in the public good case $x^{NM} = x_{PG}{}^m$.

Now consider the *private good* case. Suppose $\bar{q}_{M2} = \tilde{q}_{A2} = 0$. Firm 1, being an unconstrained monopolist, chooses x^m, which is the solution to $x^{1/r} = (a+bx)/2$. This follows from expression (5), letting $c = q_{A2} = 0$. The threshold advertising level that makes $q_{M2} = q_{A2} = 0$ rational for consumers is $\hat{x} = a(2-c^2-c)/(bc)$. This follows from expression (3) for $q_{M2} = q_{A2} = 0$. It can be shown that $x^m > \hat{x}$ if and only if $c > c^{***}$ where c^{***} is defined implicitly by

$$a^r = \frac{a(2-c^2-c)}{bc}\left[\frac{2c}{2-c^2}\right]^r .$$

We can verify that $c^*(0) = c^{***}(0)$ and that $c^{***}(1) = \left(-1+\sqrt{2b^2-8b+9}\right)/(2-b) > c^*(1)$. It can also be shown that $c^{***}(r) \geq c^*(r)$ for $r \in (0, 1]$. Thus, for $c \geq c^{***}$, zero quantity expectations for the small firm are supported by unconstrained monopoly behavior by the large firm, i.e., $x^{NM} = x^m$.

For $c \in [c^*, c^{***}]$ the natural monopolist's behavior is constrained by the presence of firm 2. By rewriting expression (3) we obtain the following condition determining the duopoly level of advertising, x^d, given $\bar{q}_{A2} = \bar{q}_{M2} = 0$:

$$x^{1/r} = \frac{bx(2-c^2)}{(4-c^2)(1-c^2)} + \frac{a}{(2-c)(c+1)}.$$

By construction, $\hat{x} = x^d$ for $c = c^*$ and $\hat{x} = x^m$ for $c = c^{***}$. Since $\hat{x}$ decreases monotonically in c, we have that $x^m \leq \hat{x} \leq x^d$ when $c \in [c^*, c^{***}]$. Now, what expectations regarding firm 1 are consistent with $\bar{q}_{A2} = \bar{q}_{M2} = 0$? If $c \in [c^*, c^{***}]$ and $\bar{q}_{A2} = \bar{q}_{M2} = 0$ then firm 1 optimally sets p_{M1} so that $x = \hat{x}$. If $x > \hat{x}$, firm 1 can raise p_{M1} (towards the monopoly level) without threat of competition, while if $x < \hat{x}$ (whereby firm 2 faces a positive demand), firm 1 would want to lower p_{M1} (towards the duopoly level). Thus, $x = x^{NM} = \hat{x}$ and $\bar{q}_{A2} = \bar{q}_{M2} = 0$ is a rational expectations equilibrium for $c \in [c^*, c^{***}]$. If $c < \hat{c}$, then $\hat{x} > x^d$ which is inconsistent with existence of a natural monopoly equilibrium.

Step III: Finally we show that there exists a unique symmetric equilibrium for $b < 0$. If $b < 0$ then no $c \in [0, 1]$ satisfies (A4) and (A5). Since $c^*(0)$ and $c^*(1)$ decrease in b, it follows that $c^*, c^{**} > 1$ for $b < 0$. Thus, only symmetric equilibria exist. The equations determining symmetric equilibria (A3) yield unique positive equilibrium quantities also for $b < 0$ and $r > 1$. □

Proof of Proposition 3: The expressions in (A4) can be rewritten as

$$\frac{b[ra]^r}{a(1-c)} = \frac{c+2}{[r(2-c^2+c)+c^2+c-2]^{1-r}} \quad \text{and} \quad \frac{b[ra]^r}{a(1-c_{PG})} = \frac{1}{[r(c_{PG}+1)-(1-c_{PG})]^{1-r}}$$

respectively. No $\{r, c\} \in [0,1]^2$ satisfies these equalities simultaneously. Hence, $c^{**}(r)$ in the private good case cannot intersect the corresponding function in the public good case and, by example, we know that $c^{**}(r)$ is smaller in the latter case. The same argument applies for $c^*(r)$. □

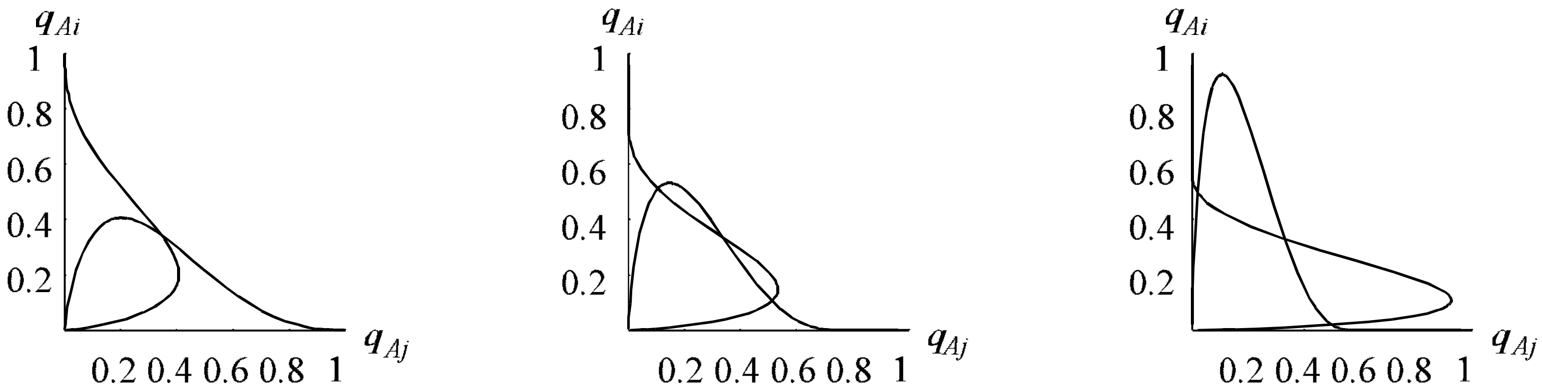

Figure 1. Consistent beliefs for different degrees of product differentiation ($c = \{0.75, 0.67, 0.55\}$, $a = 0.1$, $b = 1$ and $r = 0.4$).

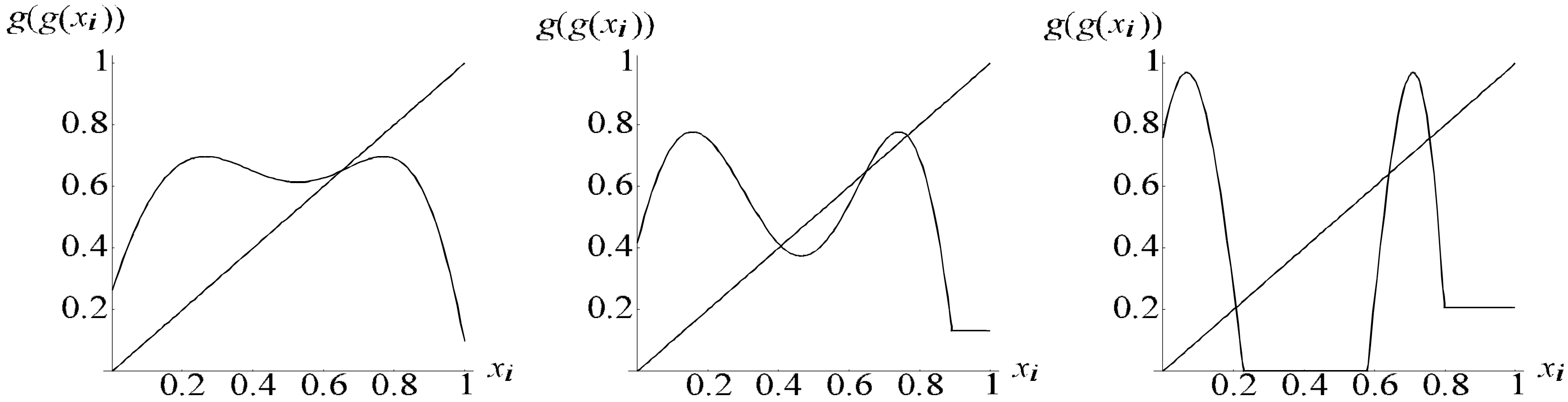

Figure 2. Composite functions corresponding to the graphs in *Figure 1*.

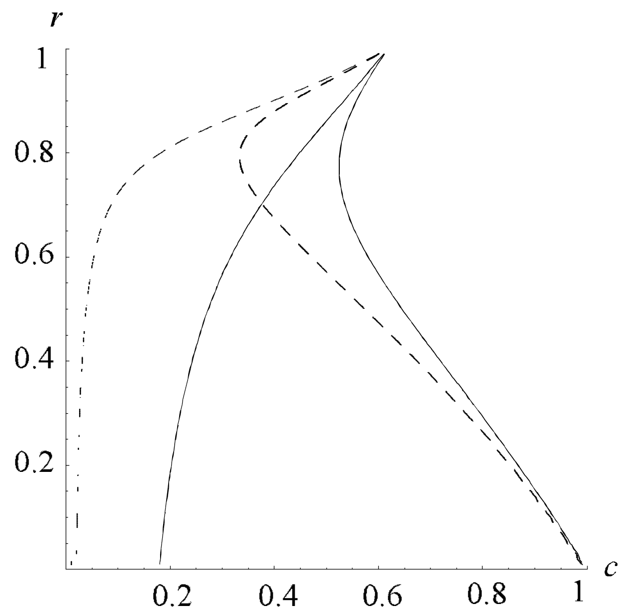

Figure 3. The threshold functions c^* and c^{**} for a = 0.1 (hashed lines) and a = 0.1 (solid lines) when b = 1.

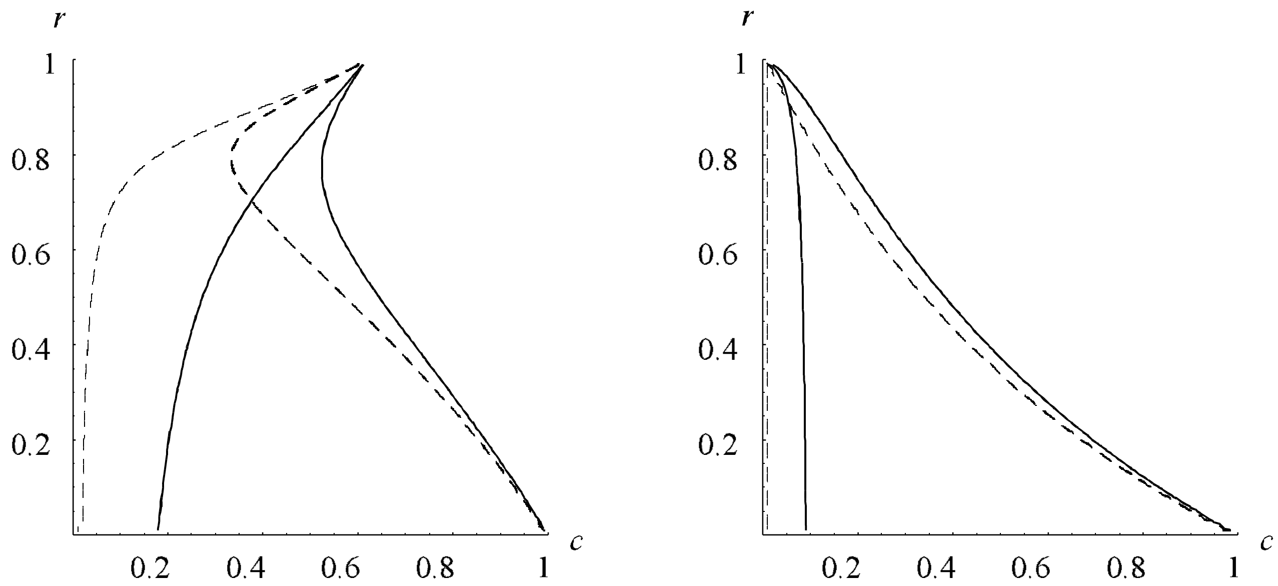

Figure 4. Threshold functions for the private good (left panel- same as *Figure 3*) and the public good case (right panel).

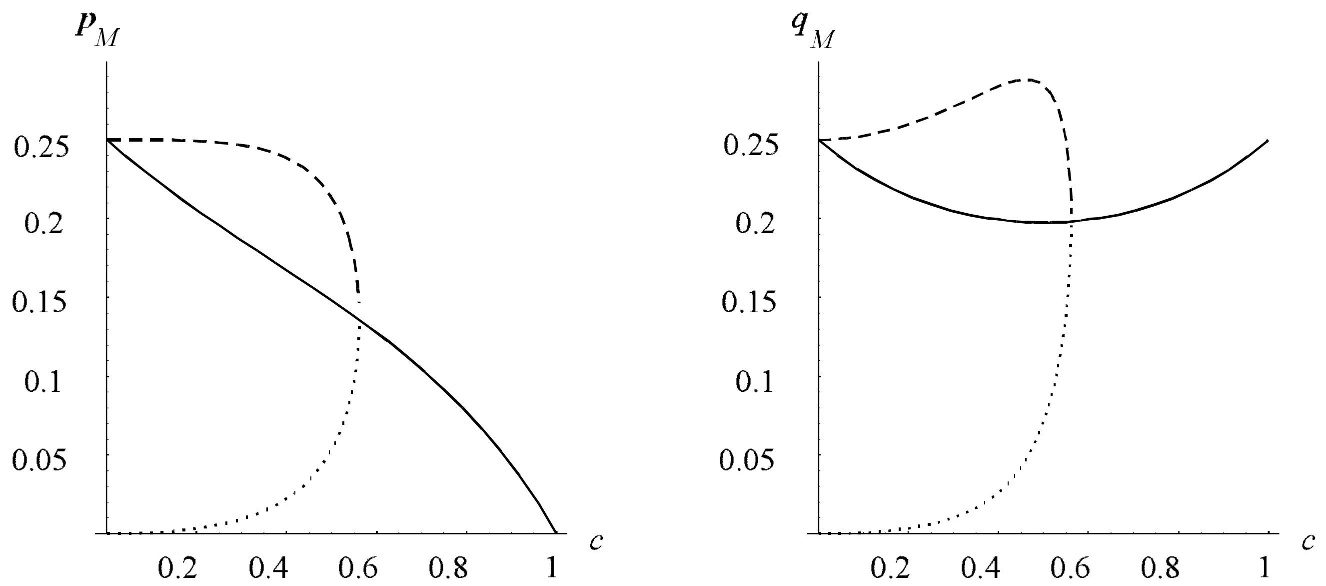

Figure 5. Equilibrium prices and quantities as functions of *c*. Solid lines represent the symmetric equilibrium, hashed lines the large firm in an (non-monopolistic) asymmetric equilibrium and dotted lines the small firm.

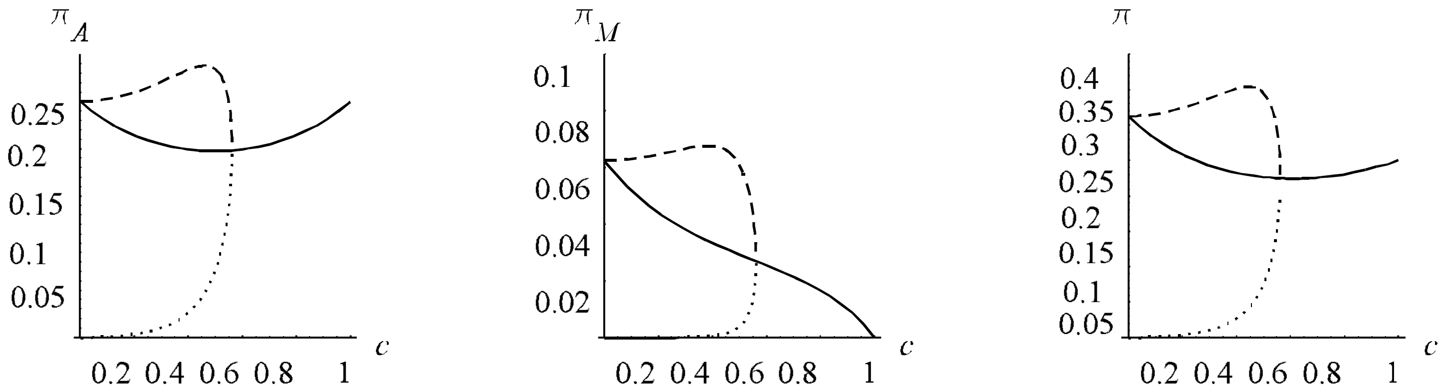

Figure 6. Equilibrium profit levels in the advertising market, the media market and total profits.